# Chapter 8 Solution

## Exercise 27

**1.** $\because L_1$ and $L_2$ are perpendicular.

$\therefore \begin{pmatrix} k-1 \\ 20 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} k+2 \\ k-2 \\ k \end{pmatrix} = 0$ (M1) for setting equation

$(k-1)(k+2)+(20)(k-2)+(-10)(k)=0$ (A1) for correct approach

$k^2+k-2+20k-40-10k=0$

$k^2+11k-42=0$

$(k+14)(k-3)=0$

$k=-14$ or $k=3$ A2

[4]

**2.** $\because$ The angle between $L_1$ and $L_2$ is not perpendicular.

$\therefore \begin{pmatrix} -4k \\ k \\ 0 \end{pmatrix} \cdot \begin{pmatrix} k \\ 1 \\ 1 \end{pmatrix} \neq 0$ (M1) for setting equation

$(-4k)(k)+(k)(1)+(0)(1)\neq 0$ (A1) for correct approach

$-4k^2+k\neq 0$

$4k^2-k\neq 0$

$k(4k-1)\neq 0$

$\therefore k\neq 0$ and $k\neq \frac{1}{4}$ A2

[4]

**3.** $L_1:\begin{cases} x = 7+4s \\ y = 5+3s \\ z = -s \end{cases}$, $L_2:\begin{cases} x = 15+6t \\ y = -8-5t \\ z = 1 \end{cases}$ (M1) for valid approach

$-s = 1$ (A1) for correct approach

$s = -1$

$\therefore \begin{cases} x = 7+4(-1) = 3 \\ y = 5+3(-1) = 2 \\ z = -(-1) = 1 \end{cases}$ (M1) for substitution

Thus, the coordinates of P are $(3, 2, 1)$. A2

[5]

**4.** $L_1:\begin{cases} x = k+3s \\ y = -5-4s \\ z = -4-3s \end{cases}$, $L_2:\begin{cases} x = 5+t \\ y = 3+2t \\ z = 2+t \end{cases}$ (M1) for valid approach

$-4-3s = 2+t$

$t = -6-3s$

$-5-4s = 3+2t$

$\therefore -5-4s = 3+2(-6-3s)$ (M1) for substitution

$-5-4s = -9-6s$

$2s = -4$

$s = -2$ A1

$t = -6-3(-2)$

$t = 0$ A1

$k+3s = 5+t$

$\therefore k+3(-2) = 5+0$

$k = 11$ A1

[5]

## Exercise 28

**1.** (a)

| | |
|---|---|
| $\overrightarrow{RP}=\overrightarrow{OP}-\overrightarrow{OR}$ | (M1) for valid approach |
| $\overrightarrow{RP}=\overrightarrow{OP}-(\overrightarrow{OQ}+\overrightarrow{QR})$ | |
| $\overrightarrow{RP}=\overrightarrow{OP}-(\overrightarrow{OQ}+\frac{3}{2}\overrightarrow{OQ})$ | (A1) for correct approach |
| $\overrightarrow{RP}=\overrightarrow{OP}-\frac{5}{2}\overrightarrow{OQ}$ | |
| $\overrightarrow{RP}=\mathbf{p}-\frac{5}{2}\mathbf{q}$ | A1 |

[3]

(b)

| | |
|---|---|
| $\overrightarrow{RT}=\overrightarrow{RP}+\overrightarrow{PT}$ | (M1) for valid approach |
| $\overrightarrow{RT}=\overrightarrow{RP}+\frac{4}{5}\overrightarrow{PQ}$ | |
| $\overrightarrow{RT}=\overrightarrow{RP}+\frac{4}{5}(\overrightarrow{OQ}-\overrightarrow{OP})$ | |
| $\overrightarrow{RT}=\overrightarrow{RP}+\frac{4}{5}\overrightarrow{OQ}-\frac{4}{5}\overrightarrow{OP}$ | (A1) for correct approach |
| $\overrightarrow{RT}=\mathbf{p}-\frac{5}{2}\mathbf{q}+\frac{4}{5}\mathbf{q}-\frac{4}{5}\mathbf{p}$ | (A1) for substitution |
| $\overrightarrow{RT}=\frac{1}{5}\mathbf{p}-\frac{17}{10}\mathbf{q}$ | A1 |

[4]

(c)

| | |
|---|---|
| $\overrightarrow{TS}=\overrightarrow{RS}-\overrightarrow{RT}$ | (M1) for valid approach |
| $\overrightarrow{TS}=\overrightarrow{OS}-\overrightarrow{OR}-\overrightarrow{RT}$ | |
| $\overrightarrow{TS}=\frac{3}{5}\overrightarrow{OP}-\frac{5}{2}\overrightarrow{OQ}-\overrightarrow{RT}$ | (A1) for correct approach |
| $\overrightarrow{TS}=\frac{3}{5}\mathbf{p}-\frac{5}{2}\mathbf{q}-\left(\frac{1}{5}\mathbf{p}-\frac{17}{10}\mathbf{q}\right)$ | (A1) for substitution |
| $\overrightarrow{TS}=\frac{3}{5}\mathbf{p}-\frac{5}{2}\mathbf{q}-\frac{1}{5}\mathbf{p}+\frac{17}{10}\mathbf{q}$ | |
| $\overrightarrow{TS}=\frac{2}{5}\mathbf{p}-\frac{4}{5}\mathbf{q}$ | A1 |

