

Chapter 19 Solution

Exercise 77

1. (a) $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$ (A1) for correct formula
 $9k + k + 0.1 + 0.4 = 1$
 $10k = 0.5$
 $k = 0.05$ A1 N2 [2]
- (b) $E(X) = 9k(0) + k + 0.1(2) + 0.4(3)$ (A1) for correct formula
 $E(X) = 0 + 0.05 + 0.2 + 1.2$ (A1) for substitution
 $E(X) = 1.45$ A1 N3 [3]
2. (a) $P(X = 0) + P(X = 20) + P(X = 40) + P(X = 60) = 1$ (A1) for correct formula
 $\frac{1}{10} + \frac{1}{5} + \frac{2}{5} + k = 1$
 $\frac{7}{10} + k = 1$
 $k = \frac{3}{10}$ A1 N2 [2]
- (b) $E(X) = \frac{1}{10}(0) + \frac{1}{5}(20) + \frac{2}{5}(40) + 60k$ (A1) for correct formula
 $E(X) = 0 + 4 + 16 + 60\left(\frac{3}{10}\right)$ (A1) for substitution
 $E(X) = 38$ A1 N3 [3]

3. (a) $P(X = 1) + P(X = 2) + P(X = k) = 1$ (A1) for correct formula
- $$\frac{1}{14} + \frac{4}{14} + \frac{k^2}{14} = 1$$
- $$5 + k^2 = 14$$
- (A1) for simplification
- $$k^2 = 9$$
- $$k = 3$$
- A1 N3 [3]
- (b) $E(X) = \frac{1}{14}(1) + \frac{4}{14}(2) + \frac{k^2}{14}(k)$ (A1) for correct formula
- $$E(X) = \frac{1}{14} + \frac{8}{14} + \frac{3^3}{14}$$
- (A1) for substitution
- $$E(X) = \frac{18}{7}$$
- A1 N3 [3]
- (c) $P(Y = 3) = \left(\frac{1}{14}\right)\left(\frac{4}{14}\right) + \left(\frac{4}{14}\right)\left(\frac{1}{14}\right)$ (A1) for correct formula
- $$P(Y = 3) = \frac{2}{49}$$
- A1 N2 [2]
4. (a) $P(X = k) + P(X = k + 1) + P(X = k + 2) + P(X = 8) = 1$ (A1) for correct formula
- $$\frac{k}{2} + \frac{1}{8} + \frac{k}{4} + \frac{1}{8} = 1$$
- $$\frac{3k}{4} + \frac{1}{4} = 1$$
- (A1) for simplification
- $$3k + 1 = 4$$
- $$k = 1$$
- A1 N3 [3]
- (b) $E(X) = \frac{k}{2}(k) + \frac{1}{8}(k + 1) + \frac{k}{4}(k + 2) + \frac{1}{8}(8)$ (A1) for correct formula
- $$E(X) = \frac{1}{2} + \frac{2}{8} + \frac{3}{4} + 1$$
- (A1) for substitution
- $$E(X) = \frac{5}{2}$$
- A1 N3 [3]
- (c) $P(Y = 2) = \left(\frac{1}{2}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{2}\right)$ (A1) for correct formula
- $$P(Y = 2) = \frac{1}{8}$$
- A1 N2 [2]

Exercise 78

1. (a) $P(X = 4) + P(X = 8) + P(X = 12) = 1$
 $10k^2 + k + 20k^2 = 1$ (A1) for substitution
 $30k^2 + k - 1 = 0$
 $(6k - 1)(5k + 1) = 0$ (A1) for factorization
 $k = \frac{1}{6}$ or $k = -\frac{1}{5}$ (*Rejected*) A1 N3

[3]

(b) $P(X = 12 | X > 6) = \frac{P(X = 12 \cap X > 6)}{P(X > 6)}$ (M1) for valid approach

$$P(X = 12 | X > 6) = \frac{P(X = 12)}{P(X > 6)}$$

$$P(X = 12 | X > 6) = \frac{20\left(\frac{1}{6}\right)^2}{20\left(\frac{1}{6}\right)^2 + \frac{1}{6}}$$
 (A1) for substitution

$$P(X = 12 | X > 6) = \frac{10}{13}$$
 A1 N3

[3]

