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Chapter

1

Standard Form

SUMMARY POINTs

✓ Standard Form: A number in the form $(\pm)a\times10^{k}$, where $1\le a<10$ and k is an integer

Solutions of Chapter 1

1 Paper 1 – Express Quantities in Standard Form

Example

A rectangle is 3250 cm long and 2720 cm wide.

(a) Find the perimeter of the rectangle, giving your answer in the form $a\times10^k$, where $1\le a<10$ and $k\in\mathbb{Z}$.

[2]

(b) Find the area of the rectangle, giving your answer in the form $a\times10^k$, where $1\le a<10$ and $k\in\mathbb{Z}$.

[2]

Solution

(a) The required perimeter

$=2(3250+2750)$ (M1) for correct formula

$=12000$

$=1.2\times10^4$ cm A1 N2

[2]

(b) The required area

$=3250\times2750$ (M1) for correct formula

$=8937500$

$=8.9375\times10^6$ cm^2 A1 N2

[2]

Exercise 1

1. For this question, give all the answers correct to 3 significant figures.

The diameter of a circle is 1730 cm.

(a) Find the circumference of the circle, giving your answer in the form $a\times10^k$, where $1\le a<10$ and $k\in\mathbb{Z}$.

[2]

(b) Find the area of the circle, giving your answer in the form $a\times10^k$, where $1\le a<10$ and $k\in\mathbb{Z}$.

[2]

2. The base length and the height of a right-angled triangle are 3348 cm and 14880 cm respectively.

(a) Find the length of the hypotenuse of the triangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

(b) Find the area of the triangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

3. The base length and the area of a rectangle are 5476 cm and 22489932 cm^2 respectively.

(a) Find the height of the rectangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

(b) Find the length of the diagonal of the rectangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

4. The height and the area of a right-angled triangle are 8283 cm and 331320000 cm^2 respectively.

(a) Find the base length of the triangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

(b) Find the length of the hypotenuse of the triangle, giving your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$.

[2]

Chapter

2

Approximation and Error

SUMMARY POINTs

✓ Summary of rounding methods:

2.71828	Correct to 3 significant figures	Correct to 3 decimal places
Round off	2.7**2**	2.71**8**

✓ Consider a quantity measured as Q and correct to the nearest unit d:

$\frac{1}{2}d$: Maximum absolute error

$Q-\frac{1}{2}d \le A < Q+\frac{1}{2}d$: Range of the actual value A

$Q-\frac{1}{2}d$: Lower bound (Least possible value) of A

$Q+\frac{1}{2}d$: Upper bound of A

$\frac{\text{Maximum absolute error}}{Q}\times 100\%$: Percentage error

Solutions of Chapter 2

2 Paper 1 – Rounding and Percentage Error

Example

$A = \dfrac{(2\sin(z))(\sqrt{x+17})}{64xy^2}$, where $x = 10$, $y = 0.5$ and $z = 60^\circ$.

(a) Calculate the **exact** value of A.

[1]

(b) Give your answer to A correct to two significant figures.

[1]

(c) Write down an inequality representing the error interval of this estimate.

[2]

Casey estimates the value of A to be 0.055.

(d) Calculate the percentage error in Casey's estimate.

[2]

Solution

(a) 0.05625 A1 N1

[1]

(b) 0.056 A1 N1

[1]

(c) $0.0555 \le A < 0.0565$ A2 N2

[2]

(d) The percentage error

$= \left|\dfrac{0.055 - 0.05625}{0.05625}\right| \times 100\%$ (A1) for correct substitution

$= 2.222222222\%$

$= 2.22\%$ A1 N2

[2]

Exercise 2

1. $B=\dfrac{x\sqrt{y}}{\cos(90^\circ - z)}$, where $x=1.125$, $y=1.5625$ and $z=30^\circ$.

(a) Calculate the **exact** value of B.

[1]

(b) Give your answer to B correct to three significant figures.

[1]

(c) Write down an inequality representing the error interval of this estimate.

[2]

Julie estimates the value of B to be 2.84.

(d) Calculate the percentage error in Julie's estimate.

[2]

2. The lengths of the four sides of a quadrilateral are 5.278 cm, 4.812 cm, 4.118 cm and 3.756 cm respectively.

(a) Calculate the **exact** perimeter of the quadrilateral.

[2]

The lengths of all four sides are estimated by rounding off, correct to 1 decimal place.

(b) Write down the upper bound and the lower bound of the error interval of the estimate of the longest side.

[2]

(c) Calculate the percentage error in the estimate of the perimeter.

[2]

3. The dimensions of a rectangular snack box are 15.75 cm, 8.95 cm and 7.15 cm.

(a) Calculate the **exact** volume of the box.

[2]

The lengths of all sides of the box are estimated by rounding off, correct to the nearest cm.

(b) Write down the upper bound and the lower bound of the error interval of the estimate of the shortest side.

[2]

(c) Calculate the percentage error in the estimate of the volume.

[2]

4. For a farm in the shape of a right-angled triangle, the lengths of the hypotenuse and the shortest side are 39.063125 km and 10.937675 km respectively.

(a) Calculate L, the **exact** length of the remaining side.

[2]

The lengths of all sides of the farm are estimated by rounding off, correct to the nearest 0.01 km.

(b) Write down an inequality representing the error interval of the estimate of the remaining side.

[2]

(c) Calculate the percentage error in the estimate of the area.

[2]

Chapter

6

Systems of Equations

SUMMARY POINTs

✓ $\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$: 2×2 system

✓ $\begin{cases} ax+by+cz=d \\ ex+fy+gz=h \\ ix+jy+kz=l \end{cases}$: 3×3 system

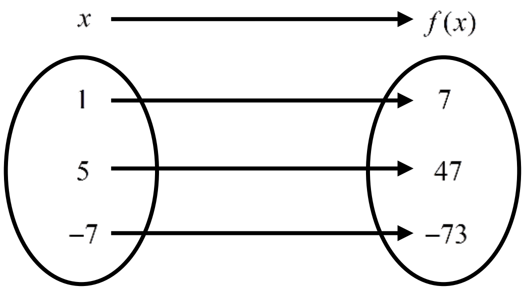
✓ The above systems can be solved by $\boxed{\text{PlySmlt2}}$ in TI-84 Plus CE

Solutions of Chapter 6

14 Paper 1 – Mapping Diagrams

Example

The mapping diagram below represents the function $f(x) = ax + b$.



(a) Write down two equations in terms of a and b.

[2]

(b) Find the values of a and b.

[2]

(c) Find the value of c when $f(c) = c$.

