

AA HL Practice Set 2 Paper 2 Solution

Section A

1. $\left(kx - \frac{4}{x}\right)^8 = (kx)^8 + \binom{8}{1}(kx)^7\left(-\frac{4}{x}\right) + \binom{8}{2}(kx)^6\left(-\frac{4}{x}\right)^2$
 $+ \binom{8}{3}(kx)^5\left(-\frac{4}{x}\right)^3 + \binom{8}{4}(kx)^4\left(-\frac{4}{x}\right)^4 + \dots$ (M1)(A1) for correct approach
- $\left(kx - \frac{4}{x}\right)^8 = k^8x^8 + 8k^7x^7\left(-\frac{4}{x}\right) + 28k^6x^6\left(\frac{16}{x^2}\right)$
 $+ 56k^5x^5\left(-\frac{64}{x^3}\right) + 70k^4x^4\left(\frac{256}{x^4}\right) + \dots$ (A1) for simplification
- $\left(kx - \frac{4}{x}\right)^8 = k^8x^8 - 32k^7x^6 + 448k^6x^4$ A1
 $- 3584k^5x^2 + 17920k^4 + \dots$
- $\therefore 448k^6 : 17920k^4 = 9 : 40$ A1
- $\frac{448k^6}{17920k^4} = \frac{9}{40}$
- $\frac{k^2}{40} = \frac{9}{40}$
- $k = -3$ or $k = 3$ (Rejected) A1

[6]

2. (a) $A = 2\pi r^2 + 2\pi rh + 2\pi r^2$ (M2) for setting equation
 $135\pi = 4\pi r^2 + 2\pi r(3.5)$ (A1) for substitution
 $135 = 4r^2 + 7r$
 $4r^2 + 7r - 135 = 0$ (M1) for quadratic equation
 $(4r + 27)(r - 5) = 0$
 $4r + 27 = 0$ or $r - 5 = 0$
 $r = -\frac{27}{4}$ (*Rejected*) or $r = 5$ mm A1
- [5]
- (b) The volume
 $= \frac{4}{3}\pi r^3 + \pi r^2 h$ (M1) for valid approach
 $= \frac{4}{3}\pi(5)^3 + \pi(5)^2(3.5)$
 $= 798.4881328 \text{ mm}^3$
 $= 798 \text{ mm}^3$ A1
- [2]
3. (a) (i) $\cos \hat{ABC} = \frac{r^2 + (1.75r)^2 - (1.5r)^2}{2(r)(1.75r)}$ M1A1
 $\cos \hat{ABC} = \frac{1.8125r^2}{3.5r^2}$ A1
 $\cos \hat{ABC} = \frac{29}{56}$ AG
- (ii) $\hat{ABC} = 1.026452178 \text{ rad}$
 $\hat{ABC} = 1.03 \text{ rad}$ A1
- [4]
- (b) $\frac{1}{2}(BC)^2(\hat{DBC}) = 9.89$ (M1) for setting equation
 $\frac{1}{2}r^2(\pi - 1.026452178) = 9.89$ (A1) for substitution
 $r^2 = 9.35162474$
 $r = 3.058042632$
 $r = 3.06$ A1
- [3]

4. $X \sim B\left(5, \frac{2p}{p+2p+10}\right)$ (R1) for correct distribution

The standard deviation of X

$$= \sqrt{5 \left(\frac{2p}{3p+10} \right) \left(1 - \frac{2p}{3p+10} \right)}$$
 (A1) for substitution

$$= \sqrt{5 \left(\frac{2p}{3p+10} \right) \left(\frac{p+10}{3p+10} \right)}$$

$$\therefore \sqrt{5 \left(\frac{2p}{3p+10} \right) \left(\frac{p+10}{3p+10} \right)} > \frac{11}{10}$$
 (M1) for valid approach

$$5 \left(\frac{2p}{3p+10} \right) \left(\frac{p+10}{3p+10} \right) > \frac{121}{100}$$
 M1

$$5 \left(\frac{2p}{3p+10} \right) \left(\frac{p+10}{3p+10} \right) - \frac{121}{100} > 0$$
 A1