2. (a) $P(X = 12) + P(X = 24) + P(X = 30) + P(X = 36) = 1$
 $k + 7k^2 + 8k^2 + k = 1$ (A1) for substitution
 $15k^2 + 2k - 1 = 0$
 $(5k - 1)(3k + 1) = 0$ (A1) for factorization
 $k = \frac{1}{5}$ or $k = -\frac{1}{3}$ (*Rejected*) A1 N3

[3]

(b) $P(X = 24 | X > 20) = \frac{P(X = 24 \cap X > 20)}{P(X > 20)}$ (M1) for valid approach

$$P(X = 24 | X > 20) = \frac{P(X = 24)}{P(X > 20)}$$

$$P(X = 24 | X > 20) = \frac{7\left(\frac{1}{5}\right)^2}{7\left(\frac{1}{5}\right)^2 + 8\left(\frac{1}{5}\right)^2 + \frac{1}{5}}$$
 (A1) for substitution

$$P(X = 24 | X > 20) = \frac{7}{20}$$
 A1 N3

[3]

3. (a) $P(X = 7) + P(X = 14) + P(X = 21)$
 $+ P(X = 28) + P(X = 35) = 1$
 $k + 3k + 10k^2 + 6k^2 + 5k^2 = 1$ (A1) for substitution
 $21k^2 + 4k - 1 = 0$
 $(7k - 1)(3k + 1) = 0$ (A1) for factorization
 $k = \frac{1}{7}$ or $k = -\frac{1}{3}$ (*Rejected*) A1 N3

[3]

(b) $P(X < 15 | X < 25) = \frac{P(X < 15 \cap X < 25)}{P(X < 25)}$ (M1) for valid approach

$$P(X < 15 | X < 25) = \frac{P(X < 15)}{P(X < 25)}$$

$$P(X < 15 | X < 25) = \frac{\frac{1}{7} + 3\left(\frac{1}{7}\right)}{\frac{1}{7} + 3\left(\frac{1}{7}\right) + 10\left(\frac{1}{7}\right)^2}$$
 (A1) for substitution

$$P(X < 15 | X < 25) = \frac{14}{19}$$
 A1 N3

[3]

4. (a) $P(X = 0) + P(X = 1) + P(X = 2)$
 $+ P(X = 3) + P(X = 4) + P(X = 5) = 1$
 $k^2 + k + 4k^2 + 8k^2 + 4k + k^2 = 1$ (A1) for substitution
 $14k^2 + 5k - 1 = 0$
 $(7k - 1)(2k + 1) = 0$ (A1) for factorization
 $k = \frac{1}{7}$ or $k = -\frac{1}{2}$ (*Rejected*) A1 N3

[3]

(b) $P(2 < X \leq 4 | 1 < X \leq 4) = \frac{P(2 < X \leq 4 \cap 1 < X \leq 4)}{P(1 < X \leq 4)}$ (M1) for valid approach

$$P(2 < X \leq 4 | 1 < X \leq 4) = \frac{P(2 < X \leq 4)}{P(1 < X \leq 4)}$$

$$P(2 < X \leq 4 | 1 < X \leq 4) = \frac{8\left(\frac{1}{7}\right)^2 + 4\left(\frac{1}{7}\right)}{4\left(\frac{1}{7}\right)^2 + 8\left(\frac{1}{7}\right) + 4\left(\frac{1}{7}\right)}$$
 (A1) for substitution

$$P(2 < X \leq 4 | 1 < X \leq 4) = \frac{9}{10}$$
 A1 N3

[3]

Exercise 79

1. (a) $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$ (A1) for correct formula
 $0.2 + 0.3 + a + b = 1$
 $a + b = 0.5$ A1 N2 [2]
- (b) $E(X) = 2.62$
 $0.2(1) + 0.3(2) + 3a + 4b = 2.62$ (A1) for correct formula
 $3a + 4b = 1.82$ A1 N2 [2]
- (c) $a = 0.18, b = 0.32$ A2 N2 [2]
2. (a) $P(X < 45) = 0.9$
 $P(X = 20) + P(X = 30) + P(X = 40) = 0.9$ (A1) for correct formula
 $0.1 + a + b = 0.9$
 $a + b = 0.8$ A1 N2 [2]
- (b) $E(X) = 33$
 $0.1(20) + 30a + 40b + 0.1(50) = 33$ (A1) for correct formula
 $30a + 40b = 26$ A1 N2 [2]
- (c) $a = 0.6, b = 0.2$ A2 N2 [2]

