Exercise 1.1

(a) The required hypotenuse

$= \sqrt{1107^2 + 4920^2}$ — Pythagoras' theorem (A1)

$= 5043 \text{ mm}$

$= 5.043 \times 10^3 \text{ mm}$ — $a = 5.043$ & $k = 3$ (A1)

(b) The required perimeter

$= 1107 + 4920 + 5043$ — The sum of 3 sides (A1)

$= 11070 \text{ mm}$

$= 11000 \text{ mm}$ — Round off to 2 sig. fig.

$= 1.1 \times 10^4 \text{ mm}$ — $a = 1.1$ & $k = 4$ (A1)

(c) The required area

$= \frac{(1107)(4920)}{2}$ — $\frac{\text{Base length} \times \text{Height}}{2}$ (A1)

$= 2723220 \text{ mm}^2$

$= 2723000 \text{ mm}^2$ — Round off to 4 sig. fig.

$= 2.723 \times 10^6 \text{ mm}^2$ — $a = 2.723$ & $k = 6$ (A1)

Exercise 1.2

(a) $d=\frac{38}{10}-\frac{39}{10}$ — $d=u_2-u_1$

$d=-\frac{1}{10}$ — (A1)

(b) $u_{12}=u_1+(12-1)d$ — $u_n=u_1+(n-1)d$

$u_{12}=\frac{39}{10}+(12-1)\left(-\frac{1}{10}\right)$ — Correct approach (A1)

$u_{12}=\frac{39}{10}-\frac{11}{10}$

$u_{12}=\frac{28}{10}$

$u_{12}=\frac{14}{5}$ — (A1)

(c) $u_n=\frac{7}{10}$ — Set up an equation

$\therefore \frac{39}{10}+(n-1)\left(-\frac{1}{10}\right)=\frac{7}{10}$ — Correct equation (A1)

$-\frac{1}{10}(n-1)=-\frac{32}{10}$

$n-1=32$

$n=33$ — (A1)

(d) The sum of the first n terms

$=S_n$

$=\frac{n}{2}[2u_1+(n-1)d]$ — $S_n=\frac{n}{2}[2u_1+(n-1)d]$ (M1)

$=\frac{n}{2}\left[2\left(\frac{39}{10}\right)+(n-1)\left(-\frac{1}{10}\right)\right]$ — $u_1=\frac{39}{10}$ & $d=-\frac{1}{10}$ (A1)

$=\frac{n}{2}\left(\frac{78}{10}-\frac{1}{10}n+\frac{1}{10}\right)$

$=\frac{n}{2}\left(-\frac{1}{10}n+\frac{79}{10}\right)$

$=-\frac{1}{20}n^2+\frac{79}{20}n$ — (A1)

(e) The required sum

$= S_{10}$

$= -\frac{1}{20}(10)^2 + \frac{79}{20}(10)$ $n = 10$ (M1)

$= -5 + \frac{79}{2}$

$= \frac{69}{2}$ (A1)

Exercise 1.3

(a) $u_m = 0$ — Set up an equation

$\therefore 120 + (m-1)(-1.25) = 0$ — Correct equation (A1)

$-1.25(m-1) = -120$

$m - 1 = 96$

$m = 97$ — (A1)

(b) The sum of the first n terms

$= S_n$

$= \frac{n}{2}\left[2u_1 + (n-1)d\right]$ — $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$ (M1)

$= \frac{n}{2}\left[2(120) + (n-1)(-1.25)\right]$

$= \frac{n}{2}(240 - 1.25n + 1.25)$

$= \frac{n(241.25 - 1.25n)}{2}$

By considering the graph of $y = \frac{n(241.25 - 1.25n)}{2}$, the coordinates of the maximum point are $(96.5, 5820.1563)$, and the graph passes through $(96, 5820)$ and $(97, 5820)$. — GDC approach (M1)

Thus, the maximum value is 5820. — (A1)

Exercise 1.4

(a) (i) $k\log x-\log x=\frac{1}{5}\log x-k\log x$ $d=u_2-u_1=u_3-u_2$ (A1)