By considering the graph of

$$y = 5 \left(\frac{2p}{3p+10} \right) \left(\frac{p+10}{3p+10} \right) - \frac{121}{100},$$

$$5.3435147 < p < 25.443002.$$

Thus, the greatest value of p is 25. A1

[6]

5. $v = \int (8 - 8t) dt$ (M1) for indefinite integral

$$v = 8t - 8 \left(\frac{1}{2} t^2 \right) + C$$
 A1

$$v = 8t - 4t^2 + C$$

The initial velocity

$$= 8(0) - 4(0)^2 + C$$
 (M1) for valid approach

$$= C$$

The difference between the velocities is 4 ms^{-1}

$$\therefore 8t - 4t^2 + C = C + 4 \text{ or } \therefore 8t - 4t^2 + C = C - 4$$
 (A1) for correct approach

$$4t^2 - 8t + 4 = 0 \text{ or } 4t^2 - 8t - 4 = 0$$
 A2

$$4(t-1)^2 = 0 \text{ or } t = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-4)}}{2(4)}$$

$$t = 1 \text{ or } t = 2.414213562, t = -0.4142135624 \text{ (Rejected)}$$

$$\therefore m = 1 \text{ or } m = 2.41$$
 A2

[8]

6. (a) By using row operations, the system

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & 3 \\ 1 & -4 & -6 & -17 \\ 3 & 1 & 2 & 21 \end{array} \right) \text{ is reduced to}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array} \right).$$

(M1) for valid approach

Thus, the coordinates of P are (5, 4, 1).

A3

[4]

(b) $\mathbf{r} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -4 \\ -6 \end{pmatrix}$

A2

[2]

7. (a) $\{x: -5 \leq x \leq 1\}$

A2

[2]

(b) $f(x) = 2 - (x-1)^2$

$$y = 2 - (x-1)^2$$

$$\Rightarrow x = 2 - (y-1)^2$$

(M1) for swapping variables

$$(y-1)^2 = 2 - x$$

$$y-1 = \sqrt{2-x} \text{ (Rejected) or } y-1 = -\sqrt{2-x}$$

A1

$$y = -\sqrt{2-x} + 1$$

$$\therefore f^{-1}(x) = -\sqrt{2-x} + 1$$

A1

[3]

(c) $(g^{-1} \circ f^{-1})(x) = \frac{x}{3}$

$$f^{-1}(x) = g\left(\frac{x}{3}\right)$$

M1

$$g\left(\frac{x}{3}\right) = -\sqrt{2-x} + 1$$

$$g\left(3\left(\frac{x}{3}\right)\right) = -\sqrt{2-3x} + 1$$

A1

$$\therefore g(x) = -\sqrt{2-3x} + 1$$

A1

[3]

8. $\frac{2\pi}{B} = 2(4-0)$
 $\frac{2\pi}{B} = 8$
 $B = \frac{\pi}{4}$ A1
- $5 + \pi = A \sec \frac{\pi}{4}(0) + C$
 $5 + \pi = A + C$
 $C = 5 + \pi - A$
- $5 - \pi = A \sec \frac{\pi}{4}(4) + C$
 $\therefore 5 - \pi = A(-1) + 5 + \pi - A$ (M1) for substitution
 $-2\pi = -2A$
 $A = \pi$ A1
 $C = 5 + \pi - \pi$
 $C = 5$ A1
- [4]
9. (a) The total number of possible ways
 $= \frac{14!}{14 \times 2}$ (A2) for correct formula
 $= 3113510400$ A1
- [3]
- (b) The number of possible ways
 $= 3113510400 - \frac{2! \times 13!}{13 \times 2}$ (A2) for correct formula
 $= 2634508800$ A1
- [3]