[2]

Solution

(a) $a + b = 7$ A1 N1

$5a + b = 47$ A1 N1

(or $-7a + b = -73$)

[2]

(b) $a = 10$, $b = -3$ A2 N2

[2]

(c) $c = 10c - 3$ (M1) for substitution

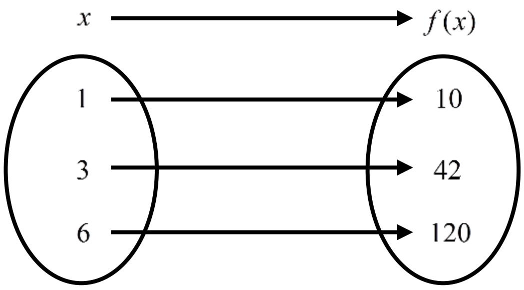
$-9c = -3$

$c = \frac{1}{3}$ A1 N2

[2]

Exercise 14

1. The mapping diagram below represents the function $f(x) = ax^2 + bx$.

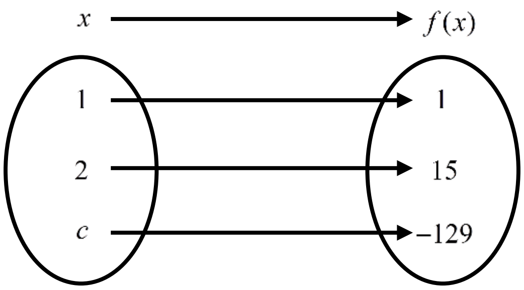


(a) Write down two equations in terms of a and b. [2]

(b) Find the values of a and b. [2]

(c) Find the equation of the axis of symmetry of f. [2]

2. The mapping diagram below represents the function $f(x) = ax^3 + b$.

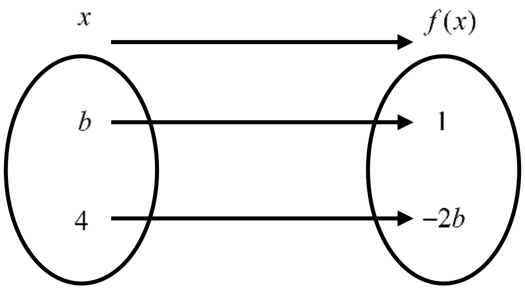


(a) Find the values of a and b. [4]

(b) Hence, find the value of c. [2]

6

3. The mapping diagram below represents the function $f(x)=\dfrac{a}{3-x}$.

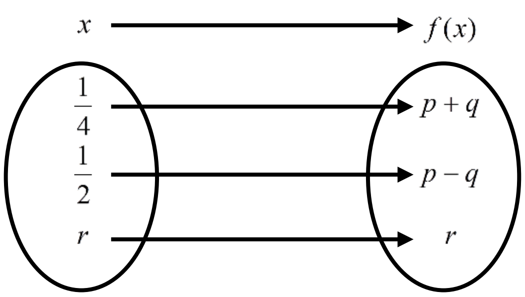


(a) Find the values of a and b.
[4]

(b) List the elements in the domain of f.
[1]

(c) Write down the equation of the horizontal asymptote.
[1]

4. The mapping diagram below represents the function $f(x)=\dfrac{1}{x^2}$.



(a) Find the values of p, q and r.
[6]

(b) List the elements in the range of f.
[1]

(c) Write down the equation of the vertical asymptote.
[1]

15 Paper 1 – 2×2 Systems

Example

3000 citizens attended a carnival. Let x be the number of adults attending the carnival and y be the number of children attending the carnival.

(a) Write down an equation in x and y.

[1]

The cost of an adult ticket and a child ticket were set to be USD 18 and USD 8 respectively. The total cost of tickets sold in the carnival was USD 36000.

(b) Write down another equation in x and y.

[1]

(c) Write down the values of x and y.

[2]

(d) Find the total cost for a group of 4 adults and 7 children.

[2]

Solution

(a) $x+y=3000$ A1 N1

[1]

(b) $18x+8y=36000$ A1 N1

[1]

(c) $x=1200$, $y=1800$ A2 N2

[2]

(d) The total cost

$=18(4)+8(7)$ (M1) for substitution

$=$ USD 128 A1 N2

[2]

6

Exercise 15

1. The Division 1 Football League in Japan has 30 matches for every participating team. The champion of the League last year is undefeated for all 30 matches. Let x and y be the number of wins and draws of the champion team respectively.

(a) Write down an equation in x and y.

[1]

A team gets 3 points for a win and 1 point for a draw. The champion team got 82 points last year.

(b) Write down another equation in x and y.

[1]

(c) Write down the values of x and y.

[2]

(d) Find the total points of an undefeated team if the number of wins and the number of draws are the same.

[2]

2. The length, L cm, of a heated copper plate is given by $L = a + bT$, where T °C is the temperature measured. At 190 °C, the length of the copper plate is 8.25 cm.

(a) Write down an equation in a and b.

[1]

At 220 °C, the length of the copper plate is 9.21 cm.

(b) Write down another equation in a and b.

[1]

(c) Write down the values of a and b.

[2]

(d) Find the temperature of the copper plate when its length is 98.5 mm.

[2]

3. The number of flats y can be modelled by the equation $y = pt + q$, where t is the number of years after 1 January, 2010.

On 1 January 2012, the number of flats is 18000. On 1 January 2017, the number of flats is 22000.

(a) Find the values of p and q.

[4]

(b) State the meaning of p in this context.

[1]

(c) State the meaning of q in this context.

[1]

4. Laurent can buy 10 CDs and 7 DVDs for USD 76.5, and he can buy 8 CDs and 11 DVDs for USD 90.9.

(a) Find the price of one CD and that of one DVD.

[4]

(b) Laurent wants to buy 7 CDs and 10 DVDs. Find the amount of change if he pays USD 100 at the counter.

[2]

16 Paper 1 – 3×3 Systems

Example

The total profit $\$y$ of selling plastic boxes can be modelled by the equation $y=ax^2+bx+c$, where x is the number of plastic boxes ordered from a factory, and a, b and c are real numbers. The total profits of selling 100 plastic boxes, 200 plastic boxes and 400 plastic boxes are \$122, \$424 and \$1628 respectively.

(a) (i) Show that $10000a+100b+c=122$.

(ii) Show that $40000a+200b+c=424$.

(iii) Write down the third equation in a, b and c.

[3]

(b) Hence, find the values of a, b and c.

[4]

Solution

(a) (i) $122=a(100)^2+b(100)+c$ A1

$10000a+100b+c=122$ AG N0

(ii) $424=a(200)^2+b(200)+c$ A1

$40000a+200b+c=424$ AG N0

(iii) $160000a+400b+c=1628$ A1 N1

[3]

(b) $\begin{cases}10000a+100b+c=122\\40000a+200b+c=424\\160000a+400b+c=1628\end{cases}$ (M1) for valid approach

$a=\dfrac{1}{100}$, $b=\dfrac{1}{50}$ and $c=20$ A3 N4

[4]

Exercise 16

1. The height h metres of a parachute t seconds after released from a helicopter can be modelled by $h = at^2 + bt + c$, and a, b and c are real numbers. The heights of a parachute at different times are shown in the following table:

t (s)	1	3	6
h (m)	998	982	928

(a) (i) Show that $a + b + c = 998$.

(ii) Show that $9a + 3b + c = 982$.

(iii) Write down the third equation in a, b and c.

[3]

(b) Hence, find the values of a, b and c.