[4]

(d) $\vec{TS} - \lambda \vec{RT} = 1.2\mu\mathbf{q}$

$\therefore \frac{2}{5}\mathbf{p} - \frac{4}{5}\mathbf{q} - \lambda\left(\frac{1}{5}\mathbf{p} - \frac{17}{10}\mathbf{q}\right) = 1.2\mu\mathbf{q}$ (M1) for substitution

$\frac{2}{5}\mathbf{p} - \frac{4}{5}\mathbf{q} - \frac{1}{5}\lambda\mathbf{p} + \frac{17}{10}\lambda\mathbf{q} = 1.2\mu\mathbf{q}$

$\left(\frac{2}{5} - \frac{1}{5}\lambda\right)\mathbf{p} + \left(-\frac{4}{5} + \frac{17}{10}\lambda\right)\mathbf{q} = 1.2\mu\mathbf{q}$ (A1) for simplification

$\frac{2}{5} - \frac{1}{5}\lambda = 0$

$2 - \lambda = 0$

$\lambda = 2$ A1

$-\frac{4}{5} + \frac{17}{10}(2) = 1.2\mu$ (M1) for substitution

$2.6 = 1.2\mu$

$\mu = \frac{13}{6}$ A1

[5]

**2.** (a) Let $\text{PS}:\text{SQ} = a:b$.

| | |
|---|---|
| $\overrightarrow{\text{OS}} = \overrightarrow{\text{OP}} + \overrightarrow{\text{PS}}$ | M1 |
| $\overrightarrow{\text{OS}} = \overrightarrow{\text{OP}} + \frac{a}{a+b}\overrightarrow{\text{PQ}}$ | A1 |
| $\overrightarrow{\text{OS}} = \overrightarrow{\text{OP}} + \frac{a}{a+b}(\overrightarrow{\text{OQ}} - \overrightarrow{\text{OP}})$ | |
| $\overrightarrow{\text{OS}} = \overrightarrow{\text{OP}} + \frac{a}{a+b}\overrightarrow{\text{OQ}} - \frac{a}{a+b}\overrightarrow{\text{OP}}$ | |
| $\overrightarrow{\text{OS}} = \frac{b}{a+b}\overrightarrow{\text{OP}} + \frac{a}{a+b}\overrightarrow{\text{OQ}}$ | A1 |
| $\therefore \alpha + \beta = \frac{b}{a+b} + \frac{a}{a+b}$ | M1 |
| $\therefore \alpha + \beta = \frac{b+a}{a+b}$ | |
| $\alpha + \beta = 1$ | AG |

[4]

(b)

| | |
|---|---|
| $\overrightarrow{\text{OH}} = \overrightarrow{\text{OP}} + \overrightarrow{\text{PH}}$ | (M1) for valid approach |
| $\overrightarrow{\text{OH}} = \overrightarrow{\text{OP}} + \frac{3}{5}\overrightarrow{\text{PR}}$ | (A1) for correct approach |
| $\overrightarrow{\text{OH}} = \overrightarrow{\text{OP}} + \frac{3}{5}(\overrightarrow{\text{OR}} - \overrightarrow{\text{OP}})$ | |
| $\overrightarrow{\text{OH}} = \overrightarrow{\text{OP}} + \frac{3}{5}\left(\frac{1}{4}\overrightarrow{\text{OQ}} - \overrightarrow{\text{OP}}\right)$ | (A1) for correct approach |
| $\overrightarrow{\text{OH}} = \mathbf{p} + \frac{3}{5}\left(\frac{1}{4}\mathbf{q} - \mathbf{p}\right)$ | (A1) for substitution |
| $\overrightarrow{\text{OH}} = \mathbf{p} + \frac{3}{20}\mathbf{q} - \frac{3}{5}\mathbf{p}$ | |
| $\overrightarrow{\text{OH}} = \frac{2}{5}\mathbf{p} + \frac{3}{20}\mathbf{q}$ | A1 |

[5]

(c) Let $\overrightarrow{OS} = c\overrightarrow{OH}$, $c \neq 0$.

$\overrightarrow{OS} = c\left(\frac{2}{5}\mathbf{p} + \frac{3}{20}\mathbf{q}\right)$

$\overrightarrow{OS} = \frac{2}{5}c\mathbf{p} + \frac{3}{20}c\mathbf{q}$ (M1) for valid approach

$\therefore \frac{2}{5}c + \frac{3}{20}c = 1$ A1

$\frac{11}{20}c = 1$

$c = \frac{20}{11}$ A1

$\therefore \overrightarrow{OS} = \frac{2}{5}\left(\frac{20}{11}\right)\mathbf{p} + \frac{3}{20}\left(\frac{20}{11}\right)\mathbf{q}$

$\overrightarrow{OS} = \frac{8}{11}\mathbf{p} + \frac{3}{11}\mathbf{q}$ A1

[4]

(d) $\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$ M1

$\overrightarrow{RS} = \overrightarrow{OS} - \frac{1}{4}\overrightarrow{OQ}$

$\overrightarrow{RS} = \frac{8}{11}\mathbf{p} + \frac{3}{11}\mathbf{q} - \frac{1}{4}\mathbf{q}$ A1

$\overrightarrow{RS} = \frac{8}{11}\mathbf{p} + \frac{1}{44}\mathbf{q}$

Therefore, $\overrightarrow{RS}$ is not a multiple of $\mathbf{p}$. R1

Thus, RS and OP are not parallel. AG

[3]

