AA SL Practice Set 6 Paper 1 Solution

Section A

1. $f'(x)$

$= 3(e^x + 2)^2(e^x + 0)$ (M1) for chain rule

$= 3e^x(e^x + 2)^2$ (A1) for correct derivative

$f'(0)$

$= 3e^0(e^0 + 2)^2$ (M1) for substitution

$= 27$ A1

[4]

2. (a) $y = 3x - 4$

$\Rightarrow x = 3y - 4$ (M1) for swapping variables

$x + 4 = 3y$

$\therefore f^{-1}(x) = \dfrac{x+4}{3}$ A1

[2]

(b) (i) The gradient of g is 3.

The equation of g:

$g(x) - 2 = 3(x - 0)$ (A1) for substitution

$g(x) - 2 = 3x$

$g(x) = 3x + 2$ A1

(ii) The gradient of h

$= \dfrac{-1}{3}$

$= -\dfrac{1}{3}$ (A1) for correct value

The equation of h:

$h(x) - (-1) = -\dfrac{1}{3}(x - 4)$ (A1) for substitution

$h(x) + 1 = -\dfrac{1}{3}x + \dfrac{4}{3}$

$h(x) = -\dfrac{1}{3}x + \dfrac{1}{3}$ A1

[5]

3. (a) L.H.S.

$= (2mn)^2 + (m^2 - n^2)^2$

$= 4m^2n^2 + m^4 - 2m^2n^2 + n^4$ M1

$= m^4 + 2m^2n^2 + n^4$ A1

$= (m^2 + n^2)^2$

$=$R.H.S.

$\therefore (2mn)^2 + (m^2 - n^2)^2 = (m^2 + n^2)^2$ AG

[2]

(b) Note that the width of the rectangle is $2mn$. R1

The area of the rectangle

$= (2mn)(m^2 - n^2)$

$= 2m^3n - 2mn^3$ A1

[2]

4. (a) The coordinates of the centre

$= \left(\frac{1+9}{2}, \frac{3+11}{2}, \frac{-2+2}{2} \right)$ (A1) for substitution

$= (5, 7, 0)$ A1

[2]

(b) The radius r

$= \frac{1}{2}\sqrt{(9-1)^2 + (11-3)^2 + (2-(-2))^2}$ (A1) for substitution

$= \frac{1}{2}\sqrt{144}$

$= 6$ A1

[2]

(c) The slant height l

$= \text{BC}$

$= \sqrt{(9-3)^2 + (11-7)^2 + (2-4)^2}$ (A1) for substitution

$= \sqrt{56}$

The exact total surface area

$= \pi r^2 + \pi rl$

$= \pi(6)^2 + \pi(6)(\sqrt{56})$ (A1) for substitution

$= (36 + 6\sqrt{56})\pi$ A1

[3]

5. (a) 0.35 A1

[1]

(b) If B is a subset of A, $\mathrm{P}(A \cap B) = 0.5$. (A1) for correct value

If $\mathrm{P}(A \cup B) = 1$:

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for setting equation

$1 = 0.7 + 0.5 - \mathrm{P}(A \cap B)$ (A1) for substitution

$\mathrm{P}(A \cap B) = 0.2$

$\therefore 0.2 \le \mathrm{P}(A \cap B) \le 0.5$ A1

[4]

6. (a) $g'(x) = \dfrac{\cos x}{\pi + \sin x}$

$g(x)$

$= \displaystyle\int \frac{\cos x}{\pi + \sin x} \mathrm{d}x$ (M1) for indefinite integral

Let $u = \pi + \sin x$

$\dfrac{\mathrm{d}u}{\mathrm{d}x} = \cos x \Rightarrow \mathrm{d}u = \cos x \mathrm{d}x$ (A1) for substitution

$= \displaystyle\int \frac{1}{u} \mathrm{d}u$ A1

$= \ln u + C$

$= \ln(\pi + \sin x) + C$

$\therefore 0 = \ln\left(\pi + \sin\left(\dfrac{\pi}{2}\right)\right) + C$ (M1) for substitution

$0 = \ln(\pi + 1) + C$

$C = -\ln(\pi + 1)$

$\therefore g(x) = \ln(\pi + \sin x) - \ln(\pi + 1)$ A1

[5]

(b) $\ln \dfrac{\pi}{\pi + 1}$ A1

[1]

Section B

7. (a) (i) $\log_2 64$

$= \log_2 2^6$ (A1) for correct approach

$= 6$ A1

(ii) $\log_3 \frac{1}{81}$

$= \log_3 3^{-4}$ (A1) for correct approach

$= -4$ A1

(iii) $\log_{125} 5$

$= \log_{125} 125^{\frac{1}{3}}$ (A1) for correct approach

$= \frac{1}{3}$ A1

(iv) $\frac{\ln 8}{\ln \sqrt{2}}$

$= \log_{\sqrt{2}} 8$ (A1) for correct approach

$= \log_{\sqrt{2}} (\sqrt{2})^6$ (A1) for correct approach

$= 6$ A1

[9]

(b) $\log_{ab} a + \log_{ab} b = \log_{ab} ab$ (A1) for correct approach

$\log_{ab} a - 10 = 1$ (A1) for correct approach

$\log_{ab} a = 11$ A1

[3]

(c) $\log_{\frac{c}{d}} c - \log_{\frac{c}{d}} d = \log_{\frac{c}{d}} \frac{c}{d}$ A1

$5 - \log_{\frac{c}{d}} d = 1$

$\log_{\frac{c}{d}} d = 4$ M1

$\therefore \left(\frac{c}{d}\right)^4 = d$ M1

$\frac{c^4}{d^4} = d$

$c^4 = d^5$ AG

[3]

8. (a) (i) $x = -3$ A1

(ii) $\{y : y \leq 8, y \in \mathbb{R}\}$ A1

[2]

(b) Let $f(x) = a(x-h)^2 + k$.

