Solución de Práctica de Prueba 2 de AE NS Set 3

Sección A

1. (a) (i) 6 A1

(ii) 6 A1

(iii) El rango

$=18-3$ (M1) por enfoque válido

$=15$ A1

[4]

(b) (i) La media

$$=\frac{(3)(12)+(6)(20)+(9)(12)+(12)(8)+(15)(4)+(18)(4)}{12+20+12+8+4+4}$$ (M1) por enfoque válido

$=8,2$ A1

(ii) La varianza

$=4,308131846^2$ (M1) por enfoque válido

$=18,6$ A1

[4]

2. (a) $f(x)=g(x)$

$\pi e^{-x^2}=1+\dfrac{1}{\pi e^{-x^2}}$ (M1) por ecuación

$\pi e^{-x^2}-1-\dfrac{e^{x^2}}{\pi}=0$

Considerando el gráfico de $y=\pi e^{-x^2}-1-\dfrac{e^{x^2}}{\pi}$,

$x=-0,814566$ o $x=0,8145662$.

$\therefore a=-0,815$, $b=0,815$ A2

[3]

(b) El área requerida

$=\int_{-0,814566}^{0,8145662}(f(x)-g(x))\mathrm{d}x$ (A1) por integral correcta

$=\int_{-0,814566}^{0,8145662}\left(\pi e^{-x^2}-1-\dfrac{e^{x^2}}{\pi}\right)\mathrm{d}x$

$=1,890606422$ (A1) por valor correcto

$=1,89$ A1

[3]

3. Observe que $f(0)=-1$.

$-1=\sqrt{2}\,\mathrm{sen}\left(\dfrac{\pi}{6}(0+h)\right)$ (M1) por ecuación

$-\dfrac{1}{\sqrt{2}}=\mathrm{sen}\left(\dfrac{\pi}{6}h\right)$ (A1) por enfoque correcto

$\dfrac{\pi}{6}h=-\dfrac{3\pi}{4}$ o $\dfrac{\pi}{6}h=-\dfrac{\pi}{4}$ (A1) por enfoque correcto

$h=-4,5$ (*Rechazada*) o $h=-1,5$ A1

$\therefore h=-1,5$ A1

[5]

4. (a) (i) $\frac{1}{2}$ A1

(ii) 3 A1

(iii) -4 A1

[3]

(b) Las coordenadas de P'

$=\left(\frac{2}{2}+3,-5(8-4)\right)$ (A2) por enfoque correcto

$=(4,-20)$ A2

[4]

5. (a) $\cos\theta=\frac{AB}{r}$

$AB=r\cos\theta$ A1

[1]

(b) $\operatorname{sen}\theta=\frac{AE}{r}$

$AE=r\operatorname{sen}\theta$ A1

El área del triángulo ABE

$=\frac{(AB)(AE)}{2}$

$=\frac{(r\cos\theta)(r\operatorname{sen}\theta)}{2}$ M1

$=\frac{1}{2}r^2\operatorname{sen}\theta\cos\theta$ A1

$=\frac{1}{2}r^2\left(\frac{1}{2}\operatorname{sen}2\theta\right)$ A1

$=\frac{r^2\operatorname{sen}2\theta}{4}$ AG

[4]

(c) $A\hat{E}B+B\hat{E}C+C\hat{E}D=\pi$ M1

$\left(\frac{\pi}{2}-\theta\right)+B\hat{E}C+\left(\frac{\pi}{2}-\theta\right)=\pi$ A1

$\pi-2\theta+B\hat{E}C=\pi$

$B\hat{E}C=2\theta$ AG

[2]

6. (a) Sea $\frac{x^2+2x+4}{(x-3)(x-7)} \equiv A+\frac{B}{x-3}+\frac{C}{x-7}$, donde A, B y C son constantes.

$$\frac{x^2+2x+4}{(x-3)(x-7)} \equiv \frac{A(x-3)(x-7)}{(x-3)(x-7)} + \frac{B(x-7)}{(x-3)(x-7)} + \frac{C(x-3)}{(x-3)(x-7)}$$ M1

$$\frac{x^2+2x+4}{(x-3)(x-7)} \equiv \frac{Ax^2-10Ax+21A+Bx-7B+Cx-3C}{(x-3)(x-7)}$$

$$x^2+2x+4 \equiv Ax^2+(-10A+B+C)x+(21A-7B-3C)$$ A1

$A=1$ A1

$2=-10(1)+B+C$

$C=12-B$

$4=21A-7B-3C$

$\therefore 4=21(1)-7B-3(12-B)$ A1

$4=21-7B-36+3B$

$19=-4B$

$B=-\frac{19}{4}$ A1

$\therefore C=12-\left(-\frac{19}{4}\right)$

$C=\frac{67}{4}$ A1

[6]