**3.** (a) $\mathbf{p}\cdot\mathbf{q}=|\mathbf{p}||\mathbf{q}|\cos \text{P}\hat{\text{O}}\text{Q}$ (M1) for valid approach

$24=(8)(6)\cos \text{P}\hat{\text{O}}\text{Q}$ (A1) for substitution

$\cos \text{P}\hat{\text{O}}\text{Q}=\frac{1}{2}$

$\text{P}\hat{\text{O}}\text{Q}=60^\circ$ A1

[3]

(b) $\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=\overrightarrow{\text{OR}}\cdot(\overrightarrow{\text{OQ}}-\overrightarrow{\text{OP}})$ M1

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=\left(\frac{1}{6}\mathbf{p}+\frac{5}{9}\mathbf{q}\right)\cdot(\mathbf{q}-\mathbf{p})$

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=\frac{1}{6}\mathbf{p}\cdot\mathbf{q}-\frac{1}{6}\mathbf{p}\cdot\mathbf{p}+\frac{5}{9}\mathbf{q}\cdot\mathbf{q}-\frac{5}{9}\mathbf{q}\cdot\mathbf{p}$ A1

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=\frac{1}{6}\mathbf{p}\cdot\mathbf{q}-\frac{1}{6}|\mathbf{p}|^2+\frac{5}{9}|\mathbf{q}|^2-\frac{5}{9}\mathbf{p}\cdot\mathbf{q}$ A1

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=\frac{1}{6}(24)-\frac{1}{6}(8)^2+\frac{5}{9}(6)^2-\frac{5}{9}(24)$ A1

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=4-\frac{32}{3}+20-\frac{40}{3}$

$\overrightarrow{\text{OR}}\cdot\overrightarrow{\text{PQ}}=0$

$\therefore \text{OR}\perp\text{PQ}$ AG

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=(\overrightarrow{\text{OR}}-\overrightarrow{\text{OP}})\cdot\overrightarrow{\text{OQ}}$ M1

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=\left(\frac{1}{6}\mathbf{p}+\frac{5}{9}\mathbf{q}-\mathbf{p}\right)\cdot\mathbf{q}$

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=\left(-\frac{5}{6}\mathbf{p}+\frac{5}{9}\mathbf{q}\right)\cdot\mathbf{q}$

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=-\frac{5}{6}\mathbf{p}\cdot\mathbf{q}+\frac{5}{9}\mathbf{q}\cdot\mathbf{q}$ A1

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=-\frac{5}{6}\mathbf{p}\cdot\mathbf{q}+\frac{5}{9}|\mathbf{q}|^2$ A1

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=-\frac{5}{6}(24)+\frac{5}{9}(6)^2$ A1

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=-20+20$

$\overrightarrow{\text{PR}}\cdot\overrightarrow{\text{OQ}}=0$

$\therefore \text{PR}\perp\text{OQ}$ AG

[8]

(c) $\overrightarrow{SH} = \lambda \overrightarrow{OR}$

$\overrightarrow{SH} = \lambda\left(\frac{1}{6}\mathbf{p} + \frac{5}{9}\mathbf{q}\right)$

$\left|\overrightarrow{SH}\right| = \lambda\left|\frac{1}{6}\mathbf{p} + \frac{5}{9}\mathbf{q}\right|$ (M1) for setting equation

$\therefore \frac{\sqrt{39}}{3} = \lambda\sqrt{\left(\frac{1}{6}\mathbf{p} + \frac{5}{9}\mathbf{q}\right)\cdot\left(\frac{1}{6}\mathbf{p} + \frac{5}{9}\mathbf{q}\right)}$ (A1) for correct approach

$\frac{\sqrt{39}}{3} = \lambda\sqrt{\frac{1}{36}\mathbf{p}\cdot\mathbf{p} + \frac{5}{27}\mathbf{p}\cdot\mathbf{q} + \frac{25}{81}\mathbf{q}\cdot\mathbf{q}}$

$\frac{\sqrt{39}}{3} = \lambda\sqrt{\frac{1}{36}|\mathbf{p}|^2 + \frac{5}{27}\mathbf{p}\cdot\mathbf{q} + \frac{25}{81}|\mathbf{q}|^2}$

$\frac{\sqrt{39}}{3} = \lambda\sqrt{\frac{1}{36}(8)^2 + \frac{5}{27}(24) + \frac{25}{81}(6)^2}$ (A1) for substitution

$\frac{\sqrt{39}}{3} = \lambda\sqrt{\frac{52}{3}}$

$\frac{\sqrt{39}}{3} = \frac{\sqrt{156}}{3}\lambda$ M1

$\lambda = \frac{1}{2}$ A1

[5]

**4.** (a) $QU \perp OP$

$\therefore \vec{QU} \cdot \vec{OP} = 0$ M1

$(\vec{OU} - \vec{OQ}) \cdot \vec{OP} = 0$

$(\vec{OU} - \mathbf{q}) \cdot \mathbf{p} = 0$ A1

$\vec{OU} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{p} = 0$

$\mathbf{p} \cdot \mathbf{q} = \vec{OU} \cdot \mathbf{p}$

$PU \perp OQ$

$\therefore \vec{PU} \cdot \vec{OQ} = 0$ M1

$(\vec{OU} - \vec{OP}) \cdot \vec{OQ} = 0$

$(\vec{OU} - \mathbf{p}) \cdot \mathbf{q} = 0$ A1

$\vec{OU} \cdot \mathbf{q} - \mathbf{p} \cdot \mathbf{q} = 0$

$\mathbf{p} \cdot \mathbf{q} = \vec{OU} \cdot \mathbf{q}$

$\therefore \mathbf{p} \cdot \mathbf{q} = \vec{OU} \cdot \mathbf{p} = \vec{OU} \cdot \mathbf{q}$ AG

