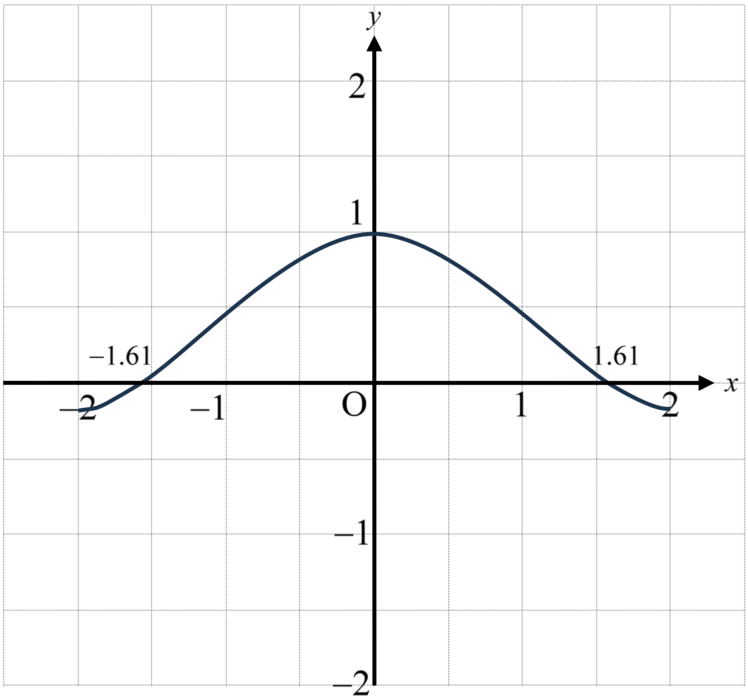
AA SL Practice Set 8 Paper 2 Solution

Section A

1. (a) For approximately correct shape A1
For correct axes intercepts A1
For approximately correct endpoints A1

[3]



(b) (i) $-1.61 < x < 1.61$ A1

(ii) $x = 0$ A1

[2]

2. (a) $P(X=0)+P(X=2)+P(X=4)+P(X=6)=1$ (M1) for valid approach

$0.4+0.1+0.5-6k+0.1+4k=1$

$1.1-2k=1$ (A1) for correct approach

$-2k=-0.1$

$k=0.05$ A1

[3]

(b) (i) $E(X)$

$=(0)(0.4)+(2)(0.1)+(4)(0.5-6(0.05))$

$+(6)(0.1+4(0.05))$ (A1) for correct approach

$=2.8$ A1

(ii) $P(X<6)$

$=1-P(X=6)$ (M1) for valid approach

$=1-(0.1+4(0.05))$

$=0.7$ A1

[4]

3. (a) The lower quartile

$=\frac{18+22}{2}$ (M1) for valid approach

$=20$ A1

[2]

(b) (i) 30 A1

(ii) $20+k+30\le 100$ (A1) for correct inequality

$k\le 50$ A1

[3]

4. $QR^2=PQ^2+PR^2-2(PQ)(PR)\cos Q\hat{P}R$ (M1) for cosine rule

$11^2=18^2+PR^2-2(18)(PR)\cos 0.555$ (A1) for substitution

$PR^2-(36\cos 0.555)PR+203=0$

$PR=9.727286483$ or $PR=20.86912937$ (A1) for correct values

The required difference

$=\frac{1}{2}(18)(20.86912937)\sin 0.555$

$-\frac{1}{2}(18)(9.727286483)\sin 0.555$ (A1) for substitution

$=52.84007515$

$=52.8$ A1

[5]

5. (a) $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ (M1) for valid approach

$\mathrm{P}(A \cap B)$

$= [\mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')][\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)]$ (A1) for correct approach

$\mathrm{P}(A \cap B) = [\mathrm{P}(A \cap B) + 0.12][\mathrm{P}(A \cap B) + 0.42]$

$\mathrm{P}(A \cap B) = \mathrm{P}(A \cap B)^2 + 0.54\mathrm{P}(A \cap B) + 0.0504$

$\mathrm{P}(A \cap B)^2 - 0.46\mathrm{P}(A \cap B) + 0.0504 = 0$ (A1) for correct equation

$\mathrm{P}(A \cap B) = 0.28$ or $\mathrm{P}(A \cap B) = 0.18$ A1A1

[5]

(b) $\mathrm{P}(B' \mid A')$

$= \dfrac{\mathrm{P}(B' \cap A')}{\mathrm{P}(A')}$

$= \dfrac{1 - \mathrm{P}(A \cup B)}{1 - \mathrm{P}(A)}$ (A1) for correct approach

$= \dfrac{1 - 0.12 - 0.28 - 0.42}{1 - 0.12 - 0.28}$

$= 0.3$ A1

[2]

6. (a) (i) $\dfrac{1}{4}$ A1

(ii) -3 A1

(iii) 2 A1

[3]

(b) The coordinates of P'

$= \left(-\left(\dfrac{5}{4} - 3 \right), 7(6+2) \right)$ (A1)(A1) for correct approach

