Chapter 7 Solution

Exercise 17

1.	(a)	$d = \dfrac{u_5 - u_1}{5-1}$	(M1) for finding d	
		$d = \dfrac{-1-27}{4}$		
		$d = -7$	A1 N2	
				[2]
	(b)	$u_{25} = u_1 + (25-1)d$	(A1) for correct formula	
		$u_{25} = 27 + (25-1)(-7)$		
		$u_{25} = -141$	A1 N2	
				[2]
	(c)	$S_{25} = \dfrac{25}{2}\left[2u_1 + (25-1)d\right]$	(A1) for correct formula	
		$S_{25} = \dfrac{25}{2}\left[2(27) + (25-1)(-7)\right]$		
		$S_{25} = -1425$	A1 N2	
				[2]
2.	(a)	$d = \dfrac{u_7 - u_1}{7-1}$	(M1) for finding d	
		$d = \dfrac{6.5-3.5}{6}$		
		$d = 0.5$	A1 N2	
				[2]
	(b)	$u_{42} = u_1 + (42-1)d$	(A1) for correct formula	
		$u_{42} = 3.5 + (42-1)(0.5)$		
		$u_{42} = 24$	A1 N2	
				[2]
	(c)	$S_{84} = \dfrac{84}{2}\left[2u_1 + (84-1)d\right]$	(A1) for correct formula	
		$S_{84} = \dfrac{84}{2}\left[2(3.5) + (84-1)(0.5)\right]$		
		$S_{84} = 2037$	A1 N2	
				[2]

3. (a) $d = \dfrac{u_{10} - u_2}{10 - 2}$ (M1) for finding d

$d = \dfrac{24 - 0}{8}$

$d = 3$ A1 N2

[2]

(b) $u_4 = u_2 + 2d$ (A1) for correct formula

$u_4 = 0 + (2)(3)$

$u_4 = 6$ A1 N2

[2]

(c) $S_{10} = \dfrac{10}{2}\left[2u_1 + (10-1)d\right]$ (A1) for correct formula

$S_{10} = \dfrac{10}{2}\left[2(-3) + (10-1)(3)\right]$

$S_{10} = 105$ A1 N2

[2]

4. (a) $d = \dfrac{u_8 - u_3}{8 - 3}$ (M1) for finding d

$d = \dfrac{-\dfrac{22}{3} - \left(-\dfrac{2}{3}\right)}{5}$

$d = -\dfrac{4}{3}$ A1 N2

[2]

(b) $u_{11} = u_8 + 3d$ (A1) for correct formula

$u_{11} = -\dfrac{22}{3} + 3\left(-\dfrac{4}{3}\right)$

$u_{11} = -\dfrac{34}{3}$ A1 N2

[2]

(c) $S_{40} = \dfrac{40}{2}\left[2u_1 + (40-1)d\right]$ (A1) for correct formula

$S_{40} = \dfrac{40}{2}\left[2(2) + (40-1)\left(-\dfrac{4}{3}\right)\right]$ (A1) for substitution

$S_{40} = -960$ A1 N3

[3]

Exercise 18

1. (a) $d = 1.9 - 1.5$

$d = 0.4$ (A1) for correct value

The required length

$= 1.5 + (20-1)(0.4)$ (M1) for substitution

$= 9.1\text{ m}$ A1 N3

[3]

(b) The perimeter

$= S_{20}$ (M1) for valid approach

$= \frac{20}{2}\left[2(1.5) + (20-1)(0.4)\right]$ (A1) for substitution

$= 106\text{ m}$ A1 N3

[3]

2. (a) $d = 23 - 12$

$d = 11$ (A1) for correct value

$221 = 12 + (n-1)(11)$ (M1) for substitution

$209 = 11(n-1)$

$19 = n - 1$

$n = 20$ A1 N3

[3]