$k-1=\frac{1}{5}-k$

$2k=\frac{6}{5}$

$k=\frac{3}{5}$ (A1)

(ii) The common difference

$=\frac{3}{5}\log x-\log x$ $d=u_2-u_1$ (A1)

$=-\frac{2}{5}\log x$ (A1)

(b) (i) $k\log x\div\log x=\frac{1}{5}\log x\div k\log x$ $r=u_2\div u_1=u_3\div u_2$ (A1)

$k=\frac{1}{5k}$

$5k^2=1$

$k^2=\frac{1}{5}$

$k=\frac{1}{\sqrt{5}}$ or $k=-\frac{1}{\sqrt{5}}$ (A1)(A1)

(ii) The sum of the first n terms

$$= S_n$$

$$= \frac{u_1(1-r^n)}{1-r}$$ $S_n = \frac{u_1(1-r^n)}{1-r}$ (M1)

$$= \frac{(\log x)\left(1-\left(-\frac{1}{\sqrt{5}}\right)^n\right)}{1-\left(-\frac{1}{\sqrt{5}}\right)}$$ $u_1 = \log x$ & $r = -\frac{1}{\sqrt{5}}$ (A1)

$$= \frac{(\log x)\left(1-\left(-\frac{1}{5^{0.5}}\right)^n\right)}{1+\frac{1}{5^{0.5}}}$$ $\sqrt{5} = 5^{0.5}$ (M1)

$$= \frac{(\log x)(1-(-5^{-0.5})^n)}{1+5^{-0.5}}$$

$$= \frac{1-(-1)^n 5^{-0.5n}}{1+5^{-0.5}} \log x$$ (A1)

(iii) $S_\infty = \frac{5+\sqrt{5}}{2}$ Set up an equation

$$\therefore \frac{\log x}{1-\frac{1}{\sqrt{5}}} = \frac{5+\sqrt{5}}{2}$$ Correct equation (A1)

$$\log x = \frac{1}{2}(5+\sqrt{5})\left(1-\frac{1}{\sqrt{5}}\right)$$

$$\log x = \frac{1}{2}(5-\sqrt{5}+\sqrt{5}-1)$$

$$\log x = 2$$ Simplify the R.H.S. (A1)

$$x = 10^2$$

$$x = 100$$ (A1)

Exercise 1.5

(a)	$r=\frac{7}{16}\div\frac{7}{12}$	$r=u_2\div u_1$
	$r=\frac{3}{4}$	(A1)
(b)	$u_7=u_1\times r^{7-1}$	$u_n=u_1\times r^{n-1}$
	$u_7=\frac{7}{12}\times\left(\frac{3}{4}\right)^{7-1}$	$u_1=\frac{7}{12}$ & $r=\frac{3}{4}$ (A1)
	$u_7=0.1038208008$	
	$u_7=0.104$	(A1)
(c)	$u_n=\frac{189}{1024}$	Set up an equation
	$\therefore \frac{7}{12}\times\left(\frac{3}{4}\right)^{n-1}=\frac{189}{1024}$	Correct equation (A1)
	$\frac{7}{12}\times\left(\frac{3}{4}\right)^{n-1}-\frac{189}{1024}=0$	
	By considering the graph of $y=\frac{7}{12}\times\left(\frac{3}{4}\right)^{n-1}-\frac{189}{1024}$, the horizontal intercept is 5.	
	$\therefore n=5$	(A1)
(d)	The sum of the first n terms	
	$=S_n$	
	$=\frac{u_1(1-r^n)}{1-r}$	$S_n=\frac{u_1(1-r^n)}{1-r}$ (M1)
	$=\frac{\frac{7}{12}\left(1-\left(\frac{3}{4}\right)^n\right)}{1-\frac{3}{4}}$	$u_1=\frac{7}{12}$ & $r=\frac{3}{4}$ (A1)
	$=\frac{7}{3}\left(1-\left(\frac{3}{4}\right)^n\right)$	(A1)