[4]

(b) $\vec{OV} = \frac{2}{3}\vec{OH}$ (A1) for correct approach

$\vec{OV} = \frac{2}{3}(\vec{OP} + \vec{PH})$ (M1) for valid approach

$\vec{OV} = \frac{2}{3}\left(\vec{OP} + \frac{1}{2}\vec{PQ}\right)$ (A1) for correct approach

$\vec{OV} = \frac{2}{3}\left(\vec{OP} + \frac{1}{2}(\vec{OQ} - \vec{OP})\right)$

$\vec{OV} = \frac{2}{3}\left(\mathbf{p} + \frac{1}{2}(\mathbf{q} - \mathbf{p})\right)$ (A1) for substitution

$\vec{OV} = \frac{2}{3}\left(\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}\right)$

$\vec{OV} = \frac{1}{3}\mathbf{p} + \frac{1}{3}\mathbf{q}$ A1

[5]

(c) $\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=\mathbf{q}\cdot(2(\overrightarrow{OV}-\overrightarrow{OW})-(\overrightarrow{OU}-\overrightarrow{OV}))$ M1A1

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=\mathbf{q}\cdot(3\overrightarrow{OV}-2\overrightarrow{OW}-\overrightarrow{OU})$ A1

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=3\mathbf{q}\cdot\overrightarrow{OV}-2\mathbf{q}\cdot\overrightarrow{OW}-\mathbf{q}\cdot\overrightarrow{OU}$ M1

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=3\mathbf{q}\cdot\left(\frac{1}{3}\mathbf{p}+\frac{1}{3}\mathbf{q}\right)$
M1A1
$-2|\mathbf{q}|\left|\overrightarrow{OW}\right|\cos W\hat{O}K-\mathbf{p}\cdot\mathbf{q}$

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=\mathbf{p}\cdot\mathbf{q}+|\mathbf{q}|^2-2|\mathbf{q}|\left(\frac{|\mathbf{q}|}{2}\right)-\mathbf{p}\cdot\mathbf{q}$ M1A1

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=\mathbf{p}\cdot\mathbf{q}+|\mathbf{q}|^2-|\mathbf{q}|^2-\mathbf{p}\cdot\mathbf{q}$

$\mathbf{q}\cdot(2\overrightarrow{WV}-\overrightarrow{VU})=0$ AG

[8]

## Exercise 29

**1.** (a) $\frac{1}{2}[(\mathbf{p}+\mathbf{q})\times(\mathbf{q}+\mathbf{r})]\cdot(\mathbf{r}+\mathbf{p})$

$=\frac{1}{2}[(\mathbf{p}+\mathbf{q})\times\mathbf{q}+(\mathbf{p}+\mathbf{q})\times\mathbf{r}]\cdot(\mathbf{r}+\mathbf{p})$ M1

$=\frac{1}{2}[\mathbf{p}\times\mathbf{q}+\mathbf{q}\times\mathbf{q}+\mathbf{p}\times\mathbf{r}+\mathbf{q}\times\mathbf{r}]\cdot(\mathbf{r}+\mathbf{p})$

$=\frac{1}{2}[\mathbf{p}\times\mathbf{q}+\mathbf{0}+\mathbf{p}\times\mathbf{r}+\mathbf{q}\times\mathbf{r}]\cdot(\mathbf{r}+\mathbf{p})$ A1

$=\frac{1}{2}[\mathbf{p}\times\mathbf{q}\cdot(\mathbf{r}+\mathbf{p})+\mathbf{p}\times\mathbf{r}\cdot(\mathbf{r}+\mathbf{p})+\mathbf{q}\times\mathbf{r}\cdot(\mathbf{r}+\mathbf{p})]$ M1

$=\frac{1}{2}[(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}+(\mathbf{p}\times\mathbf{q})\cdot\mathbf{p}+(\mathbf{p}\times\mathbf{r})\cdot\mathbf{r}$
$+(\mathbf{p}\times\mathbf{r})\cdot\mathbf{p}+(\mathbf{q}\times\mathbf{r})\cdot\mathbf{r}+(\mathbf{q}\times\mathbf{r})\cdot\mathbf{p}]$ M1

$=\frac{1}{2}[(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}+\mathbf{0}+\mathbf{0}+\mathbf{0}+\mathbf{0}+(\mathbf{q}\times\mathbf{r})\cdot\mathbf{p}]$ A1

$=\frac{1}{2}[(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}+(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}]$ A1

$=\frac{1}{2}[2(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}]$

$=(\mathbf{p}\times\mathbf{q})\cdot\mathbf{r}$ AG

[6]

(b) $\mathbf{p}\times(\mathbf{q}\times\mathbf{r})+(\mathbf{p}\cdot\mathbf{q})\mathbf{r}$