(b) $y=1$ A1

[1]

7. (a) (i) $\begin{cases} x+2y-z=1 \\ 2x-y+az=0 \\ x+3y+2z=b \end{cases}$

$\rightarrow \begin{cases} x+2y-z=1 \\ -5y+(a+2)z=-2 \\ y+3z=b-1 \end{cases}$ M1

$(R_2-2R_1 \,\&\, R_3-R_1)$

$\rightarrow \begin{cases} x+2y-z=1 \\ -5y+(a+2)z=-2 \quad (R_3+0,2R_2) \\ (0,2a+3,4)z=b-1,4 \end{cases}$ A1

El Sistema no tiene solución cuando

$0,2a+3,4=0$ y $b-1,4\neq 0$.

$a=-17$ y $b\neq 1,4$ A1

(ii) El Sistema tiene solución única cuando

$0,2a+3,4\neq 0$.

$\therefore a\neq -17$ y $b\in\mathbb{R}$ A1

[4]

(b) $\begin{cases} x+2y-z=1 \\ 2x-y+3z=0 \\ x+3y+2z=3 \end{cases}$

Resolviendo el sistema, $x=-0,2$, $y=0,8$ y

$z=0,4$. A2

[2]

8. $\mathbf{r}=(-1+2\lambda+4\mu)\mathbf{i}+(3+\lambda)\mathbf{j}+(-1+5\mu)\mathbf{k}$

$\mathbf{r}=(-\mathbf{i}+3\mathbf{j}-\mathbf{k})+\lambda(2\mathbf{i}+\mathbf{j})+\mu(4\mathbf{i}+5\mathbf{k})$ (M1) por enfoque válido

$\mathbf{n}=(2\mathbf{i}+\mathbf{j})\times(4\mathbf{i}+5\mathbf{k})$

$$\mathbf{n}=\begin{pmatrix}(1)(5)-(0)(0)\\(0)(4)-(2)(5)\\(2)(0)-(1)(4)\end{pmatrix}$$

$\mathbf{n}=5\mathbf{i}-10\mathbf{j}-4\mathbf{k}$ (A1) por valores correctos

La ecuación cartesiana del plano π:

$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(5\mathbf{i}-10\mathbf{j}-4\mathbf{k})=(-\mathbf{i}+3\mathbf{j}-\mathbf{k})\cdot(5\mathbf{i}-10\mathbf{j}-4\mathbf{k})$ M1A1

$5x-10y-4z=(-1)(5)+(3)(-10)+(-1)(-4)$

$5x-10y-4z=-31$ A1

[5]

9. (a) $f(0) = \arctan\frac{\pi}{2}(0) = 0$ (A1) por valor correcto

$$f'(x) = \left(\frac{1}{1+\left(\frac{\pi}{2}x\right)^2}\right)\left(\frac{\pi}{2}\right)$$

$$f'(x) = \frac{2\pi}{4+\pi^2x^2}$$

$$f'(0) = \frac{2\pi}{4+\pi^2(0)^2} = \frac{\pi}{2}$$ (A1) por valor correcto

$$f''(x) = \frac{(4+\pi^2x^2)(0)-(2\pi)(2\pi^2x)}{(4+\pi^2x^2)^2}$$ (M1) por enfoque válido

$$f''(x) = -\frac{4\pi^3x}{(4+\pi^2x^2)^2}$$

$$f''(0) = -\frac{4\pi^3(0)}{(4+\pi^2(0)^2)^2} = 0$$ (A1) por valor correcto

$$f^{(3)}(x) = -\frac{(4+\pi^2x^2)^2(4\pi^3) - (4\pi^3x)(2)(4+\pi^2x^2)(2\pi^2x)}{(4+\pi^2x^2)^4}$$ (M1) por enfoque válido

$$f^{(3)}(0) = -\frac{(4+0)^2(4\pi^3)-0}{4^4} = -\frac{\pi^3}{4}$$ (A1) por valor correcto

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f^{(3)}(0) + \cdots$$

$$f(x) = 0 + x\left(\frac{\pi}{2}\right) + \frac{x^2}{2}(0) + \frac{x^3}{6}\left(-\frac{\pi^3}{4}\right) + \cdots$$

$$f(x) = \frac{\pi}{2}x - \frac{\pi^3}{24}x^3 + \cdots$$ A1

[7]

Sección B

10. (a) $a = -0,0807147258$

$a = -0,0807$ A1

$b = 3,177202711$

$b = 3,18$ A1

[2]