(e) The required sum

$= S_{15}$

$= \frac{7}{3}\left(1 - \left(\frac{3}{4}\right)^{15}\right)$ — $n = 15$ (M1)

$= 2.302151924$

$= 2.30$ — (A1)

(f) The common ratio is $\frac{3}{4}$ which is between -1 and 1. — (R1)

(g) $S_n < 2.3315$ — Set up an inequality

$\therefore \frac{7}{3}\left(1 - \left(\frac{3}{4}\right)^{n}\right) < 2.3315$ — Correct inequality (A1)

$\frac{7}{3}\left(1 - \left(\frac{3}{4}\right)^{n}\right) - 2.3315 < 0$

By considering the graph of

$y = \frac{7}{3}\left(1 - \left(\frac{3}{4}\right)^{n}\right) - 2.3315$, the graph is below the horizontal axis when $n < 24.850062$. — GDC approach (M1)

$\therefore$ The greatest value of n is 24. — (A1)

Exercise 1.6

(a) The amount of money after one year

$= P\left(1+\frac{8\%}{4}\right)^{(4)(1)}$ — $FV = PV\left(1+\frac{r\%}{k}\right)^{kn}$ (M1)

$\therefore R = \left(1+\frac{8\%}{4}\right)^{(4)(1)}$ — $R = \frac{FV}{PV}$ (A1)

$R = 1.08243216$ — (A1)

(b) $FV = 2.5P$ — Set up an equation

$\therefore P\left(1+\frac{8\%}{4}\right)^{4n} = 2.5P$ — Correct equation (A1)

$\left(1+\frac{8\%}{4}\right)^{4n} = 2.5$

$\left(1+\frac{8\%}{4}\right)^{4n} - 2.5 = 0$

By considering the graph of

$y = \left(1+\frac{8\%}{4}\right)^{4n} - 2.5$, the horizontal intercept is

11.567792. — GDC approach (M1)

$\therefore$ The required year is 2036. — (A1)

(c) (i) The amount

$= PV\left(1+\frac{r\%}{k}\right)^{kn}$ — $FV = PV\left(1+\frac{r\%}{k}\right)^{kn}$ (M1)

$= 10000\left(1+\frac{8\%}{4}\right)^{(4)(4)}$ — $r = 8$, $k = 4$ & $n = 4$ (A1)

$= \$13727.85705$

$= \$13700$ — (A1)

(ii) The interest

$= 13727.85705 - 10000$ — $I = FV - PV$ (M1)

$= \$3727.857051$

$= \$3730$ — (A1)

(d) Let t be the number of years required for the amount of money in April's account exceeding that in Bea's account.

$$10000\left(1+\frac{8\%}{4}\right)^{4t} > 15000\left(1+\frac{3\%}{2}\right)^{2t}$$ Correct inequality (A1)

$$10000\left(1+\frac{8\%}{4}\right)^{4t} - 15000\left(1+\frac{3\%}{2}\right)^{2t} > 0$$

By considering the graph of

$y = 10000\left(1+\frac{8\%}{4}\right)^{4t} - 15000\left(1+\frac{3\%}{2}\right)^{2t}$, the graph is above the horizontal axis when $t > 8.2022693$. GDC approach (M1)

$\therefore$ The minimum number of complete years is 9. (A1)

(e) Let T be the number of years required for the sum of the amount of money in April's account and that in Bea's account first reaches $\$30500$.

$$10000\left(1+\frac{8\%}{4}\right)^{4T} + 15000\left(1+\frac{3\%}{2}\right)^{2T} \ge 30500$$ Correct inequality (A1)

$$10000\left(1+\frac{8\%}{4}\right)^{4T} + 15000\left(1+\frac{3\%}{2}\right)^{2T} - 30500 \ge 0$$

By considering the graph of

$y = 10000\left(1+\frac{8\%}{4}\right)^{4T} + 15000\left(1+\frac{3\%}{2}\right)^{2T} - 30500$, the graph is above the horizontal axis when $T > 3.9210584$. GDC approach (M1)