$=\mathbf{p}\times((q_1\mathbf{i}+q_2\mathbf{j}+q_3\mathbf{k})\times(r_1\mathbf{i}+r_2\mathbf{j}+r_3\mathbf{k}))$
$+((p_1\mathbf{i}+p_2\mathbf{j}+p_3\mathbf{k})\cdot(q_1\mathbf{i}+q_2\mathbf{j}+q_3\mathbf{k}))\mathbf{r}$ M1

$$=\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix}\times\begin{pmatrix}q_2r_3-q_3r_2\\q_3r_1-q_1r_3\\q_1r_2-q_2r_1\end{pmatrix}+(p_1q_1+p_2q_2+p_3q_3)\begin{pmatrix}r_1\\r_2\\r_3\end{pmatrix}$$ A2

$$=\begin{pmatrix}p_2(q_1r_2-q_2r_1)-p_3(q_3r_1-q_1r_3)\\p_3(q_2r_3-q_3r_2)-p_1(q_1r_2-q_2r_1)\\p_1(q_3r_1-q_1r_3)-p_2(q_2r_3-q_3r_2)\end{pmatrix}+\begin{pmatrix}(p_1q_1+p_2q_2+p_3q_3)r_1\\(p_1q_1+p_2q_2+p_3q_3)r_2\\(p_1q_1+p_2q_2+p_3q_3)r_3\end{pmatrix}$$ A2

$$=\begin{pmatrix}p_2q_1r_2-p_2q_2r_1-p_3q_3r_1+p_3q_1r_3\\p_3q_2r_3-p_3q_3r_2-p_1q_1r_2+p_1q_2r_1\\p_1q_3r_1-p_1q_1r_3-p_2q_2r_3+p_2q_3r_2\end{pmatrix}+\begin{pmatrix}p_1q_1r_1+p_2q_2r_1+p_3q_3r_1\\p_1q_1r_2+p_2q_2r_2+p_3q_3r_2\\p_1q_1r_3+p_2q_2r_3+p_3q_3r_3\end{pmatrix}$$ M1

$$=\begin{pmatrix}p_1q_1r_1+p_2q_1r_2+p_3q_1r_3\\p_1q_2r_1+p_2q_2r_2+p_3q_2r_3\\p_1q_3r_1+p_2q_3r_2+p_3q_3r_3\end{pmatrix}$$ A1

$$=\begin{pmatrix}(p_1r_1+p_2r_2+p_3r_3)q_1\\(p_1r_1+p_2r_2+p_3r_3)q_2\\(p_1r_1+p_2r_2+p_3r_3)q_3\end{pmatrix}$$

$=(p_1r_1+p_2r_2+p_3r_3)\mathbf{q}$ A1

$=(\mathbf{p}\cdot\mathbf{r})\mathbf{q}$ AG

[8]

(c) Similarly, $\mathbf{q}\times(\mathbf{r}\times\mathbf{p})+(\mathbf{q}\cdot\mathbf{r})\mathbf{p}=(\mathbf{q}\cdot\mathbf{p})\mathbf{r}$ and

$\mathbf{r}\times(\mathbf{p}\times\mathbf{q})+(\mathbf{r}\cdot\mathbf{p})\mathbf{q}=(\mathbf{r}\cdot\mathbf{q})\mathbf{p}$. A1

$\therefore \mathbf{p}\times(\mathbf{q}\times\mathbf{r})+\mathbf{q}\times(\mathbf{r}\times\mathbf{p})+\mathbf{r}\times(\mathbf{p}\times\mathbf{q})$

$=((\mathbf{p}\cdot\mathbf{r})\mathbf{q}-(\mathbf{p}\cdot\mathbf{q})\mathbf{r})+((\mathbf{q}\cdot\mathbf{p})\mathbf{r}-(\mathbf{q}\cdot\mathbf{r})\mathbf{p})$

$+((\mathbf{r}\cdot\mathbf{q})\mathbf{p}-(\mathbf{r}\cdot\mathbf{p})\mathbf{q})$ A1

$=(\mathbf{p}\cdot\mathbf{r})\mathbf{q}-(\mathbf{p}\cdot\mathbf{q})\mathbf{r}+(\mathbf{q}\cdot\mathbf{p})\mathbf{r}-(\mathbf{q}\cdot\mathbf{r})\mathbf{p}$

$+(\mathbf{r}\cdot\mathbf{q})\mathbf{p}-(\mathbf{r}\cdot\mathbf{p})\mathbf{q}$

$=(\mathbf{r}\cdot\mathbf{p})\mathbf{q}-(\mathbf{p}\cdot\mathbf{q})\mathbf{r}+(\mathbf{p}\cdot\mathbf{q})\mathbf{r}-(\mathbf{q}\cdot\mathbf{r})\mathbf{p}$

$+(\mathbf{q}\cdot\mathbf{r})\mathbf{p}-(\mathbf{r}\cdot\mathbf{p})\mathbf{q}$ A1

$=\mathbf{0}$ AG

[3]

**2.** (a) $\mathbf{q}\times\mathbf{r}=\mathbf{q}\times(\mathbf{0}-\mathbf{p}-\mathbf{q})$ A1

$\mathbf{q}\times\mathbf{r}=\mathbf{q}\times(-\mathbf{p}-\mathbf{q})$

$\mathbf{q}\times\mathbf{r}=\mathbf{q}\times(-\mathbf{p})-\mathbf{q}\times\mathbf{q}$ M1