3. (a) $P(X < 15) = 0.5$
 $P(X = 0) + P(X = 10) = 0.5$ (A1) for correct formula
 $0.1 + a = 0.5$
 $a = 0.4$ A1 N2 [2]
- (b) $P(X = 0) + P(X = 10) + P(X = 20) + P(X = 30) = 1$
 $0.1 + 0.4 + b + c = 1$ (A1) for correct formula
 $b + c = 0.5$
 $E(X) = 16$
 $0.1(0) + 10(0.4) + 20b + 30c = 16$ (A1) for correct formula
 $20b + 30c = 12$
By solving the system $\begin{cases} b + c = 0.5 \\ 20b + 30c = 12 \end{cases}$, $b = 0.3$
and $c = 0.2$. A2 N4 [4]
- (c) $P(Y = 50) = (0.3)(0.2) + (0.2)(0.3)$ (A1) for correct formula
 $P(Y = 50) = 0.12$ A1 N2 [2]
4. (a) $P(X = 0) + P(X = 3) + P(X = 6) + P(X = 9) = 1$
 $0.4 + 0.3 + c = 1$ (A1) for correct formula
 $c = 0.3$ A1 N2 [2]
- (b) $P(2 < X < 7) = 0.3$
 $P(X = 3) + P(X = 6) = 0.3$ (A1) for correct formula
 $a + b = 0.3$
 $E(X) = 4.2$
 $0.4(0) + 3a + 6b + 9(0.3) = 4.2$ (A1) for correct formula
 $3a + 6b = 1.5$
By solving the system $\begin{cases} a + b = 0.3 \\ 3a + 6b = 1.5 \end{cases}$, $a = 0.1$
and $b = 0.2$. A2 N4 [4]
- (c) $P(Y = 36) = (0.2)(0.2)$ (A1) for correct formula
 $P(Y = 36) = 0.04$ A1 N2 [2]

Exercise 80

1. (a) (i) $P(F \cap S) = (0.6)(0.6)$ (A1) for substitution
 $P(F \cap S) = 0.36$ A1 N2
- (ii) $P(S) = P(F \cap S) + P(F' \cap S)$ (M1) for valid approach
 $P(S) = 0.36 + (0.4)(0.6)$
 $P(S) = 0.6$ A1 N2
- [4]
- (b) (i) The required probability
 $= P(F' \cap S')$
 $= (0.4)(0.4)$ (A1) for substitution
 $= 0.16$ A1 N2
- (ii) The required probability
 $= P(F | S')$ (M1) for valid approach
 $= \frac{P(F \cap S')}{P(S')}$
 $= \frac{(0.6)(0.4)}{1 - 0.6}$ (A1) for substitution
 $= 0.6$ A1 N3
- [5]
- (c)
- | | | | |
|------------|------|------|------|
| X | 2 | 5 | 8 |
| $P(X = x)$ | 0.16 | 0.48 | 0.36 |
- A3 N3
- [3]
- (d) The expected value
 $= (2)(0.16) + (5)(0.48) + (8)(0.36)$ (A1) for substitution
 $= 5.6$ A1 N2
- [2]

2. (a) (i) $P(S \cap L') = (0.2)(0.3)$ (A1) for substitution
 $P(S \cap L') = 0.06$ A1 N2
- (ii) $P(L') = P(S \cap L') + P(S' \cap L')$ (M1) for valid approach
 $P(L') = 0.06 + (0.8)(0.6)$
 $P(L') = 0.54$ A1 N2
- [4]
- (b) (i) The required probability
 $= P(S' \cap L')$
 $= (0.8)(0.6)$ (A1) for substitution
 $= 0.48$ A1 N2
- (ii) The required probability
 $= P(S | L)$ (M1) for valid approach
 $= \frac{P(S \cap L)}{P(L)}$
 $= \frac{(0.2)(0.7)}{1 - 0.54}$ (A1) for substitution
 $= \frac{7}{23}$ A1 N3
- [5]
- (c)
- | | | | |
|------------|------|------|------|
| X | 0 | 10 | 25 |
| $P(X = x)$ | 0.09 | 0.42 | 0.49 |
- A3 N3
- [3]
- (d) The expected value
 $= (0)(0.09) + (10)(0.42) + (25)(0.49)$ (A1) for substitution
 $= 16.45$ A1 N2
- [2]