Section B

10. (a) $P(L > 59.2) = 0.12$ (M1) for valid approach
 $P\left(Z > \frac{59.2 - \mu}{3.5}\right) = 0.12$ (A1) for correct approach
 $\frac{59.2 - \mu}{3.5} = 1.174986791$ A1
 $59.2 - \mu = 4.11245377$
 $\mu = 55.08754623$
 $\mu = 55.1$ A1
- [4]
- (b) $P(L < q) = 0.55$
 $P\left(Z < \frac{q - 55.08754623}{3.5}\right) = 0.55$ (A1) for correct approach
 $\frac{q - 55.08754623}{3.5} = 0.1256613375$
 $q - 55.08754623 = 0.4398146813$
 $q = 55.52736091$ A1
 $\therefore q = 55.5$ A1
- [3]
- (c) (i) $X \sim B(10, 0.55)$ (R1) for correct distribution
 $E(X) = (10)(0.55)$ (A1) for substitution
 $E(X) = 5.5$ A1
- (ii) $P(X > 5) = 1 - P(X \leq 5)$ (M1) for valid approach
 $P(X > 5) = 1 - 0.4955954083$ A1
 $P(X > 5) = 0.5044045917$
 $P(X > 5) = 0.504$ A1
- [6]
- (d) $m\left(\frac{55\%}{55\% + 33\%}\right)(0.8) + m\left(\frac{33\%}{55\% + 33\%}\right)(1.1)$ (M1)(A1) for correct approach
 $= (949)(1000)$
 $0.5m + 0.4125m = 949000$ A1
 $0.9125m = 949000$
 $m = 1040000$ A1
- [4]

11. (a) When $0 \leq t \leq 1$,
- $$s(t) = \int \pi t dt \quad (\text{M1) for valid approach}$$
- $$s(t) = \frac{\pi}{2} t^2 + C \quad (\text{A1) for correct value}$$
- $$s(0) = -\frac{\pi}{2}$$
- $$\therefore \frac{\pi}{2} (0)^2 + C = -\frac{\pi}{2}$$
- $$C = -\frac{\pi}{2}$$
- $$s(1) = \frac{\pi}{2} (1)^2 - \frac{\pi}{2}$$
- $$s(1) = 0 \quad (\text{A1) for correct value}$$
- When $1 < t \leq 5$,
- $$s(t) = \int \pi e^{1-t} dt \quad (\text{M1) for valid approach}$$
- $$s(t) = -\pi e^{1-t} + D \quad (\text{A1) for correct value}$$
- $$s(1) = 0$$
- $$\therefore -\pi e^{1-1} + D = 0$$
- $$D = \pi \quad (\text{A1) for correct value}$$
- $$s(5) = -\pi e^{1-5} + \pi$$
- $$s(5) = -\pi e^{-4} + \pi$$
- $$s(5) = \frac{\pi}{e^4} (e^4 - 1) \quad (\text{A1) for correct value}$$
- $$\therefore s(t) = \begin{cases} \frac{\pi}{2} t^2 - \frac{\pi}{2} & 0 \leq t \leq 1 \\ -\pi e^{1-t} + \pi & 1 < t \leq 5 \\ \frac{\pi}{e^4} (e^4 - 1) & t > 5 \end{cases} \quad \text{A1}$$

[8]

$$(b) \quad a(t) = \begin{cases} \pi(1) & 0 \leq t \leq 1 \\ \pi e^{1-t}(-1) & 1 < t \leq 5 \\ 0 & \text{otherwise} \end{cases} \quad \text{(M1) for valid approach}$$

$$a(t) = \begin{cases} \pi & 0 \leq t \leq 1 \\ -\pi e^{1-t} & 1 < t \leq 5 \\ 0 & \text{otherwise} \end{cases} \quad \text{(A1) for correct values}$$

$$a(t) < -3$$

$$-\pi e^{1-t} < -3$$

(M1) for setting inequality

$$3 - \pi e^{1-t} < 0$$

By considering the graph of $y = 3 - \pi e^{1-t}$,

$$t < 1.0461176.$$

$$\therefore 1 < t < 1.05$$

A1

[4]

$$(c) \quad (i) \quad \frac{ds}{dt} = \pi e^{1-t}$$

$$\frac{dv}{dt} = -\pi e^{1-t}$$

$$\therefore \frac{ds}{dv}$$

$$= \frac{ds}{dt} \div \frac{dv}{dt}$$

M1

$$= \frac{\pi e^{1-t}}{-\pi e^{1-t}}$$

A1

$$= -1$$

AG

$$(ii) \quad \frac{dt}{dv}$$

$$= 1 \div \frac{dv}{dt}$$

M1

$$= \frac{1}{-\pi e^{1-t}}$$

A1

$$= -\frac{1}{\pi} e^{t-1}$$

AG

[4]