(b) $\log y = -0,0807147258\sqrt{9} + 3,177202711$ (M1) por enfoque válido

$\log y = 2,935058534$

$y = 10^{2,935058534}$ (M1) por enfoque válido

$y = 861,1098035$

$y = 861$ A1

[3]

(c) $\log y = -0,0807147258\sqrt{x} + 3,177202711$

$y = 10^{-0,0807147258\sqrt{x}+3,177202711}$ (M1) por enfoque válido

$y = 10^{-0,0807147258\sqrt{x}} \cdot 10^{3,177202711}$ (A1) por enfoque correcto

$y = 10^{3,177202711} \cdot (10^{-0,0807147258})^{\sqrt{x}}$ A1

$k = 10^{3,177202711}$ (A1) por enfoque correcto

$k = 1503,843735$

$k = 1500$ A1

$m = 10^{-0,0807147258}$ (A1) por enfoque correcto

$m = 0,8303960491$

$m = 0,830$ A1

[7]

11. (a) $a = \dfrac{v^2 + 64}{240}$

$\dfrac{dv}{dt} = \dfrac{v^2 + 64}{240}$

$\dfrac{1}{v^2 + 64}\,dv = \dfrac{1}{240}\,dt$ (M1) por enfoque válido

$\int \dfrac{1}{v^2 + 64}\,dv = \int \dfrac{1}{240}\,dt$ (A1) por enfoque correcto

$\dfrac{1}{8}\arctan\dfrac{v}{8} = \dfrac{1}{240}t + C$ A1

$\arctan\dfrac{v}{8} = \dfrac{1}{30}t + C$

$\dfrac{v}{8} = \tan\left(\dfrac{1}{30}t + C\right)$

$v = 8\tan\left(\dfrac{1}{30}t + C\right)$ A1

$0 = 8\tan\left(\dfrac{1}{30}(0) + C\right)$ (M1) por sustitución

$C = 0$ (A1) por valor correcto

$\therefore v = 8\tan\dfrac{1}{30}t$ A1

[7]

(b) $\arctan\dfrac{v}{8} = \dfrac{1}{30}t$

$\arctan\left(\dfrac{1}{8}\cdot\dfrac{8}{3}\sqrt{3}\right) = \dfrac{1}{30}t$ (M1) por ecuación

$\arctan\dfrac{\sqrt{3}}{3} = \dfrac{1}{30}t$

$\dfrac{\pi}{6} = \dfrac{1}{30}t$ (A1) por enfoque correcto

$t = 5\pi$ s A1

[3]

(c) $\dfrac{dv}{dt} = \dfrac{v^2 + 64}{240}$

$\dfrac{dv}{ds} \cdot \dfrac{ds}{dt} = \dfrac{v^2 + 64}{240}$ A1

$v\dfrac{dv}{ds} = \dfrac{v^2 + 64}{240}$ A1

$\dfrac{240v}{v^2 + 64} dv = ds$ M1

$\int \dfrac{240v}{v^2 + 64} dv = \int ds$ A1

$s = \int \dfrac{240v}{v^2 + 64} dv$ AG

[4]

(d) $s = \int \dfrac{240v}{v^2 + 64} dv$

Sea $u = v^2 + 64$. (M1) por sustitución

$\dfrac{du}{dv} = 2v \Rightarrow 240v dv = 120 du$

$\therefore s = \int \dfrac{1}{u} \cdot 120 du$ A1

$s = 120 \ln|u| + D$

$s = 120 \ln(v^2 + 64) + D$ A1

$0 = 120 \ln(0^2 + 64) + D$ (M1) por sustitución

$D = -120 \ln 64$ (A1) por valor correcto

$\therefore s = 120 \ln(v^2 + 64) - 120 \ln 64$

$s = 120 \ln\left(\left(\dfrac{8}{3}\sqrt{3}\right)^2 + 64\right) - 120 \ln 64$

$s = 34{,}52184869 \text{ m}$

$s = 34{,}5 \text{ m}$ A1

[6]

12. (a) $\left(\cos\frac{\theta}{7}+\mathrm{i}\,\mathrm{sen}\frac{\theta}{7}\right)^7$

$=\cos^7\frac{\theta}{7}+\binom{7}{1}\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}+\binom{7}{2}\mathrm{i}^2\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+\binom{7}{3}\mathrm{i}^3\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+\binom{7}{4}\mathrm{i}^4\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ A2

$+\binom{7}{5}\mathrm{i}^5\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}+\binom{7}{6}\mathrm{i}^6\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$

$+\mathrm{i}^7\mathrm{sen}^7\frac{\theta}{7}$

$=\cos^7\frac{\theta}{7}+7\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$-35\mathrm{i}\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ A1

$+21\mathrm{i}\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}-\mathrm{i}\,\mathrm{sen}^7\frac{\theta}{7}$