[4]

2. 8400 citizens attended a concert. Let x, y and z be the number of children, the number of adults and the number of elderly people attending the concert respectively. It is also given that the sum of the number of children and the number of elderly people is less than the number of adults by 6288.

(a) (i) Write down the first equation in x, y and z.

(ii) Show that $x - y + z = -6288$.

[2]

The cost of a child ticket, an adult ticket and an elderly ticket were set to be \$42, \$84 and \$21 respectively. The total cost of tickets sold in the concert was \$655872.

(b) Write down the third equation in x, y and z.

[1]

(c) Hence, find the values of x, y and z.

[4]

3. The total price $\$P$ of selling x apples, y oranges and z pears can be modelled by $P = ax + by + cz$, where a, b and c are the unit prices of apples, oranges and pears respectively. The following table shows the total prices of different combinations of apples, oranges and pears:

$\$P$	x	y	z
\$150	10	12	13
\$178	14	8	19
\$230	22	23	7

(a) (i) Write down the system of three equations in a, b and c.

(ii) Hence, find the values of a, b and c.

[5]

(b) Find the total price of 30 apples and 35 pears.

[2]

4. In a football league, each team gains x points for a win, y points for a draw and z points for a loss. There are 46 matches in a year. The following table shows the performances of two teams in 2019:

Team	Win	Draw	Loss	Points
A	30	16	0	152
B	23	15	8	114
C	11	17	18	60

(a) (i) Write down the system of three equations in x, y and z.

(ii) Hence, find the values of x, y and z.

[5]

(b) Interpret the meaning of z.

[1]

Chapter

7

Arithmetic Sequences

SUMMARY POINTs

✓ Properties of an arithmetic sequence u_n:

1. u_1: First term
2. $d = u_2 - u_1 = u_n - u_{n-1}$: Common difference
3. $u_n = u_1 + (n-1)d$: General term (n th term)
4. $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right] = \frac{n}{2}\left[u_1 + u_n\right]$: The sum of the first n terms

✓ $\sum_{r=1}^{n} u_r = u_1 + u_2 + u_3 + \cdots + u_{n-1} + u_n$: Summation sign

Solutions of Chapter 7

17 Paper 1 – Finding u_n and S_n with Given Terms

Example

In an arithmetic sequence, $u_1 = 3$ and $u_4 = 15$.

(a) Find d.

[2]

(b) Find u_{30}.

[2]

(c) Find S_{30}.

[2]

Solution

(a) $d = \frac{u_4 - u_1}{3}$ (M1) for finding d

$d = \frac{15-3}{3}$

$d = 4$ A1 N2

[2]

(b) $u_{30} = u_1 + (30-1)d$

$u_{30} = 3 + (30-1)(4)$ (A1) for correct substitution

$u_{30} = 119$ A1 N2

[2]

(c) $S_{30} = \frac{30}{2}\left[2u_1 + (30-1)d\right]$

$S_{30} = \frac{30}{2}\left[2(3) + (30-1)(4)\right]$ (A1) for correct substitution

$S_{30} = 1830$ A1 N2

[2]

Exercise 17

1. In an arithmetic sequence, $u_1 = 27$ and $u_5 = -1$.

(a) Find d. [2]

(b) Find u_{25}. [2]

(c) Find S_{25}. [2]

2. In an arithmetic sequence, $u_1 = 3.5$ and $u_7 = 6.5$.

(a) Find d. [2]

(b) Find u_{42}. [2]

(c) Find S_{84}. [2]

3. In an arithmetic sequence, the second term is 0 and the tenth term is 24.

(a) Find the common difference. [2]

(b) Find the fourth term. [2]

(c) Find the sum of the first ten terms of the sequence. [2]

4. In an arithmetic sequence, the third term is $-\frac{2}{3}$ and the eighth term is $-\frac{22}{3}$.

(a) Find the common difference. [2]

(b) Find the eleventh term. [2]

(c) Find the sum of the first forty terms of the sequence. [3]

18 Paper 1 – Real Life Problems

Example

Josie pays money into a saving scheme each year, for 50 years. In the first year she pays \$4100, and her payments increases by the same amount each year. It is given that she pays \$4350 in the second year.

(a) Find the amount that Josie will pay in the 15th year.

[3]

(b) Find the exact total amount that Josie will pay in over the 50 years.

[3]

Solution

(a)	$d = 4350 - 4100$	(M1) for finding d
	$d = 250$	
	The required amount	
	$= u_{15}$	(M1) for valid approach
	$= 4100 + (15-1)(250)$	
	$= \$7600$	A1 N3
		[3]
(b)	The total amount	
	$= S_{50}$	(M1) for valid approach
	$= \frac{50}{2}\left[2(4100) + (50-1)(250)\right]$	(A1) for substitution
	$= \$511250$	A1 N3
		[3]

Exercise 18

1. The lengths of the sides of a 20-sided polygon form an arithmetic sequence. The two shortest sides are 1.5 m and 1.9 m respectively.

(a) Find the length of the longest side of the polygon.

[3]

(b) Find the perimeter of the polygon.

[3]

2. Consider the following sequence of figures.

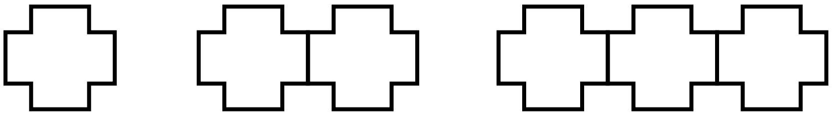


Figure 1 Figure 2 Figure 3

Figure 1 contains 12 edges and Figure 2 contains 23 edges.

(a) Given that Figure n contains 221 edges, find n.

[3]

(b) Find the total number of edges of Figure 18, Figure 19 and Figure 20.

[3]

3. Workers are drilling for oil in a large-scale project in Saudi Arabia. The costs of oil drilling to a depth of 10 m and a depth of 20 m are \$7500 and \$9300 respectively. It costs an extra constant amount for every subsequent extra depth of 10 m.

(a) Find the cost of drilling to a depth of 100 m.

[3]

(b) The budget of the large-scale project is \$340000. Find the greatest possible depth to be drilled.

[4]

4. In the theatre of a town hall, there are 24 rows in total. The number of seats in each row forms an arithmetic sequence. The number of seats in the second row and the third row are 30 and 32 respectively.

(a) Find the number of seats in the 24th row.

[3]

(b) The price of a ticket in the theatre is \$75. Find the total income of the theatre if all tickets are sold.

[4]

19 Paper 2 – Miscellaneous Problems

Example

In an arithmetic sequence, $S_{30} = 5430$ and $u_3 = 31$. Let u_1 and d be the first term and the common difference of the arithmetic sequence respectively.

(a) (i) Write down the first equation connecting u_1 and d.

(ii) Write down the second equation connecting u_1 and d.

(iii) Hence, write down the values of u_1 and d.

[6]

(b) Find u_n in terms of n.

[2]

(c) Find the smallest value of n such that $u_n > 888$.

[3]

(d) Find S_n in terms of n.