$\therefore$ The required year is 2028. (A1)

Exercise 1.7

$$(1+px)^n$$

$$=1^n+C_1^n 1^{n-1}(px)^1+C_2^n 1^{n-2}(px)^2+\cdots+(px)^n$$

Binomial theorem (M1)

$$=1+(n)(1)(px)+\left(\frac{(n)(n-1)}{(2)(1)}\right)(1)(p^2x^2)+\cdots+p^nx^n$$

$C_1^n=n$ & $C_2^n=\dfrac{n(n-1)}{2}$ (A1)

$$=1+npx+\frac{n(n-1)p^2}{2}x^2+\cdots+p^nx^n$$

$$npx=\frac{10}{3}x$$

$$p=\frac{10}{3n}$$

Make p the subject (A1)

$$\frac{n(n-1)p^2}{2}x^2=\frac{40}{9}x^2$$

$$\therefore \frac{n(n-1)}{2}\left(\frac{10}{3n}\right)^2=\frac{40}{9}$$

Substitution (M1)

$$\frac{n(n-1)}{2}\left(\frac{100}{9n^2}\right)=\frac{40}{9}$$

$$\frac{50(n-1)}{9n}=\frac{40}{9}$$

$$50n-50=40n$$

$$-50=-10n$$

$$n=5$$

(A1)

$$\therefore p=\frac{10}{3(5)}$$

$$p=\frac{2}{3}$$

(A1)

$$C_3^n 1^{n-3}(px)^3=qx^3$$

$$\therefore C_3^5(1)\left(\frac{2}{3}\right)^3x^3=qx^3$$

$$q=\frac{80}{27}$$

(A1)

Exercise 1.8

(a) The general term

$=\frac{1}{2x^2}\cdot C_r^n(1)^{n-r}(x^3)^r$ $C_r^n a^{n-r} b^r$ (M1)

$=\frac{1}{2x^2}\cdot C_r^n x^{3r}$

$=\frac{1}{2}C_r^n x^{3r-2}$ Term in x^{3r-2} (A1)

Consider the term in x^4.

$\therefore 3r-2=4$ Correct equation (A1)

$3r=6$

$r=2$ $r=2$ (A1)

The coefficient of x^4 is 3

$\therefore \frac{1}{2}C_2^n=3$ Correct equation (A1)

$\frac{1}{2}\left(\frac{(n)(n-1)}{(2)(1)}\right)=3$

$\frac{n(n-1)}{4}-3=0$

By considering the graph of $y=\frac{n(n-1)}{4}-3$, the horizontal intercept is 4.

$\therefore n=4$ (A1)

(b) $\frac{(1+x^3)^n}{2x^2}$

$=\frac{1}{2x^2}\left[\cdots+C_3^4 1^{4-3}(x^3)^3+\cdots\right]$ Binomial theorem (M1)

$=\frac{1}{2x^2}\left[\cdots+4x^9+\cdots\right]$

$=\cdots+2x^7+\cdots$

Thus, the coefficient of x^7 is 2. (A1)

Exercise 1.9

(a) L.H.S.

$=(3n)^2+(3n+3)^2$ — Starts from L.H.S. (M1)

$=9n^2+9n^2+18n+9$ — $(a+b)^2 \equiv a^2+2ab+b^2$ (A1)

$=18n^2+18n+9$

$=$R.H.S.

$\therefore (3n)^2+(3n+3)^2=18n^2+18n+9$ — (AG)

(b) $3n$ and $3n+3$ are consecutive multiples of 3. — Consecutive multiples (R1)

$(3n)^2+(3n+3)^2=18n^2+18n+9$ — Proved in (a) (A1)

Also, $18n^2+18n+9=9(2n^2+2n+1)$ which is divisible by 9. — $9(2n^2+2n+1)$ (R1)

Thus, the sum of the squares of any two consecutive multiples of 3 is divisible by 9. — (AG)