Analysis and Approaches Higher Level for IBDP Mathematics Practice Paper Set 1 – Paper 2 (120 Minutes)

Question – Answer Book

Instructions

1. This paper consists of **TWO** sections: A and B.

2. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

3. A graphic display calculator is needed.

4. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.

5. Supplementary answer sheets and graph papers will be supplied on request.

6. Unless otherwise specified, **ALL** working must be clearly shown.

7. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

8. The diagrams in this paper are **NOT** necessarily drawn to scale.

9. Information to be read before you start the exam:

	Marker's Use Only	Examiner's Use Only	
Question Number	**Marks**	**Marks**	**Maximum Mark**
Section A			
1			**6**
2			**7**
3			**7**
4			**7**
5			**7**
6			**5**
7			**6**
8			**5**
9			**7**
Section A Total			**57**
Section B			
10			**14**
11			**20**
12			**19**
Section B Total			**53**
Overall			
Paper 2 Total			**110**

Section A (57 marks)

1. Let $f(x) = 3x + 7$ and $g(x) = 2\sqrt{x}$.

(a) Find $f^{-1}(x)$.

[2]

(b) Find $(f \circ g)(x)$.

[2]

(c) Hence, find $(f \circ g)(529)$.

[2]

2. The radius and the vertical height of a metal circular cone are both equal to $18\,\mathrm{cm}$.

(a) Find the volume of the circular cone expressing your answer in the form $a \times 10^k$, $1 \le a < 10$ and $k \in \mathbb{Z}$.

[3]

16 identical circular cones are to be melted down and remoulded into 27 identical metal hemispheres.

(b) Find the ratio of the radius of the cone to the radius of the hemisphere.

[4]

3. In a geometric series, $u_1 = 4.5$ and $u_2 = 5.4$.

(a) Find the value of r.

[2]

(b) Find the value of S_{12}.

[2]

(c) Find the greatest value of n such that $u_n < 678$.

[3]

4. A population of eagles P_t can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $20P_1 - 17P_0 = 0$. It is given that one decade is equivalent to ten years.

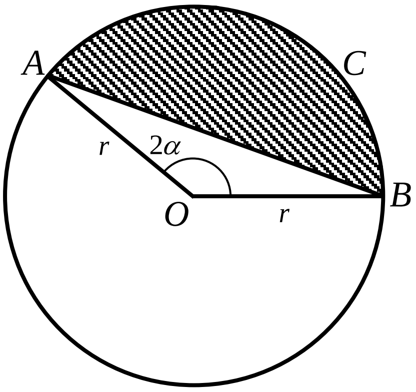
(a) Show that $k = \ln 0.85$.

[2]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} \leq 0.5$.

[5]

5. Consider the following circle with centre O and radius r.



The points A, B and C are on the circumference such that $\mathrm{A\hat{O}B} = 2\alpha$, $0 < \alpha < \dfrac{\pi}{2}$.

(a) Show that $\mathrm{AB} = r\sqrt{2(1-\cos 2\alpha)}$.

[2]

Let P be the perimeter of the shaded region.

(b) Show that $P = 2r(\alpha + \sin\alpha)$.

[5]

6. Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$$x+3y+2z=1$$
$$5x+7y+z=2$$
$$32x+24y-17z=5$$

[5]

7. (a) Using the extended binomial expansion, find the first three terms of the expansion of $\dfrac{1-x}{1+ax}$ in ascending powers of x up to x^2, giving the answer in terms of a.

[3]

It is given that the sum of the coefficient of x^2 and the constant term is 21, where $a>0$.

(b) (i) Find the value of a.

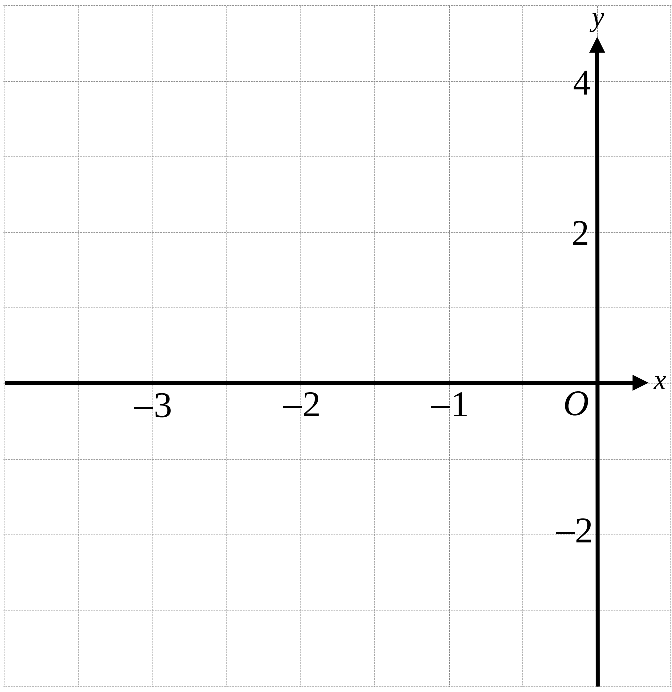
(ii) Hence, write down the coefficient of x.