[2]

Solution

(a) (i) $S_{30} = 5430$

$\frac{30}{2}\left[2u_1 + (30-1)d\right] = 5430$ (M1) for valid approach

$30u_1 + 435d = 5430$ A1 N2

(ii) $u_3 = 31$

$u_1 + (3-1)d = 31$ (M1) for valid approach

$u_1 + 2d = 31$ A1 N2

(iii) $u_1 = 7$ A1 N1

$d = 12$ A1 N1

[6]

(b) $u_n = 7 + (n-1)(12)$ (M1) for valid approach

$u_n = 12n - 5$ A1 N2

[2]

(c) $u_n > 888$

$12n - 5 > 888$ (M1) for setting inequality

$12n > 893$

$n > \frac{893}{12}$ (A1) for correct value

Thus, the smallest value of n is 75. A1 N3

[3]

(d) $S_n = \frac{n}{2}[2(7) + (n-1)(12)]$ (M1) for valid approach

$S_n = \frac{n}{2}(12n + 2)$

$S_n = 6n^2 + n$ A1 N2

[2]

Exercise 19

7

1. In an arithmetic sequence, $u_4 = 43$ and $S_{80} = -5320$. Let u_1 and d be the first term and the common difference of the arithmetic sequence respectively.

(a) (i) Write down the first equation connecting u_1 and d.

(ii) Write down the second equation connecting u_1 and d.

(iii) Hence, write down the values of u_1 and d.

[6]

(b) Find u_n in terms of n.

[2]

(c) Find the greatest value of n such that $u_n > 0$.

[3]

(d) Find S_n in terms of n.

[2]

(e) Find the value of n such that $S_n = -4425$.

[3]

2. In an arithmetic sequence, $u_{11} = 17$ and $S_{96} = 4512$. Let u_1 and d be the first term and the common difference of the arithmetic sequence respectively.

(a) (i) Write down the first equation connecting u_1 and d.

(ii) Write down the second equation connecting u_1 and d.

(iii) Hence, write down the values of u_1 and d.

[6]

(b) Find u_n in terms of n.

[2]

(c) Find the greatest value of n such that $u_n < 147$.

[3]

(d) Find $u_{49} + u_{50} + u_{51} + \cdots + u_{95} + u_{96}$.

[3]

(e) Find the value of n such that $S_n = 4949$.

[3]

3. In an arithmetic sequence, the general term is given by $u_n = 63 - 3n$, and the sum of the first n terms is given by S_n.

(a) (i) Write down the values of u_1 and u_2.

(ii) Hence, write down the common difference of the sequence.

[3]

(b) Find the smallest value of n such that $u_n \leq -21$.

[3]

(c) (i) Write down the values of S_1.

(ii) Find S_n in terms of n.

[3]

(d) Find the value of n such that $S_n = 0$.

[3]

(e) Find the value of $\sum_{r=11}^{20} u_r$.

[3]

4. In an arithmetic sequence, the general term is given by u_n, and the sum of the first n terms is given by $S_n = \dfrac{n^2 + 19n}{28}$.

(a) (i) Write down the values of S_1 and S_2.

(ii) Hence, write down the values of u_1 and u_2.

(iii) Find the common difference of the sequence.

[6]

(b) Find the smallest value of n such that $\sum_{r=1}^{n} u_r > 100$.

[3]

(c) Find the value of $\sum_{r=50}^{100} u_r$.

[3]

(d) Find u_n in terms of n.

[2]

(e) Find the value of n such that $u_n + 2u_{n+1} = \dfrac{58}{7}$.

[3]

Chapter

8

Geometric Sequences

SUMMARY POINTs

✓ Properties of a geometric sequence u_n:

1. u_1: First term
2. $r = u_2 \div u_1 = u_n \div u_{n-1}$: Common ratio
3. $u_n = u_1 \times r^{n-1}$: General term (nth term)
4. $S_n = \dfrac{u_1(1-r^n)}{1-r}$: The sum of the first n terms

Solutions of Chapter 8

20 Paper 1 –Finding u_n and S_n with given terms

Example

The first three terms of a geometric sequence are 54, 18 and 6.

(a) Write down the value of r, the common ratio of the geometric sequence.

[1]

(b) Find u_7.

[2]

(c) Find the sum of the first 10 terms of this sequence, giving the answer correct to four decimal places.

[3]

Solution

(a) $r = \dfrac{1}{3}$ A1 N1

[1]

(b) $u_7 = u_1 \cdot r^{7-1}$

$u_7 = 54 \cdot \left(\dfrac{1}{3}\right)^{7-1}$ (A1) for substitution

$u_7 = \dfrac{2}{27}$ A1 N2

[2]

(c) $S_{10} = \dfrac{u_1(1-r^{10})}{1-r}$ (M1) for valid approach

$S_{10} = \dfrac{54\left(1-\left(\dfrac{1}{3}\right)^{10}\right)}{1-\dfrac{1}{3}}$ (A1) for substitution

$S_{10} = 80.99862826$

$S_{10} = 80.9986$ A1 N3

[3]

8

Exercise 20

1. The first three terms of a geometric sequence are 1024, 256 and 64.

(a) Write down the value of r, the common ratio of the geometric sequence. [1]

(b) Find u_8. [2]

(c) Find the sum of the first twelve terms of this sequence, giving the answer correct to the nearest integer. [3]

2. The first three terms of a geometric sequence are $u_1 = 576$, $u_2 = 768$ and $u_3 = 1024$.

(a) Write down the value of r. [1]

(b) Find the value of $\sum_{n=1}^{7} u_n$, giving the answer correct to the nearest integer. [3]

(c) Find the value of n such that $243u_n = 1048576$. [3]

3. The first three terms of a geometric sequence are $u_1 = 1.024$, $u_2 = 1.28$ and $u_3 = 1.6$.

(a) Find the value of r. [2]

(b) Find the least value of n such that $u_n > 5$. [3]

(c) Find the value of $\sum_{n=1}^{10} u_n$. [2]

4. The first three terms of a geometric sequence are $u_1 = 1.5$, $u_2 = 2.4$ and $u_3 = 3.84$.

(a) Find the value of r. [2]

(b) Find the value of the eighth term of the sequence, giving the answer correct to two decimal places. [2]

(c) Find the greatest value of n such that $S_n < 100$. [3]

21 Paper 1 – Real Life Problems

Example

A population of bees is growing at a rate of 6% per year. There were 160 bees at the beginning of 2002.

(a) Find the number of bees at the beginning of 2006.

[2]

(b) Find the number of bees n complete years after the beginning of 2002.

[2]

(c) It is assumed that each bee can produce 100 units of honey per year. Find the total amount of honey produced in a complete six-year period after the beginning of 2002.

[3]

Solution

(a) The number of bees

$=u_5$ (M1) for valid approach

$=160\times(1+6\%)^{5-1}$

$=201.9963136$

$=202$ A1 N2

[2]

(b) The number of bees

$=u_n$ (M1) for valid approach

$=160\times(1+6\%)^{n-1}$

$=160\times1.06^{n-1}$ A1 N2

[2]

(c) The total amount of honey

$=100S_6$ (M1) for valid approach

$=100\left(\dfrac{160(1-1.06^6)}{1-1.06}\right)$ (A1) for substitution

$=111605.0966$

$=112000$ units A1 N3

[3]

8

Exercise 21

1. The lengths of the sides of a 10-sided polygon form a geometric sequence. The three shortest sides are 1 m, 1.1 m and 1.21 m respectively.