$\mathbf{q}\times\mathbf{r}=-\mathbf{q}\times\mathbf{p}-\mathbf{q}\times\mathbf{q}$

$\mathbf{q}\times\mathbf{r}=-(-\mathbf{p}\times\mathbf{q})-\mathbf{0}$ A1

$\mathbf{q}\times\mathbf{r}=\mathbf{p}\times\mathbf{q}$

$\mathbf{r}\times\mathbf{p}=(\mathbf{0}-\mathbf{p}-\mathbf{q})\times\mathbf{p}$ A1

$\mathbf{r}\times\mathbf{p}=(-\mathbf{p}-\mathbf{q})\times\mathbf{p}$

$\mathbf{r}\times\mathbf{p}=-\mathbf{p}\times\mathbf{p}-\mathbf{q}\times\mathbf{p}$ M1

$\mathbf{r}\times\mathbf{p}=\mathbf{0}-(-\mathbf{p}\times\mathbf{q})$ A1

$\mathbf{r}\times\mathbf{p}=\mathbf{p}\times\mathbf{q}$

$\therefore \mathbf{p}\times\mathbf{q}=\mathbf{q}\times\mathbf{r}=\mathbf{r}\times\mathbf{p}$ AG

[6]

(b) $\mathbf{p}=\lambda\mathbf{q}$

$\mathbf{r}=\mathbf{0}-\mathbf{p}-\mathbf{q}$

$\mathbf{r}=-\lambda\mathbf{q}-\mathbf{q}$ A1

$\therefore \mathbf{p}\times\mathbf{q}-\mathbf{q}\times\mathbf{r}-\mathbf{r}\times\mathbf{p}$

$=\lambda\mathbf{q}\times\mathbf{q}-\mathbf{q}\times(-\lambda\mathbf{q}-\mathbf{q})-(-\lambda\mathbf{q}-\mathbf{q})\times\lambda\mathbf{q}$ M1

$=\lambda\mathbf{q}\times\mathbf{q}-\mathbf{q}\times(-\lambda\mathbf{q})+\mathbf{q}\times\mathbf{q}-(-\lambda\mathbf{q})\times\lambda\mathbf{q}+\mathbf{q}\times\lambda\mathbf{q}$

$=\lambda\mathbf{q}\times\mathbf{q}+\lambda\mathbf{q}\times\mathbf{q}+\mathbf{q}\times\mathbf{q}+\lambda^2\mathbf{q}\times\mathbf{q}+\lambda\mathbf{q}\times\mathbf{q}$ A1

$=(\lambda^2+3\lambda+1)\mathbf{q}\times\mathbf{q}$

$=(\lambda^2+3\lambda+1)\mathbf{0}$

$=\mathbf{0}$ AG

[3]

(c) $\mathbf{p}\times\mathbf{q}=\mathbf{q}\times\mathbf{r}=\mathbf{r}\times\mathbf{p}$

$|\mathbf{p}\times\mathbf{q}|=|\mathbf{q}\times\mathbf{r}|=|\mathbf{r}\times\mathbf{p}|$ M1

$|\mathbf{p}||\mathbf{q}|\sin \mathrm{Q\hat{R}P}=|\mathbf{q}||\mathbf{r}|\sin \mathrm{R\hat{P}Q}=|\mathbf{p}||\mathbf{r}|\sin \mathrm{P\hat{Q}R}$ A1

$\frac{|\mathbf{p}||\mathbf{q}|\sin \mathrm{Q\hat{R}P}}{|\mathbf{p}||\mathbf{q}||\mathbf{r}|}=\frac{|\mathbf{q}||\mathbf{r}|\sin \mathrm{R\hat{P}Q}}{|\mathbf{p}||\mathbf{q}||\mathbf{r}|}=\frac{|\mathbf{p}||\mathbf{r}|\sin \mathrm{P\hat{Q}R}}{|\mathbf{p}||\mathbf{q}||\mathbf{r}|}$ M1

$\frac{\sin \mathrm{Q\hat{R}P}}{|\mathbf{r}|}=\frac{\sin \mathrm{R\hat{P}Q}}{|\mathbf{p}|}=\frac{\sin \mathrm{P\hat{Q}R}}{|\mathbf{q}|}$

$\therefore \frac{\sin \mathrm{R\hat{P}Q}}{|\mathbf{p}|}=\frac{\sin \mathrm{P\hat{Q}R}}{|\mathbf{q}|}=\frac{\sin \mathrm{Q\hat{R}P}}{|\mathbf{r}|}$ AG

[3]

(d) $|-\mathbf{p}|^2 + |\mathbf{q}|^2 - 2|-\mathbf{p}||\mathbf{q}|\cos Q\hat{R}P$ A1

$= |-\mathbf{p}|^2 + |\mathbf{q}|^2 - 2(-\mathbf{p})\cdot\mathbf{q}$ A1

$= |\mathbf{p}|^2 + |\mathbf{q}|^2 + 2\mathbf{p}\cdot\mathbf{q}$

$= \mathbf{p}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{q} + \mathbf{p}\cdot\mathbf{q} + \mathbf{q}\cdot\mathbf{q}$ M1

$= \mathbf{p}\cdot(\mathbf{p}+\mathbf{q}) + \mathbf{q}\cdot(\mathbf{p}+\mathbf{q})$