[3]

8. The function f is defined as $f(x)=2e^{\frac{\pi}{3}x}\cot(x-0.5)$, $-3.5\le x\le 0$.

(a) Sketch the graph of $y=f(x)$, showing clearly any asymptotes and any points of intersection with the axes.

[3]



(b) Let $g(x)=f(x)+k$, $-3.5\le x\le 0$. It is given that the equation $g(x)=0$ has two real roots. Write down the range of values of k.

[2]

9. The region R is enclosed by the graph of $y=4^{-x}$, the graph of $x=-\frac{1}{32}(y-24)^2$ and the y-axis. Find the area of R.

[7]

Section B (53 marks)

10. At 8 am on any school day, Stephen walks from his apartment to the school bus stop. The time T minutes needed for Stephen to walk to the school bus stop follows a normal distribution such that $T \sim \text{N}(16, 5^2)$.

There are two school buses which will depart at $8:12\text{ am}$ and $8:24\text{ am}$ respectively, in every school day morning. Stephen will take the first bus if he arrives at or before $8:12\text{ am}$.

(a) Find the probability that Stephen can arrive at the school bus stop before the second school bus departs.

[2]

The time U minutes needed for a school bus to travel from the school bus stop to school follows a normal distribution such that $U \sim \text{N}(\mu, 7^2)$. It is given that U and T are independent, and 99.494% of the school buses take at most 48 minutes to travel from the school bus stop to school.

(b) Find μ.

[3]

In order to be marked as on time, Stephen needs to take any one of the buses and arrive at school by 9 am.

(c) Show that, correct to five significant figures, for all school buses departing at $8:24\text{ am}$, 80.439% of them will arrive at school on time.

[2]

(d) Hence, find the probability that Stephen will not arrive at school on time.

[5]

There are twenty school days in February 2021.

(e) Find the expected number of school days that Stephen will not arrive at school on time in February 2021.

[2]

11. The Cartesian equations of the planes π_1 and π_2 are $4x+3y+kz=24$ and $4x-3y+kz=-24$ respectively, $k\in\mathbb{R}$. The vector equation of the line of intersection of the planes π_1 and π_2 is given by $\mathbf{r}=\begin{pmatrix}0\\8\\0\end{pmatrix}+t\begin{pmatrix}-3\\0\\1\end{pmatrix}$.

(a) Find k.

[4]

(b) The plane π_1 meets the x, y and z axes at $\mathrm{A}(a,0,0)$, $\mathrm{B}(0,b,0)$ and $\mathrm{C}(0,0,c)$ respectively. The plane π_2 meets the x, y and z axes at $\mathrm{A}'(\alpha,0,0)$, B and $\mathrm{C}'(0,0,-c)$ respectively, where A' is the reflection point of A about the y-z plane.

(i) Write down the values of a, b, c and α.

(ii) Hence, find the volume of the pyramid $\mathrm{A'ABC}$.

[7]

(c) (i) Find the angle between $\mathrm{AC'}$ and the x-axis.

(ii) Hence, find the size of $\mathrm{A\hat{C}'A'}$.

[5]

(d) The line L passing through C is perpendicular to the plane ABC. It is given that the line L meets the plane $\mathrm{A'BC'}$ at Q. Find the coordinates of Q, giving the answer correct to four decimal places.

[4]

12. The function f is given by $f(x)=\ln(x^2+1)$.

(a) (i) Find the first three derivatives of $f(x)$.

It is also given that $f^{(4)}(x)=-\dfrac{12(x^4-6x^2+1)}{(x^2+1)^4}$.

(ii) Using the above results to find the Maclaurin series for $f(x)$ up to and including the x^4 term.

[10]

(b) Find the Maclaurin series for $\ln\left((x^2+1)^{\sin x}\right)$ up to and including the x^5 term.

[5]

The region R is enclosed by the graph of $g(y)=y\sqrt{\ln\left((y^2+1)^{\sin y}\right)}$, the y-axis and the lines $y=0.7$ and $y=1.3$.

(c) Using (b) to find the volume of the solid of revolution formed when R is rotated through 2π about the y-axis.

[4]

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