(a) Write down the exact length of the fourth shortest side of the polygon.
[1]

(b) Find the length of the longest side of the polygon.
[2]

(c) Find the perimeter of the polygon.
[3]

2. In the theatre of a town hall, there are 12 rows in total. Each row consists of 30 seats. The price of a seat in the first row is $100. The prices of a seat in the following rows continue to decrease in the same ratio, such that the price of a seat in the second row is $90, and the price of a seat in the third row is $81, and so on. Give the answers correct to the nearest dollar in this question.

(a) Find the price of a seat in the sixth row.
[2]

(b) Find the price of a seat in the n th row.
[2]

(c) Find the total income of the theatre if all tickets are sold.
[3]

3. In a souvenir market in Khabarovsk located in East Russia, a set of 20 wooden dolls is displayed in a row and arranged in descending doll sizes. The volume of the largest doll is 24000 cm^3. The volume of the second largest doll is 95% of the previous doll, and this pattern continues.

(a) Find the total volume of all wooden dolls when they are displayed in a single row.
[3]

(b) Find the number of wooden dolls with volume less than 10000 cm^3.
[3]

4. A car is travelling in a journey. On 1st February the car travels 120 km. The distance travelled is expected to decrease by 10% each day.

(a) Find the distance travelled on the fifth day.
[2]

(b) Find the difference between the distance travelled on the fifth day and the one on the sixth day.
[2]

(c) The car needs to refuel for every 500 km travelled. Find the date when the car refuels for the second time of this journey.
[3]

22 Paper 2 – Arithmetic and Geometric Sequences

Example

Consider an arithmetic sequence $u_1, u_2, u_3, \ldots, u_n, \ldots$ where $u_1 = 264$, $u_2 = 276$ and $u_3 = 288$.

(a) Find the value of u_{17}.

[2]

(b) Find the sum of the first 23 terms of the sequence.

[3]

Now consider the sequence $v_1, v_2, v_3, \ldots, v_n, \ldots$ where $v_1 = \frac{40}{729}$, $v_2 = \frac{40}{243}$ and $v_3 = \frac{40}{81}$.

This sequence continues in the same manner.

(c) Find the value of v_{13}.

[2]

(d) Find the sum of the first 13 terms of this sequence, giving the answer correct to the nearest integer.

[3]

Let k be a positive integer such that $u_k = v_k$.

(e) Calculate the value of k.

[3]

Solution

(a) u_{17}

$= u_1 + (17-1)d$ (M1) for valid approach

$= 264 + (17-1)(12)$

$= 456$ A1 N2

[2]

(b) The sum of the first 23 terms

$= \frac{23}{2}[2u_1 + (23-1)d]$ (M1) for valid approach

$= \frac{23}{2}[2(264) + (23-1)(12)]$ (A1) for substitution

$= 9108$ A1 N2

[3]

8

(c) v_{13}

$= v_1 \times r^{13-1}$ (M1) for valid approach

$= \frac{40}{729} \times 3^{13-1}$

$= 29160$ A1 N2

[2]

(d) The sum of the first 13 terms

$= \frac{v_1(1-r^{13})}{1-r}$ (M1) for valid approach

$= \frac{\frac{40}{729}(1-3^{13})}{1-3}$ (A1) for substitution

$= 43739.97257$

$= 43740$ A1 N2

[3]

(e) $u_k = v_k$

$264 + (k-1)(12) = \frac{40}{729} \times 3^{k-1}$ (M1) for setting equation

$264 + 12k - 12 = \frac{40}{729} \times 3^{k-1}$

$252 + 12k - \frac{40}{729} \times 3^{k-1} = 0$ (A1) for correct equation

By considering the graph of

$y = 252 + 12k - \frac{40}{729} \times 3^{k-1}$, $k = 9$. A1 N2

[3]

Exercise 22

1. Kensuke purchased a new car for 24000 EUR on 1st January, 2011 and insures his car with an insurance company. In 2011, Kensuke needs to pay 1200 EUR for the insurance premium, and the amount he needs to pay is reduced by 15 EUR per year.

(a) Find the amount of insurance premium Kensuke has paid in 2014.

[2]

The insurance company also estimates the value of Kensuke's car in each year. The company estimates that the car depreciates by 15% each year, such that the value of his car is 20400 EUR in 2012.

(b) Find the exact value of the car in 2016.

[2]

(c) Find the year when the value of the car is first below 8000 EUR.

[3]

Kensuke will stop insuring his car if the amount of insurance premium is greater than the value of the car in a particular year.

(d) Find the year when Kensuke stops insuring his car.

[3]

(e) Hence, find the total amount of insurance premium Kensuke has paid in this period.

[3]

2. The table below shows the first four terms of four sequences: t_n, u_n, v_n and w_n.

n	1	2	3	4
t_n	50	100	200	400
u_n	50	70	110	170
v_n	50	1050	2050	3050
w_n	50	50	50	50

(a) State the sequence which is

(i) arithmetic;

(ii) geometric;

(iii) neither arithmetic nor geometric;

(iii) both arithmetic and geometric.

[4]

8

(b) For the arithmetic sequence,

(i) Find the value of the 100th term.

(ii) Find the sum of the first 25 terms.

[5]

(c) For the geometric sequence,

(i) Find the value of the 7th term.

(ii) Find the sum of the first 14 terms.

[5]

(d) The first m terms of the arithmetic sequence are not less than the first m terms of the geometric sequence, find the greatest value of m.

[3]

3. Giselle joins a training program in a sports centre. In the first training session Giselle runs a distance of 2400 metres. She increases her running distance by 200 metres after each training session.

(a) Find Giselle's running distance in the nth training session.

[2]

(b) In the xth training session, Giselle will first run further than 10 kilometres. Find the value of x.

[3]

(c) Find the total running distance in the first sixteen training sessions, giving the answer in the form $a \times 10^k$ km.

[3]

Giselle's best friend, Helena, followed Giselle to join the same training program, but due to different fitness conditions their running distances in each session can be different. In the first training session Helena runs a distance of 2000 metres. She increases her running distance by 5% after each training session.

(d) Find Helena's running distance in the 10th training session.

[2]

(e) After the wth training session, the total running distance of Giselle in the training program is less than that of Helena at the first time. Find the value of w.

[4]

4. A new coffee shop starts its business on 1st March, 2017. Its profit is EUR 2000 in the first month and increases by 15% every month.

(a) Find the coffee shop's profit in November 2017.

[2]

(b) Find the coffee shop's total profit in 2017.

[3]

On 1st March, 2017 a new fast food shop nearby starts its business as well. Its profit is EUR 4000 in the first month and increases by EUR 1100 every month.

(c) Find the fast food shop's profit in June 2018.