$= (\mathbf{p}+\mathbf{q})\cdot(\mathbf{p}+\mathbf{q})$ M1

$= |\mathbf{p}+\mathbf{q}|^2$

$= |-(\mathbf{p}+\mathbf{q})|^2$ M1

$= |\mathbf{r}|^2$ AG

[5]

**3.** (a) $\mathbf{r} = \lambda\mathbf{p} + \mathbf{q}$

$|\mathbf{r}|^2 = |\lambda\mathbf{p} + \mathbf{q}|^2$

$|\mathbf{r}|^2 = (\lambda\mathbf{p} + \mathbf{q}) \cdot (\lambda\mathbf{p} + \mathbf{q})$ M1

$|\mathbf{r}|^2 = \lambda\mathbf{p} \cdot \lambda\mathbf{p} + \lambda\mathbf{p} \cdot \mathbf{q} + \mathbf{q} \cdot \lambda\mathbf{p} + \mathbf{q} \cdot \mathbf{q}$ A1

$|\mathbf{r}|^2 = \lambda^2\mathbf{p} \cdot \mathbf{p} + \lambda\mathbf{p} \cdot \mathbf{q} + \lambda\mathbf{q} \cdot \mathbf{p} + \mathbf{q} \cdot \mathbf{q}$

$|\mathbf{r}|^2 = \lambda^2\mathbf{p} \cdot \mathbf{p} + \lambda\mathbf{p} \cdot \mathbf{q} + \lambda\mathbf{p} \cdot \mathbf{q} + \mathbf{q} \cdot \mathbf{q}$ M1

$|\mathbf{r}|^2 = \lambda^2 |\mathbf{p}|^2 + 2\lambda\mathbf{p} \cdot \mathbf{q} + |\mathbf{q}|^2$ AG

[3]

(b) $(\mathbf{p} + \mathbf{q} + \mathbf{r}) \cdot (\mathbf{p} \times (\mathbf{q} + \mathbf{r}))$

$= (\mathbf{p} + \mathbf{q} + \mathbf{r}) \cdot (\mathbf{p} \times \mathbf{q} + \mathbf{p} \times \mathbf{r})$ M1

$= \mathbf{p} \cdot (\mathbf{p} \times \mathbf{q}) + \mathbf{q} \cdot (\mathbf{p} \times \mathbf{q}) + \mathbf{r} \cdot (\mathbf{p} \times \mathbf{q})$
$+ \mathbf{p} \cdot (\mathbf{p} \times \mathbf{r}) + \mathbf{q} \cdot (\mathbf{p} \times \mathbf{r}) + \mathbf{r} \cdot (\mathbf{p} \times \mathbf{r})$ M1

$= \mathbf{0} + \mathbf{0} + \mathbf{r} \cdot (\mathbf{p} \times \mathbf{q}) + \mathbf{0} + \mathbf{q} \cdot (\mathbf{p} \times \mathbf{r}) + \mathbf{0}$ A1

$= (\mathbf{p} \times \mathbf{q}) \cdot \mathbf{r} - \mathbf{q} \cdot (\mathbf{r} \times \mathbf{p})$ M1

$= 0$ AG

[4]

(c) $\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \left(\frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|^2}\right)\mathbf{q} \cdot \left(\mathbf{p} - \left(\frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|^2}\right)\mathbf{q}\right)$

$\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \left(\frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|^2}\right)\mathbf{q} \cdot \mathbf{p} - \left(\frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|^2}\right)\mathbf{q} \cdot \left(\frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|^2}\right)\mathbf{q}$ M1

$\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^2} - \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^4}\mathbf{q} \cdot \mathbf{q}$ A1

$\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^2} - \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^4}|\mathbf{q}|^2$ A1

$\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^2} - \frac{(\mathbf{p} \cdot \mathbf{q})^2}{|\mathbf{q}|^2}$

$\mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = 0$ AG

[3]

(d) $\because \mathbf{v} \cdot (\mathbf{p} - \mathbf{v}) = \mathbf{0}$

$\therefore \cos\theta = \dfrac{|\mathbf{v}|}{|\mathbf{p}|}$ A1

$\sec\theta = \dfrac{|\mathbf{p}|}{|\mathbf{v}|}$ M1

$\sec^2\theta = \dfrac{|\mathbf{p}|^2}{|\mathbf{v}|^2}$ A1

$\sec^2\theta - 1 = \dfrac{|\mathbf{p}|^2}{|\mathbf{v}|^2} - 1$ M1

$\therefore \tan^2\theta = \dfrac{|\mathbf{p}|^2}{|\mathbf{v}|^2} - 1$ A1

$\tan^2\theta = \dfrac{|\mathbf{p}|^2}{|\mathbf{v}|^2} - \dfrac{|\mathbf{v}|^2}{|\mathbf{v}|^2}$

$\tan\theta = \sqrt{\dfrac{|\mathbf{p}|^2 - |\mathbf{v}|^2}{|\mathbf{v}|^2}}$ AG

[5]

**4.** (a) $\overrightarrow{PN}=\overrightarrow{PQ}+\overrightarrow{QN}$

$\overrightarrow{PN}=\mathbf{r}+\frac{\lambda}{\lambda+1}\overrightarrow{QR}$ (A1) for correct approach

$\overrightarrow{PN}=\mathbf{r}+\frac{\lambda}{\lambda+1}(\overrightarrow{PR}-\overrightarrow{PQ})$ (M1) for valid approach

$\overrightarrow{PN}=\mathbf{r}+\frac{\lambda}{\lambda+1}(\mathbf{q}-\mathbf{r})$