$f(x) = a(x-(-3))^2 + 8$ M1

$f(x) = a(x+3)^2 + 8$

$\therefore 0 = a(-5+3)^2 + 8$ M1

$0 = 4a + 8$

$-8 = 4a$

$a = -2$ A1

$\therefore f(x) = -2(x+3)^2 + 8$

$f(x) = -2(x^2 + 6x + 9) + 8$ M1

$f(x) = -2x^2 - 12x - 18 + 8$

$f(x) = -2x^2 - 12x - 10$

$f(x) = -2(x^2 + 6x + 5)$

$f(x) = -2(x+5)(x+1)$ AG

[4]

(c) (i) -1 A1

(ii) -10 A1

[2]

(d) $g(x)$

$= -f(x-p)+q$ M1

$= -\left[-2(x-p)^2 - 12(x-p) - 10\right] + q$ A1

$= -\left[-2(x^2 - 2px + p^2) - 12x + 12p - 10\right] + q$ M1

$= -\left[-2x^2 + 4px - 2p^2 - 12x + 12p - 10\right] + q$

$= 2x^2 - 4px + 2p^2 + 12x - 12p + 10 + q$

$= 2x^2 + (12-4p)x + (2p^2 - 12p + 10 + q)$

$g(x) = h(x)$ M1

$\therefore 2x^2 + (12-4p)x + (2p^2 - 12p + 10 + q)$

$= -4x + (4p + q - 30)$

$2x^2 + (16-4p)x + (2p^2 - 16p + 40) = 0$ A1

The discriminant

$= (16-4p)^2 - 4(2)(2p^2 - 16p + 40)$ M1

$= 256 - 128p + 16p^2 - 8(2p^2 - 16p + 40)$ A1

$= 256 - 128p + 16p^2 - 16p^2 + 128p - 320$

$= -64$

which is always negative for all values of p and q. R1

Thus, the graphs of g and h never intersect for all values of p and q. AG

[8]

9. (a) $f(x)=0$

$\frac{1}{\cos^2 x}-2\tan^2 x=0$

$\frac{1}{\cos^2 x}-\frac{2\sin^2 x}{\cos^2 x}=0$ (M1) for valid approach

$1-2\sin^2 x=0$

$1=2\sin^2 x$

$\sin^2 x=\frac{1}{2}$ (A1) for correct equation

$\sin x=\sqrt{\frac{1}{2}}$ or $\sin x=-\sqrt{\frac{1}{2}}$ (A1) for correct approach

$\therefore x=\frac{\pi}{4}$ or $x=\pi-\frac{\pi}{4}$

$\therefore a=\frac{\pi}{4},\ b=\frac{3\pi}{4}$ A1A1

[5]

(b) $f(x)=\frac{1-2\sin^2 x}{\cos^2 x}$

$f'(x)$

$=\frac{(\cos^2 x)\left[-2(2\sin x)(\cos x)\right]-(1-2\sin^2 x)\left[(2\cos x)(-\sin x)\right]}{(\cos^2 x)^2}$ (M1) for quotient rule

$=\frac{-4\sin x\cos^3 x-(1-2\sin^2 x)(-2\sin x\cos x)}{\cos^4 x}$ (A1) for correct approach

$=\frac{\sin x\cos x(-4\cos^2 x+2-4\sin^2 x)}{\cos^4 x}$

$=\frac{\sin x\cos x(-4(1)+2)}{\cos^4 x}$ (A1) for correct approach

$=-\frac{2\sin x}{\cos^3 x}$ A1

[4]

(c) $f'\left(\frac{\pi}{4}\right)$

$= -\frac{2\sin\frac{\pi}{4}}{\cos^3\frac{\pi}{4}}$ M1

$= -\frac{2\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)^3}$ A1

$= -\frac{2}{\frac{1}{2}}$

$= -4$ A1

The equation of the tangent at A:

$y - 0 = -4\left(x - \frac{\pi}{4}\right)$

$y = -4x + \pi$ A1

$\therefore \frac{1-2\sin^2 x}{\cos^2 x} = -4x + \pi$ M1

$\frac{\cos^2 x + \sin^2 x - 2\sin^2 x}{\cos^2 x} = -4x + \pi$ M1

$1 - \frac{\sin^2 x}{\cos^2 x} = -4x + \pi$

$1 - \tan^2 x = -4x + \pi$ A1

$1 - \pi + 4x - \tan^2 x = 0$ AG

[7]