[2]

(d) The fast food shop's profit first exceeds EUR 30000 in the m th month. Find the value of m.

[3]

(e) The fast food shop's total profit is less than the coffee shop's total profit for the first time in the n th month. Find the value of n.

[4]

8

Chapter

9

Financial Mathematics

SUMMARY POINTs

✓ Compound Interest:

PV : Present value

$r\%$: Interest rate per annum (per year)

n: Number of years

k: Number of compounded periods in one year

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$: Future value

$I = FV - PV$: Interest

✓ Inflation:

$i\%$: Inflation rate

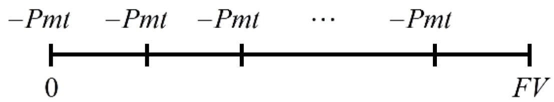
$R\%$: Interest rate compounded yearly

$(R-i)\%$: Real rate

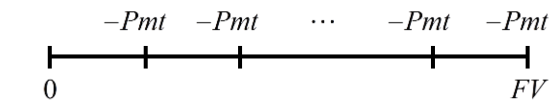
SUMMARY POINTs

✓ Annuity:

1. Payments at the beginning of each year

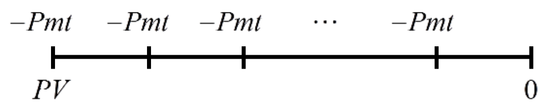


2. Payments at the end of each year

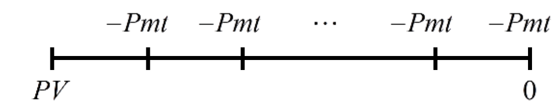


✓ Amortization:

1. Payments at the beginning of each year



2. Payments at the end of each year



Solutions of Chapter 9

23 Paper 1 – Compound Interest

Example

For this question, give all the answers correct to the nearest USD.

24000 USD is invested for 5 years at a nominal annual interest rate of 6%, compounded yearly.

(a) Find the value of P, the amount of money after 5 years.

[3]

(b) An amount of money Q is invested for 5 years at a nominal annual interest rate of 6%, compounded half-yearly. The amount of money after 5 years is P. Find the value of Q.

[3]

Solution

(a) $P=24000\left(1+\frac{6}{100}\right)^5$ (M1)(A1) for substitution

$P=32117.41386$

$P=32117\text{ USD}$ A1 N3

$\boxed{\begin{array}{l}\text{By TVM Solver:}\\ \text{N}=5\\ \text{I\%}=6\\ \text{PV}=-24000\\ \text{PMT}=0\\ \text{FV}=?\\ \text{P/Y}=1\\ \text{C/Y}=1\\ \text{PMT: END}\end{array}}$ (M1)(A1) for correct values

$P=32117\text{ USD}$ A1 N3

[3]

(b) $Q\left(1+\frac{6}{(100)(2)}\right)^{(2)(5)}=32117.41386$ (M1)(A1) for correct equation

$Q(1.03)^{10}=32117.41386$

$Q=23898.37222$

$Q=23898\text{ USD}$ A1 N3

$$\boxed{\begin{array}{l} \text{By TVM Solver:} \\ \text{N} = 5 \\ \text{I\%} = 6 \\ \text{PV} = ? \\ \text{PMT} = 0 \\ \text{FV} = 32117.41386 \\ \text{P / Y} = 1 \\ \text{C / Y} = 2 \\ \text{PMT : END} \end{array}}$$

(M1)(A1) for correct values

$Q = 23898$ USD A1 N3

[3]

Exercise 23

1. For this question, give all the answers correct to the nearest 100 EUR.

Aaron invested 360000 EUR in an account that pays a nominal annual interest rate of 3%, compounded half-yearly.

(a) Find the value of P, the amount of money after 8 years.

[3]

(b) An amount of money Q is invested for 8 years at a nominal annual interest rate of 3%, compounded monthly. The amount of money after 8 years is P. Find the value of Q.

[3]

2. \$125000 is invested for 12 years at a nominal annual interest rate of 8%, compounded yearly.

(a) Find the amount of money after 12 years, give the answer correct to the nearest 1000 dollars.

[3]

(b) Find the minimum number of complete years required for the amount of money to be doubled.

[3]

3. $\$P$ is invested for 5 years at a nominal annual interest rate of 4%, compounded quarterly. The amount of money after 5 years is 87000 AUD.

(a) Find the value of P, give the answer correct to the nearest AUD.

[3]

(b) Find the minimum number of complete years required for the amount of money to be $\$2.5P$.

[3]

4. \$640000 is invested for t_1 years at a nominal annual interest rate of 5%, compounded half-yearly. The amount after t_1 years is doubled.

Also, \$640000 is invested for t_2 years at a nominal annual interest rate of 5%, compounded quarterly. The amount after t_2 years is doubled.

Find the value of $t_1 - t_2$.

[6]

24 Paper 1 – Equivalent Rates

Example

360000 USD is invested for 6 years at a nominal annual interest rate of 4%, compounded yearly.

(a) Find the value of P, the amount of money after 6 years, give the answer correct to the nearest USD.

[3]

(b) 360000 USD is invested for 6 years at a nominal annual interest rate of $r\%$, compounded half-yearly. The amount of money after 6 years is P. Find the value of r.

[3]

Solution

(a) $P = 360000\left(1+\frac{4}{100}\right)^6$ (M1)(A1) for substitution

$P = 455514.8467$

$P = 455515\ \text{USD}$ A1 N3

$$\boxed{\begin{array}{l}\text{By TVM Solver:}\\ \text{N} = 6\\ \text{I\%} = 4\\ \text{PV} = -360000\\ \text{PMT} = 0\\ \text{FV} = ?\\ \text{P/Y} = 1\\ \text{C/Y} = 1\\ \text{PMT: END}\end{array}}$$

(M1)(A1) for correct values

$P = 455515\ \text{USD}$ A1 N3

[3]

(b) $360000\left(1+\frac{r}{(100)(2)}\right)^{(2)(6)} = 455514.8467$ (M1)(A1) for correct equation

$360000\left(1+\frac{r}{200}\right)^{12} - 455514.8467 = 0$

By considering the graph of

$y = 360000\left(1+\frac{r}{200}\right)^{12} - 455514.8467$,

$r = 3.9607805$.

9

Thus, $r = 3.96$. A1 N3

By TVM Solver:
N = 6
I% = ?
PV = −360000
PMT = 0
FV = 455514.8467
P / Y = 1
C / Y = 2
PMT : END

(M1)(A1) for correct values

Thus, $r = 3.96$. A1 N3

[3]

Exercise 24

1. Ben invested 54000 EUR in an account that pays a nominal annual interest rate of 6%, compounded monthly.

(a) Find the value of P, the amount of money after 10 years, give the answer correct to the nearest 1000 EUR.

[3]

(b) 54000 EUR is invested for 10 years at a nominal annual interest rate of $r\%$, compounded quarterly. The amount of money after 10 years is P. Find the value of r.

[3]

2. An amount of money is invested for 7 years at a nominal annual interest rate of 9%, compounded half-yearly. The amount of money after 7 years is $1600000.