$= \left(\dfrac{7}{4}, 56 \right)$ A1A1

[4]

Section B

7. (a) (i) $y = 25 - 5x$ A1

(ii) $0 < x < 5$ A1

[2]

(b) The required length

$= \sqrt{(5x)^2 + (3x)^2 + (25-5x)^2}$ (M1) for valid approach

$= \sqrt{25x^2 + 9x^2 + 625 - 250x + 25x^2}$

$= \sqrt{59x^2 - 250x + 625}$ cm A1

[2]

(c) $V = (5x)(3x)(25-5x)$ (M1) for valid approach

$V = 15x^2(25-5x)$

$V = 375x^2 - 75x^3$ A1

[2]

(d) (i) By considering the graph of $y = 375x^2 - 75x^3$, the coordinates of the maximum point are $(3.333331, 1388.8889)$. (M1) for valid approach

Thus, the maximum volume is 1390 cm^3. A1

(ii) 3.33 A1

(iii) $y = 25 - 5(3.333331)$ (M1) for substitution

$y = 8.333345$

$y = 8.33$ A1

[5]

(e) $A = 2(5x)(3x) + 2(5x)(25-5x) + 2(3x)(25-5x)$ (M1) for valid approach

$A = 30x^2 + 250x - 50x^2 + 150x - 30x^2$

$A = 400x - 50x^2$ A1

[2]

(f) By considering the graph of $y = 400x - 50x^2$, the x-coordinate of the maximum point is 4, which is not 3.33. R1

Therefore, the total surface area of the box does not attain its maximum when its volume attains its maximum.

Thus, the claim is incorrect. A1

[2]

8. (a) (i) $x=-3$ A1

(ii) $y=3$ A1

(iii) $\{y : y \neq 3, y \in \mathbb{R}\}$ A1

[3]

(b) (i) $y=\frac{1+3x}{3+x}$

$\Rightarrow x=\frac{1+3y}{3+y}$ (M1) for swapping variables

$x(3+y)=1+3y$

$3x+xy=1+3y$

$xy-3y=1-3x$ (A1) for correct approach

$y(x-3)=1-3x$

$y=\frac{1-3x}{x-3}$

$\therefore f^{-1}(x)=\frac{1-3x}{x-3}$ A1

(ii) $f(-x)$

$=\frac{1+3(-x)}{3+(-x)}$ A1

$=\frac{1-3x}{3-x}$

$-f^{-1}(x)$

$=-\frac{1-3x}{x-3}$

$=\frac{1-3x}{-(x-3)}$ A1

$=\frac{1-3x}{3-x}$

$\therefore f(-x)=-f^{-1}(x)$ AG

(iii) Reflection in the y-axis followed by reflection in the x-axis A1A1

[7]

(c) By considering the graphs of $y=\frac{1+3x}{3+x}$ and $y=\frac{1-3x}{x-3}$, the coordinates of the points of intersections are $(-1,-1)$ and $(1,1)$. (A1)(A1) for correct values

The required area

$=\int_{-1}^{1}(f(x)-f^{-1}(x))\mathrm{d}x$ (M1) for valid approach

$=\int_{-1}^{1}\left(\frac{1+3x}{3+x}-\frac{1-3x}{x-3}\right)\mathrm{d}x$ (A1) for correct integral

$=0.909645111$

$=0.910$ A1

[5]

9. (a) Let L_A be the length of a Type A nail.

The required probability

$= \mathrm{P}(L_A > 51.5)$ (M1) for valid approach

$= 0.3297231005$

$= 0.330$ A1

[2]

(b) (i) $X \sim \mathrm{B}(10, 0.08)$ (R1) for binomial distribution

$\mathrm{E}(X)$

$= (10)(0.08)$ (A1) for substitution

$= 0.8$ A1

(ii) $\mathrm{P}(X > 3)$

$= 1 - \mathrm{P}(X \le 3)$ (M1) for valid approach

$= 0.0058013245$

$= 0.00580$ A1

[5]

(c) (i) Let L_B and L_C be the lengths of a Type B nail and a Type C nail respectively.

$\mathrm{P}(L_B > 51.5) = 0.391846495$ (A1) for correct value

$\mathrm{P}(L_C > 51.5) = 0.445766878$ (A1) for correct value

The required probability

$= 0.55\mathrm{P}(L_A > 51.5) + 0.08\mathrm{P}(L_B > 51.5) + 0.37\mathrm{P}(L_C > 51.5)$ (M1) for valid approach

$= 0.55(0.3297231005) + 0.08(0.391846495) + 0.37(0.445766878)$ (A1) for substitution

$= 0.3776291697$

$= 0.378$ A1

(ii) The required probability

$$= \frac{0.37\mathrm{P}(L_C > 51.5)}{0.55\mathrm{P}(L_A > 51.5) + 0.08\mathrm{P}(L_B > 51.5) + 0.37\mathrm{P}(L_C > 51.5)}$$ (M1) for valid approach

$$= \frac{0.37(0.445766878)}{0.3776291697}$$

$= 0.4367611352$

$= 0.437$ A1

[7]