(b) The total number

$= u_{18} + u_{19} + u_{20}$ (M1) for valid approach

$= (221-22) + (221-11) + 221$ (A1) for substitution

$= 630$ A1 N3

[3]

3. (a) $d = 9300 - 7500$ (M1) for finding d

$d = 1800$

The required cost

$= u_{10}$ (M1) for valid approach

$= 7500 + (10-1)(1800)$

$= \$23700$ A1 N3

[3]

(b) $S_n < 340000$ (M1) for setting inequality

$\frac{n}{2}\left[2(7500) + (n-1)(1800)\right] < 340000$ (A1) for substitution

$900n^2 + 6600n - 340000 < 0$

By considering the graph of

$y = 900n^2 + 6600n - 340000$, $n < 16.112672$.

$\therefore n = 16$ (A1) for correct value

Thus, the greatest possible length is 160 m. A1 N4

[4]

4. (a) $d = 32 - 30$ (M1) for finding d

$d = 2$

The required number

$= u_{24}$ (M1) for valid approach

$= (30-2) + (24-1)(2)$

$= 74$ A1 N3

[3]

(b) The total number of seats in the theatre

$= S_{24}$ (M1) for valid approach

$= \frac{24}{2}\left[2(30-2) + (24-1)(2)\right]$

$= 1224$ (A1) for correct value

The total income

$= (1224)(75)$ (M1) for valid approach

$= \$91800$ A1 N4

[4]

Exercise 19

1. (a) (i) $u_4 = 43$

$u_1 + (4-1)d = 43$ (M1) for valid approach

$u_1 + 3d = 43$ A1 N2

(ii) $S_{80} = -5320$

$\frac{80}{2}\left[2u_1 + (80-1)d\right] = -5320$ (M1) for valid approach

$80u_1 + 3160d = -5320$ A1 N2

(iii) $u_1 = 52$ A1 N1

$d = -3$ A1 N1

[6]

(b) $u_n = 52 + (n-1)(-3)$ (M1) for valid approach

$u_n = -3n + 55$ A1 N2

[2]

(c) $u_n > 0$

$-3n + 55 > 0$ (M1) for setting inequality

$-3n > -55$

$n < \frac{55}{3}$ (A1) for correct value

Thus, the greatest value of n is 18. A1 N3

[3]

(d) $S_n = \frac{n}{2}[2(52) + (n-1)(-3)]$ (M1) for valid approach

$S_n = \frac{n}{2}(-3n + 107)$

$S_n = -\frac{3}{2}n^2 + \frac{107}{2}n$ A1 N2

[2]

(e) $S_n = -4425$

$-\frac{3}{2}n^2 + \frac{107}{2}n = -4425$ (M1) for setting equation

$3n^2 - 107n - 8850 = 0$ (M1) for quadratic equation

$(3n + 118)(n - 75) = 0$

$n = -\frac{118}{3}$ (*Rejected*) or $n = 75$ A1 N3

[3]

2. (a) (i) $u_{11} = 17$

$u_1 + (11-1)d = 17$ (M1) for valid approach

$u_1 + 10d = 17$ A1 N2

(ii) $S_{96} = 4512$

$\frac{96}{2}\left[2u_1 + (96-1)d\right] = 4512$ (M1) for valid approach

$96u_1 + 4560d = 4512$ A1 N2

(iii) $u_1 = 9$ A1 N1

$d = 0.8$ A1 N1

[6]

(b) $u_n = 9 + (n-1)(0.8)$ (M1) for valid approach

$u_n = 0.8n + 8.2$ A1 N2

[2]

(c) $u_n < 147$

$0.8n + 8.2 < 147$ (M1) for setting inequality

$0.8n < 138.8$

$n < 173.5$ (A1) for correct value

Thus, the greatest value of n is 173. A1 N3

[3]

(d) $u_{49} + u_{50} + u_{51} + \cdots + u_{95} + u_{96}$

$= S_{96} - S_{48}$ (M1) for valid approach

$= 4512 - \frac{48}{2}\left[2(9) + (48-1)(0.8)\right]$ (A1) for substitution

$= 3177.6$ A1 N3

[3]