(a) Find the value of P, the original amount of money invested, give the answer correct to the nearest 10000 dollars.

[3]

(b) P is invested for n years at a nominal annual interest rate of 9%, compounded yearly. The amount of money after n years is $1600000. Find the value of n.

[3]

3. Two equal amounts of money are invested in two different bank accounts, one at a nominal annual interest rate of 12%, compounded quarterly, for 4 years, with the another one at a nominal annual interest rate of 12%, compounded monthly, for n years. The amounts in both accounts at the end of the investments are equal. Find the value of n.

[4]

4. Two equal amounts of money are invested in two different bank accounts, one at a nominal annual interest rate of 5%, compounded half-yearly, for 8 years, with the another one at a nominal annual interest rate of 5%, compounded k times per year, for 7.98 years. The amounts in both accounts at the end of the investments are equal. Find the value of k.

[4]

25 Paper 1 – Rate of Inflation

Example

\$170000 is invested for 8 years at a nominal annual interest rate of $r\%$, compounded yearly. The amount of money after 8 years is \$260000.

(a) Find the value of r.

[3]

It is given that the rate of inflation during these 8 years is 2% per year.

(b) Write down the value of the real interest rate.

[1]

(c) Hence, find the real value of amount of money after 8 years.

[2]

Solution

(a) $260000 = 170000\left(1+\frac{r}{100}\right)^8$ (M1)(A1) for correct equation

$170000\left(1+\frac{r}{100}\right)^8 - 260000 = 0$

By considering the graph of

$y = 170000\left(1+\frac{r}{100}\right)^8 - 260000$,

$r = 5.454606$.

Thus, $r = 5.45$. A1 N3

$$\boxed{\begin{array}{l}\text{By TVM Solver:}\\ \text{N}=8\\ \text{I\%}=?\\ \text{PV}=-170000\\ \text{PMT}=0\\ \text{FV}=260000\\ \text{P/Y}=1\\ \text{C/Y}=1\\ \text{PMT: END}\end{array}}$$

(M1)(A1) for correct values

Thus, $r = 5.45$. A1 N3

[3]

(b) 3.45% A1 N1

[1]

(c) The real value of amount of money

9

$= 170000\left(1+\dfrac{3.454606}{100}\right)^{8}$ (A1) for substitution

$= 223073.2887$

$= \$223000$ A1 N2

$$\boxed{\begin{array}{l}\text{By TVM Solver:}\\ \text{N}=8\\ \text{I\%}=3.454606\\ \text{PV}=-170000\\ \text{PMT}=0\\ \text{FV}=?\\ \text{P/Y}=1\\ \text{C/Y}=1\\ \text{PMT: END}\end{array}}$$

(A1) for correct values

Thus, the real value is \$223000. A1 N2

[2]

Exercise 25

1. An amount of money $\$P$ is invested for 4 years at a nominal annual interest rate of 7%, compounded yearly. The amount of money after 4 years is \$300000.

(a) Find the value of P, the original amount of money invested.

[3]

It is given that the rate of inflation during these 4 years is 1.6% per year.

(b) Write down the value of the real interest rate.

[1]

(c) Hence, find the real value of amount of money after 4 years.

[2]

2. Ciana invested 8500 EUR in an account that pays a nominal annual interest rate of 12%, compounded monthly. This amount is invested for 9 years and the inflation rate in these 9 years is 1.8%.

(a) Find the real interest rate per year.

[4]

(b) Find the real value of amount of interest incurred after 9 years.

[3]

3. An amount of money \$2800 is invested for 12 years at a nominal annual interest rate of 4%, compounded yearly. After the rate of inflation is considered, the real value of amount of money after 12 years is \$4000.

(a) Find the value of the real interest rate per year.

[3]

(b) Find the rate of inflation per year.

[2]

4. Debby invested 14500 USD in an account that pays a nominal annual interest rate of 9.2%, compounded quarterly. This amount is invested for 8 years and the inflation rate in these 8 years is $i\%$.

(a) Find the real interest rate per year, giving the answer in terms of i and correct to 4 decimal places.

[4]

It is given that the real value of amount of money after 8 years is 18500 USD.

(b) Using the answer in (a), find the value of i.

[3]

9

26 Paper 1 – Annuities

Example

Kevin is going to create an annuity fund which will pay him a monthly allowance of 2000 USD for 40 years after he is retired. In the fund, interest is earned 6% per year, compounded yearly.

(a) Find the value of the annuity fund that has to be saved, giving the answer correct to the nearest USD.

[3]

(b) Find the amount of payment needed per year if Kevin wants to save his money in his fund for 30 years.

[3]

Solution

(a) By TVM Solver:

$$\boxed{\begin{array}{l} N = 40\times 12 \\ I\% = 6 \\ PV = ? \\ PMT = 2000 \\ FV = 0 \\ P/Y = 12 \\ C/Y = 1 \\ PMT : END \end{array}}$$

(M1)(A1) for correct values

$PV = -370397.2097$

Thus, the value of the annuity fund that has to be saved is 370937 USD. A1 N3

[3]

(b) By TVM Solver:

$$\boxed{\begin{array}{l} N = 30 \\ I\% = 6 \\ PV = 0 \\ PMT = ? \\ FV = 370937.2097 \\ P/Y = 1 \\ C/Y = 1 \\ PMT : END \end{array}}$$

(M1)(A1) for correct values

$PMT = -4691.951934$

Thus, the amount of payment needed per year is 4690 USD. A1 N3

[3]

Exercise 26

1. An annuity fund is created and it requires regular payments at the beginning of every year for 20 years. The value of the fund at the end of 20 years is $60000 and the interest is earned 7.5% per year.

(a) Find the value of the regular payment per year.

[3]

(b) If the value of the regular payment per year is increased by $500, find the number of years required for the investment.

[3]

2. Simon wants to accumulate an amount of money in an education fund at the end of 10 years. He deposits $1000 at the end of each month in the first five years, and deposits $1500 at the end of each month in the last five years. The interest is earned 3% per year.

(a) Find the value of the investment after five years.

[3]

(b) Find the value of the investment after ten years.

[3]

3. Teddy wants to accumulate an amount of money in an investment plans. He deposits $300 at the end of March, June, September and December every year. The interest is earned 5% per year.

9

(a) Let P be the value of the investment after fifteen years. Find the value of P.

[3]

(b) If Teddy wants to adjust the amount of deposit such that the value of the investment after thirty years is $3.5P$, find the new amount of deposit.

[3]

4. Consider the following table of the payments of the annuities X and Y, at the beginning of each month:

	Annuity X	Annuity Y
1st to 8th year	100	p
9th to 16th year	200	p

It is given that the values of the investment after sixteen years for both annuities are the same. The interest is earned 2.9% per year for both annuities.

(a) Find the value of the investment after eight years for annuity X.

[3]

(b) Find the value of the investment after sixteen years for annuity X.

[3]

(c) Hence, find the value of p.