$\therefore \cos\theta+\mathrm{i}\,\mathrm{sen}\,\theta$

$=\cos^7\frac{\theta}{7}+7\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$-35\mathrm{i}\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ M1

$+21\mathrm{i}\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}-\mathrm{i}\,\mathrm{sen}^7\frac{\theta}{7}$

$=\cos^7\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$

$+\mathrm{i}\begin{pmatrix}7\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-35\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}\\ +21\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-\mathrm{sen}^7\frac{\theta}{7}\end{pmatrix}$

$\therefore \cos\theta=\cos^7\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$ y

$\mathrm{sen}\,\theta=7\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-35\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}$. A2

$+21\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-\mathrm{sen}^7\frac{\theta}{7}$

[6]

(b) $\tan\theta$

$$= \frac{\operatorname{sen}\theta}{\cos\theta}$$

$$= \frac{7\cos^6\frac{\theta}{7}\operatorname{sen}\frac{\theta}{7} - 35\cos^4\frac{\theta}{7}\operatorname{sen}^3\frac{\theta}{7} + 21\cos^2\frac{\theta}{7}\operatorname{sen}^5\frac{\theta}{7} - \operatorname{sen}^7\frac{\theta}{7}}{\cos^7\frac{\theta}{7} - 21\cos^5\frac{\theta}{7}\operatorname{sen}^2\frac{\theta}{7} + 35\cos^3\frac{\theta}{7}\operatorname{sen}^4\frac{\theta}{7} - 7\cos\frac{\theta}{7}\operatorname{sen}^6\frac{\theta}{7}}$$ M1A1

$$= \frac{7\tan\frac{\theta}{7} - 35\tan^3\frac{\theta}{7} + 21\tan^5\frac{\theta}{7} - \tan^7\frac{\theta}{7}}{1 - 21\tan^2\frac{\theta}{7} + 35\tan^4\frac{\theta}{7} - 7\tan^6\frac{\theta}{7}}$$ A1

Sea $x = \tan\frac{\theta}{7}$.

$$\tan\theta = \frac{7x - 35x^3 + 21x^5 - x^7}{1 - 21x^2 + 35x^4 - 7x^6}$$ M1

$$x^6 - 21x^4 + 35x^2 - 7 = 0$$

$$\frac{-x(x^6 - 21x^4 + 35x^2 - 7)}{1 - 21x^2 + 35x^4 - 7x^6} = 0$$

$$\frac{7x - 35x^3 + 21x^5 - x^7}{1 - 21x^2 + 35x^4 - 7x^6} = 0$$

$\tan\theta = 0$ M1

$\theta = 0$, π, 2π, 3π, 4π, 5π o 6π

$\therefore x = \tan\frac{0}{7}$, $x = \tan\frac{\pi}{7}$, $x = \tan\frac{2\pi}{7}$, $x = \tan\frac{3\pi}{7}$,

$x = \tan\frac{4\pi}{7}$, $x = \tan\frac{5\pi}{7}$ o $x = \tan\frac{6\pi}{7}$ A1

$x = 0$ (*Rechazada*), $x = \tan\frac{\pi}{7}$, $x = \tan\frac{2\pi}{7}$,

$x = \tan\frac{3\pi}{7}$, $x = \tan\frac{4\pi}{7}$, $x = \tan\frac{5\pi}{7}$ o $x = \tan\frac{6\pi}{7}$ A1

Por tanto, la ecuación $x^6 - 21x^4 + 35x^2 - 7 = 0$ tiene seis raíces. AG

[7]

(c) (i) $\sum_{r=1}^{7} \tan\frac{r\pi}{7}$

$= \sum_{r=1}^{6} \tan\frac{r\pi}{7} + \tan\frac{7\pi}{7}$ M1

$= -\frac{0}{1} + 0$ A1

$= 0$ A1

(ii) $\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)$ M1A1

$\left(\tan\frac{4\pi}{7}\right)\left(\tan\frac{5\pi}{7}\right)\left(\tan\frac{6\pi}{7}\right) = -7$

$\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)\left(\tan\left(\pi - \frac{3\pi}{7}\right)\right)$

$\left(\tan\left(\pi - \frac{2\pi}{7}\right)\right)\left(\tan\left(\pi - \frac{\pi}{7}\right)\right) = -7$

$\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)\left(-\tan\frac{3\pi}{7}\right)$ A1

$\left(-\tan\frac{2\pi}{7}\right)\left(-\tan\frac{\pi}{7}\right) = -7$

$\left(\tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{3\pi}{7}\right)^2 = 7$

$\therefore \tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{3\pi}{7} = \sqrt{7}$ A1

[7]