(e) $S_n = 4949$

$\frac{n}{2}\left[2(9) + (n-1)(0.8)\right] = 4949$ (M1) for setting equation

$\frac{n}{2}\left[0.8n + 17.2\right] = 4949$

$0.4n^2 + 8.6n - 4949 = 0$

$2n^2 + 43n - 24745 = 0$ (M1) for quadratic equation

$(2n + 245)(n - 101) = 0$

$n = -\frac{245}{2}$ (*Rejected*) or $n = 101$ A1 N3

[3]

3. (a) (i) $u_1 = 60$, $u_2 = 57$ A2 N2

(ii) -3 A1 N1

[3]

(b) $u_n \le -21$

$63 - 3n \le -21$ (M1) for setting inequality

$-3n \le -84$

$n \ge 28$ (A1) for correct value

Thus, the smallest value of n is 28. A1 N3

[3]

(c) (i) 60 A1 N1

(ii) $S_n = \frac{n}{2}\left[2(60) + (n-1)(-3)\right]$ (M1) for valid approach

$S_n = \frac{n}{2}\left[-3n + 123\right]$

$S_n = -\frac{3}{2}n^2 + \frac{123}{2}n$ A1 N2

[3]

(d) $S_n = 0$

$-\frac{3}{2}n^2 + \frac{123}{2}n = 0$ (M1) for setting inequality

$n^2 - 41n = 0$

$n(n-41) = 0$ (A1) for factorization

$n = 0$ (*Rejected*) or $n = 41$ A1 N3

[3]

(e) $\sum_{r=11}^{20} u_r$

$= S_{20} - S_{10}$ (M1) for valid approach

$= \left[-\frac{3}{2}(20)^2 + \frac{123}{2}(20)\right] - \left[-\frac{3}{2}(10)^2 + \frac{123}{2}(10)\right]$ (A1) for substitution

$= 165$ A1 N3

[3]

4. (a) (i) $S_1 = \frac{5}{7}$, $S_2 = \frac{3}{2}$ A2 N2

(ii) $u_1 = \frac{5}{7}$, $u_2 = \frac{11}{14}$ A2 N2

(iii) The common difference

$= \frac{11}{14} - \frac{5}{7}$ (A1) for substitution

$= \frac{1}{14}$ A1 N2

[6]

(b) $\sum_{r=1}^{n} u_r > 100$

$\frac{n}{2}\left[2\left(\frac{5}{7}\right) + (n-1)\left(\frac{1}{14}\right)\right] > 100$ (A1) for substitution

$\frac{1}{28}n^2 + \frac{19}{28}n - 100 > 0$

By considering the graph of

$y = \frac{1}{28}n^2 + \frac{19}{28}n - 100$, $n > 44.261045$. (A1) for correct value

Thus, the smallest value of n is 45. A1 N3

[3]

(c) $\sum_{r=50}^{100} u_r$

$= S_{100} - S_{49}$ (M1) for valid approach

$= \left[\frac{100^2 + 19(100)}{28}\right] - \left[\frac{49^2 + 19(49)}{28}\right]$ (A1) for substitution

$= 306$ A1 N3

[3]

(d) $u_n = \frac{5}{7} + (n-1)\left(\frac{1}{14}\right)$ (M1) for valid approach

$u_n = \frac{1}{14}n + \frac{9}{14}$ A1 N2

[2]

(e) $u_n + 2u_{n+1} = \frac{58}{7}$

$u_n + 2\left(u_n + \frac{1}{14}\right) = \frac{58}{7}$ (M1) for valid approach

$3u_n = \frac{57}{7}$

$u_n = \frac{19}{7}$

$\frac{1}{14}n + \frac{9}{14} = \frac{19}{7}$ (A1) for substitution

$n = 29$ A1 N3

[3]