[2]

27 Paper 2 – Amortization

Example

Young Jae is going to buy a flat. He is suggested to choose one of the two options to repay the loan of 250000 USD with a nominal annual interest rate of 9%:

Option 1: A total of 240 equal monthly payments have to be paid at the end of each month
Option 2: A monthly payment of 2000 USD has to be paid at the end of each month until the loan is fully repaid

(a) If Young Jae selects the option 1, find

(i) the amount of monthly payment,

(ii) the total amount to be paid,

(iii) the amount of interest paid.

[7]

(b) If Young Jae selects the option 2, find

(i) the number of months to repay the loan,

(ii) the total amount to be paid,

(iii) the amount of interest paid.

[7]

(c) By considering the amounts of monthly payment in both options, state the better option for Young Jae and explain the answer.

[2]

(d) By considering the amounts of interest paid in both options, state the better option for Young Jae and explain the answer.

[2]

Solution

(a) (i) By TVM Solver:

$$\boxed{\begin{array}{l} N = 240 \\ I\% = 9 \\ PV = 250000 \\ PMT = ? \\ FV = 0 \\ P/Y = 12 \\ C/Y = 1 \\ PMT: END \end{array}}$$

(M1)(A1) for correct values

$PMT = -2193.157954$

Thus, the amount of monthly payment is 2190 USD. A1 N3

(ii) The total amount to be paid

$= (2193.157954)(240)$ (M1) for valid approach

$= 526357.909$

$= 526000$ USD A1 N2

(iii) The amount of interest paid

$= 526357.909 - 250000$ (M1) for valid approach

$= 276357.909$

$= 276000$ USD A1 N2

[7]

(b) (i) By TVM Solver:

$$\boxed{\begin{array}{l} N = ? \\ I\% = 9 \\ PV = 250000 \\ PMT = -2000 \\ FV = 0 \\ P/Y = 12 \\ C/Y = 1 \\ PMT: END \end{array}}$$

(M1)(A1) for correct values

$N = 321.9090083$

Thus, the number of months to repay the loan is 322 months. A1 N3

(ii) The total amount to be paid

$= (2000)(322)$ (M1) for valid approach

$= 644000$ USD A1 N2

(iii) The amount of interest paid
$= 644000 - 250000$ (M1) for valid approach
$= 394000$ USD A1 N2

[7]

(c) The amount of monthly payment in option 2 is less than that in option 1. R1
Thus, the option 2 is better. A1 N0

[2]

(d) The amount of interest paid in option 1 is less than that in option 2. R1
Thus, the option 1 is better. A1 N0

[2]

Exercise 27

1. Takumi is going to purchase a boat. He is suggested to choose one of the two options to repay the loan of $1900000:

Option 1: A total of 144 equal monthly payments have to be paid at the end of each month, with a nominal annual interest rate of 3.7%
Option 2: A deposit of $350000 has to be paid at the beginning of the loan, followed by monthly payments of $17500 at the end of each month until the loan is fully repaid, with a nominal annual interest rate of 3.4%

(a) If Takumi selects the option 1, find

(i) the amount of monthly payment,

(ii) the total amount to be paid,

(iii) the amount of interest paid.

[7]

(b) If Takumi selects the option 2, find

(i) the number of months to repay the loan,

(ii) the exact total amount to be paid,

(iii) the amount of interest paid.

[7]

(c) By considering the amounts of monthly payment in both options, state the better option for Takumi and explain the answer.

[2]

(d) By considering the amounts of interest paid in both options, state the better option for Takumi and explain the answer.

[2]

2. Bosco needs to settle a payment for his postgraduate programme. He is suggested by a bank to choose one of the two options to repay the loan of $40000 with a nominal annual interest rate of 4.5%:

Option 1: A deposit of $10000 has to be paid at the beginning of the loan, followed by a total of 36 equal monthly payments at the end of each month
Option 2: A monthly payment of $800 has to be paid at the end of each month until the loan is fully repaid

(a) If Bosco selects the option 1, find

(i) the amount of monthly payment,

(ii) the amount of interest paid.

[6]

(b) If Bosco selects the option 2, find

(i) the number of months to repay the loan, rounding up the answer correct to the nearest month,

(ii) the amount of interest paid.

[6]

(c) By considering the amounts of monthly payment in both options, state the better option for Bosco and explain the answer.

[2]

(d) By considering the amounts of interest paid in both options, state the better option for Bosco and explain the answer.

[2]

The bank also suggested the option 3 to Bosco, such that a total of 60 equal monthly payments of $900 have to be paid at the end of each month, with a nominal annual interest rate of r%

(e) Find the value of r.

[3]

3. Clement works in a bank and he is designing the structure of an amortization schedule for customers. He suggests the version 1 of the amortization schedule for a loan of \$10000, with a nominal annual interest rate of 2%:

Version 1: A total of 120 equal monthly payments of $\$R$ have to be paid at the end of each month in a ten-year period

(a) Find

(i) the value of R,

(ii) the amount of interest paid.

[6]

Clement then makes some amendments in the version 1, such that the version 2 of the amortization schedule is as follows:

Version 2: A total of 60 equal monthly payments of $\$R$ have to be paid at the end of each month in the first five years, then the amount of monthly payments of $\$(R+60)$ have to be paid at the end of each month, until the loan is fully repaid

(b) (i) Find the number of months to repay the loan, rounding up the answer correct to the nearest month.

(ii) Find the amount of interest paid.

(iii) Explain the reason why the version 2 of the amortization schedule is more favourable to customers than the version 1.

[9]

Clement later makes the final amendments in the version 2, such that the version 3 of the amortization schedule is as follows:

Version 3: A monthly payment of $\$1.5R$ has to be paid at the end of each month until the loan is fully repaid

(c) (i) Find the number of months to repay the loan, rounding up the answer correct to the nearest month.

(ii) Hence, write down the difference of the number of months required to repay the loan between the version 2 and the version 3.

[4]

4. Daniel is a financial analyst and he is investigating the differences of various amortization schedules when some of the factors are changed.

Firstly, he is investigating the differences between the amortization schedules with payments at the beginning of each year and at the end of each year, by considering the following two versions:

Version 1: A total of 20 equal yearly payments of $\$R_1$ have to be paid at the beginning of each year
Version 2: A total of 20 equal yearly payments of $\$R_2$ have to be paid at the end of each year

Both versions are designed for a loan of \$50000, with a nominal annual interest rate of 2%.

(a) (i) Find the value of R_1.

(ii) Find the value of R_2.

(iii) Interpret the meaning of $\$20(R_2 - R_1)$.

(iv) State which version will have the smaller total amount to be paid.

[8]

In addition, he is also investigating the differences between the amortization schedules with yearly payments and monthly payments, by considering the following additional version:

Version 3: A total of 240 equal monthly payments of $\$R_3$ have to be paid at the end of each month

The version 3 is designed for a loan of \$50000, with a nominal annual interest rate of 2%.

(b) (i) Find the value of R_3.

(ii) Interpret the meaning of $\$(240R_3 - 50000)$.

(iii) By comparing the version 2 and the version 3, determine which version will have the smaller total amount to be paid. Justify the answer.

[7]