3. (a) (i) $P(T' \cap L') = \left(1 - \frac{1}{2}\right) \left(1 - \frac{3}{10}\right)$ (A1) for substitution

$P(T' \cap L') = \frac{7}{20}$ A1 N2

(ii) $P(L') = P(T \cap L') + P(T' \cap L')$ (M1) for valid approach

$P(L') = \left(\frac{1}{2}\right) \left(1 - \frac{9}{10}\right) + \frac{7}{20}$

$P(L') = \frac{2}{5}$ A1 N2

[4]

(b) (i) The required probability
 $= P(T \cap L')$

$= \left(\frac{1}{2}\right) \left(1 - \frac{9}{10}\right)$ (A1) for substitution

$= \frac{1}{20}$ A1 N2

(ii) The required probability
 $= P(L' | T')$ (M1) for valid approach

$= \frac{P(L' \cap T')}{P(T')}$

$= \frac{\frac{7}{20}}{1 - \frac{1}{2}}$ (A1) for substitution

$= \frac{7}{10}$ A1 N3

[5]

(c)

X	0	125	250	375
$P(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

A3 N3

[3]

(d) The expected expenditure

$$= (0)\left(\frac{8}{125}\right) + (125)\left(\frac{36}{125}\right) + (250)\left(\frac{54}{125}\right)$$

$$+ (375)\left(\frac{27}{125}\right)$$

$$= \$225$$

(A1) for substitution

A1 N2

[2]

4. (a) (i) $P(R' \cap A) = (1-0.5)(0.4)$ (A1) for substitution
 $P(R' \cap A) = 0.2$ A1 N2
- (ii) $P(A) = P(R \cap A) + P(R' \cap A)$ (M1) for valid approach
 $P(A) = (0.5)(0.8) + 0.2$
 $P(A) = 0.6$ A1 N2
- (b) (i) The required probability [4]
 $= P(R \cap A')$
 $= (0.5)(1-0.8)$ (A1) for substitution
 $= 0.1$ A1 N2
- (ii) The required probability
 $= P(R | A)$ (M1) for valid approach
 $= \frac{P(R \cap A)}{P(A)}$
 $= \frac{(0.5)(0.8)}{0.6}$ (A1) for substitution
 $= \frac{2}{3}$ A1 N3
- (c) [5]
- | | | | | |
|------------|------------------|------------------|------------------|-----------------|
| X | 0 | 4 | 8 | 12 |
| $P(X = x)$ | $\frac{27}{125}$ | $\frac{54}{125}$ | $\frac{36}{125}$ | $\frac{8}{125}$ |
- A3 N3
- (d) The expected expenditure [3]
 $= (0)\left(\frac{27}{125}\right) + (4)\left(\frac{54}{125}\right) + (8)\left(\frac{36}{125}\right) + (12)\left(\frac{8}{125}\right)$ (A1) for substitution
 $= \$4.8$ A1 N2
- [2]

Exercise 81

1. (a) (i) There are 4 ways such that $X = 5$ (A1) for correct value

$$P(X = 5) = \frac{4}{36}$$

$$P(X = 5) = \frac{1}{9}$$
 A1 N2
- (ii) There are 6 ways such that $X < 5$ (A1) for correct value

$$P(X < 5) = \frac{6}{36}$$

$$P(X < 5) = \frac{1}{6}$$
 A1 N2
- (iii)
$$P(X = 4 \mid X < 6) = \frac{P(X = 4 \cap X < 6)}{P(X < 6)}$$
 (M1) for valid approach

$$P(X = 4 \mid X < 6) = \frac{P(X = 4)}{P(X < 6)}$$

$$P(X = 4 \mid X < 6) = \frac{\frac{3}{36}}{\frac{1}{9} + \frac{1}{6}}$$
 (A1) for substitution

$$P(X = 4 \mid X < 6) = \frac{3}{10}$$
 A1 N3
- (b) (i) $\frac{13}{18}$ A1 N1 [7]
- (ii) $E(X) = 0$
 $(3)P(X = 5) + (2)P(X < 5)$
 $+ (-k)P(X > 5) = 0$ (M1)(A1) for correct formula
 $(3)\left(\frac{1}{9}\right) + (2)\left(\frac{1}{6}\right) + (-k)\left(\frac{13}{18}\right) = 0$ (A1) for substitution
 $6 + 6 - 13k = 0$
 $k = \frac{12}{13}$ A1 N4 [5]