12. (a) $|z|$

$$= \left| \frac{\frac{4}{5} e^{i\theta}}{2} \right|$$

(A1) for correct approach

$$= \left| \frac{2}{5} e^{i\theta} \right|$$

$$= \frac{2}{5} |e^{i\theta}|$$

$$= \frac{2}{5} (1)$$

$$= \frac{2}{5}$$

A1

[2]

(b) $\frac{2}{1 - \frac{2}{5} e^{i\theta}}$

A2

[2]

(c) (i) $\frac{2}{1 - \frac{2}{5} e^{i\theta}}$

$$= \frac{10}{5 - 2e^{i\theta}}$$

$$= \frac{10(5 - 2e^{i(-\theta)})}{(5 - 2e^{i\theta})(5 - 2e^{i(-\theta)})}$$

M1A1

$$= \frac{50 - 20e^{i(-\theta)}}{25 - 10e^{i(-\theta)} - 10e^{i\theta} + 4}$$

M1

$$= \frac{50 - 20e^{i(-\theta)}}{29 - 10(e^{i(-\theta)} + e^{i\theta})}$$

$$= \frac{50 - 20(\cos(-\theta) + i\sin(-\theta))}{29 - 10(\cos(-\theta) + i\sin(-\theta))}$$

A1

$$+ \cos \theta + i\sin \theta)$$

$$= \frac{50 - 20(\cos \theta - i\sin \theta)}{29 - 10(\cos \theta - i\sin \theta + \cos \theta + i\sin \theta)}$$

$$= \frac{50 - 20\cos \theta + 20i\sin \theta}{29 - 10(2\cos \theta)}$$

M1

$$= \frac{(50 - 20\cos \theta) + i(20\sin \theta)}{29 - 20\cos \theta}$$

A1

$$\begin{aligned} & \frac{4}{5} \sin \theta + \frac{8}{25} \sin 2\theta + \frac{16}{125} \sin 3\theta + \dots \\ &= \frac{20 \sin \theta}{29 - 20 \cos \theta} \end{aligned} \quad \text{M1A1}$$

$$\begin{aligned} & \therefore \sin \theta + \frac{2}{5} \sin 2\theta + \frac{4}{25} \sin 3\theta + \dots \\ &= \frac{25 \sin \theta}{29 - 20 \cos \theta} \end{aligned} \quad \text{AG}$$

$$(ii) \quad 2 + \frac{4}{5} \cos \theta + \frac{8}{25} \cos 2\theta + \dots = \frac{50 - 20 \cos \theta}{29 - 20 \cos \theta} \quad \text{M1A1}$$

$$\begin{aligned} & \frac{4}{5} \cos \theta + \frac{8}{25} \cos 2\theta + \frac{16}{125} \cos 3\theta + \dots \\ &= \frac{50 - 20 \cos \theta}{29 - 20 \cos \theta} - 2 \end{aligned} \quad \text{M1}$$

$$\begin{aligned} &= \frac{50 - 20 \cos \theta}{29 - 20 \cos \theta} - \frac{2(29 - 20 \cos \theta)}{29 - 20 \cos \theta} \\ &= \frac{50 - 20 \cos \theta}{29 - 20 \cos \theta} - \frac{58 - 40 \cos \theta}{29 - 20 \cos \theta} \end{aligned} \quad \text{A1}$$

$$= \frac{50 - 20 \cos \theta - 58 + 40 \cos \theta}{29 - 20 \cos \theta} \quad \text{M1}$$

$$= \frac{-8 + 20 \cos \theta}{29 - 20 \cos \theta} \quad \text{A1}$$

$$\therefore \cos \theta + \frac{2}{5} \cos 2\theta + \frac{4}{25} \cos 3\theta + \dots \quad \text{A1}$$

$$= \frac{-10 + 25 \cos \theta}{29 - 20 \cos \theta}$$

$$= \frac{5(-2 + 5 \cos \theta)}{29 - 20 \cos \theta} \quad \text{AG}$$

[15]