2. (a) (i) There are 5 ways such that $X = 8$ (A1) for correct value
 $P(X = 8) = \frac{5}{36}$ A1 N2
- (ii) There are 10 ways such that $X > 8$ (A1) for correct value
 $P(X > 8) = \frac{10}{36}$
 $P(X > 8) = \frac{5}{18}$ A1 N2
- (iii) $P(X > 9 | X > 8) = \frac{P(X > 9 \cap X > 8)}{P(X > 8)}$ (M1) for valid approach
 $P(X > 9 | X > 8) = \frac{P(X > 9)}{P(X > 8)}$
 $P(X > 9 | X > 8) = \frac{\frac{6}{18}}{\frac{5}{18}}$ (A1) for substitution
 $P(X > 9 | X > 8) = \frac{3}{5}$ A1 N3
- [7]
- (b) (i) $\frac{7}{12}$ A1 N1
- (ii) $E(X) = 1$
 $(5)P(X = 8) + (k)P(X > 8)$
 $+ (-1)P(X < 8) = 1$ (M1)(A1) for correct formula
 $(5)\left(\frac{5}{36}\right) + (k)\left(\frac{5}{18}\right) + (-1)\left(\frac{7}{12}\right) = 1$ (A1) for substitution
 $25 + 10k - 21 = 36$
 $k = 3.2$ A1 N4
- [5]

3. (a) (i) There is only 1 way such that $X = 21$ (A1) for correct value
 $P(X = 21) = \frac{1}{9}$ A1 N2

(ii) There are 5 ways such that $X > 21$ (A1) for correct value
 $P(X > 21) = \frac{5}{9}$ A1 N2

(iii) $P(30 < X < 33 | X > 21)$
 $= \frac{P(30 < X < 33 \cap X > 21)}{P(X > 21)}$ (M1) for valid approach

$$= \frac{P(30 < X < 33)}{P(X > 21)}$$

$$= \frac{2}{\frac{5}{9}}$$

(A1) for substitution

$$= \frac{2}{5}$$

A1 N3

[7]

(b) $P(X < 21) = 1 - P(X = 21) - P(X > 21)$

$$P(X < 21) = 1 - \frac{1}{9} - \frac{5}{9}$$

(A1) for substitution

$$P(X < 21) = \frac{1}{3}$$

(A1) for correct value

$$E(X) = 8$$

$$(3k)P(X = 21) + (k)P(X > 21) + (0)P(X < 21) = 8$$
 (M1)(A1) for correct formula

$$\therefore (3k)\left(\frac{1}{9}\right) + (k)\left(\frac{5}{9}\right) + (0)\left(\frac{1}{3}\right) = 8$$

(A1) for substitution

$$3k + 5k = 72$$

$$k = 9$$

A1 N6

[6]

4. (a) (i) There is only 1 way such that $X = 33$ (A1) for correct value
 $P(X = 33) = \frac{1}{9}$ A1 N2

(ii) There are 2 ways such that $X \geq 35$ (A1) for correct value
 $P(X \geq 35) = \frac{2}{9}$ A1 N2

(iii) $P(X < 22 | X < 33) = \frac{P(X < 22 \cap X < 33)}{P(X < 33)}$ (M1) for valid approach

$P(X < 22 | X < 33) = \frac{P(X < 22)}{P(X < 33)}$
 $P(X < 22 | X < 33) = \frac{\frac{5}{6}}{\frac{9}{9}}$ (A1) for substitution

$P(X < 22 | X < 33) = \frac{5}{6}$ A1 N3

[7]

(b) $P(X < 33) = 1 - P(X = 33) - P(X > 33)$

$P(X < 33) = 1 - \frac{1}{9} - \frac{2}{9}$ (A1) for substitution

$P(X < 33) = \frac{2}{3}$ (A1) for correct value

$E(X) = -16$

$(4k)P(X = 33) + (3k)P(X > 33) + (-2k)P(X < 33)$
 $= -16$ (M1)(A1) for correct formula

$\therefore (4k)\left(\frac{1}{9}\right) + (3k)\left(\frac{2}{9}\right) + (-2k)\left(\frac{2}{3}\right) = -16$ (A1) for substitution

$4k + 6k - 12k = -144$

$k = 72$ A1 N6

[6]