$\overrightarrow{PN}=\frac{\lambda+1}{\lambda+1}\mathbf{r}+\frac{\lambda}{\lambda+1}\mathbf{q}-\frac{\lambda}{\lambda+1}\mathbf{r}$ (M1) for valid approach

$\overrightarrow{PN}=\frac{\lambda}{\lambda+1}\mathbf{q}+\frac{1}{\lambda+1}\mathbf{r}$ A1

[4]

(b) $\because \text{PN}\perp\text{QR}$

$\therefore \overrightarrow{PN}\cdot\overrightarrow{QR}=0$ M1

$\left(\frac{\lambda}{\lambda+1}\mathbf{q}+\frac{1}{\lambda+1}\mathbf{r}\right)\cdot(\mathbf{q}-\mathbf{r})=0$ A1

$(\lambda\mathbf{q}+\mathbf{r})\cdot(\mathbf{q}-\mathbf{r})=0$

$\lambda\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})+\mathbf{r}\cdot(\mathbf{q}-\mathbf{r})=0$ M1

$\lambda\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})=-\mathbf{r}\cdot(\mathbf{q}-\mathbf{r})$

$\lambda\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})=\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})$ M1

$\lambda=\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}$ AG

[4]

(c) $\overrightarrow{PN} = \frac{\lambda}{\lambda+1}\mathbf{q} + \frac{1}{\lambda+1}\mathbf{r}$

$\overrightarrow{PN} = \frac{1}{\lambda+1}(\lambda\mathbf{q} + \mathbf{r})$

$\overrightarrow{PN} = \frac{1}{\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}+1}\left(\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\mathbf{q}+\mathbf{r}\right)$ M1

$\overrightarrow{PN} = \frac{1}{\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}+\frac{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}}\left(\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\mathbf{q}+\frac{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\mathbf{r}\right)$ M1

$\overrightarrow{PN} = \frac{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})+\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\left(\frac{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\mathbf{q}+\frac{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}{\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}\mathbf{r}\right)$ A1

$\overrightarrow{PN} = \frac{(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}}{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})+\mathbf{q}\cdot(\mathbf{q}-\mathbf{r})}$ M1

$\overrightarrow{PN} = \frac{(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}}{\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})-\mathbf{q}\cdot(\mathbf{r}-\mathbf{q})}$ A1

$\overrightarrow{PN} = \frac{(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}}{(\mathbf{r}-\mathbf{q})\cdot(\mathbf{r}-\mathbf{q})}$

$\overrightarrow{PN} = \frac{(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}}{|\mathbf{r}-\mathbf{q}|^2}$ A1

$|\mathbf{r}-\mathbf{q}|^2\,\overrightarrow{PN} = (\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}$ M1

$(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{r}-|\mathbf{r}-\mathbf{q}|^2\,\overrightarrow{PN} = 0$ AG

[7]

## Exercise 30

**1.** (a) $L_1:\begin{cases} x=15+6t \\ y=11+3t \\ z=6+2t \end{cases}$, $L_2:\begin{cases} x=-3s \\ y=7+2s \\ z=8+6s \end{cases}$ (M1) for valid approach

$15+6t=-3s$

$s=-5-2t$

$11+3t=7+2s$

$\therefore 11+3t=7+2(-5-2t)$ (M1) for substitution

$11+3t=-3-4t$

$7t=-14$

$t=-2$ A1

$\therefore \begin{cases} x=15+6(-2)=3 \\ y=11+3(-2)=5 \\ z=6+2(-2)=2 \end{cases}$ (M1) for substitution

Thus, the coordinates of C are $(3, 5, 2)$. A1

[5]

(b) The coordinates of A and B are $(15, 11, 6)$ and $(0, 7, 8)$ respectively. (A1) for correct values

$\vec{CA} = (15\mathbf{i} + 11\mathbf{j} + 6\mathbf{k}) - (3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$

$\vec{CA} = 12\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$ A1

$\vec{CB} = (7\mathbf{j} + 8\mathbf{k}) - (3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$

$\vec{CB} = -3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ A1

The area of the triangle ABC

$= \frac{1}{2}\left|\vec{CA} \times \vec{CB}\right|$ (M1) for valid approach

$= \frac{1}{2}\left|(12\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})\right|$

$$= \frac{1}{2}\left|\begin{pmatrix} (6)(6) - (4)(2) \\ (4)(-3) - (12)(6) \\ (12)(2) - (6)(-3) \end{pmatrix}\right|$$

$= \frac{1}{2}\left|28\mathbf{i} - 84\mathbf{j} + 42\mathbf{k}\right|$

$= \frac{1}{2}(14)\left|2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\right|$

$= 7\sqrt{2^2 + (-6)^2 + 3^2}$ A1

$= 49$ A1

[6]

(c) The vector equation of the line $L_3$:

$$\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} + u\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$$ (A1) for correct values

$$\begin{cases} x = 3 + 2u \\ y = 5 - 6u \\ z = 2 + 3u \end{cases}$$ A1

[2]

(d) $3 + 2u = 73$

$2u = 70$

$u = 35$ (A1) for correct value

$d = 2 + 3(35)$

$d = 107$ A1

[2]

(e) $\overrightarrow{CD} = (73\mathbf{i} - 205\mathbf{j} + 107\mathbf{k}) - (3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$

$\overrightarrow{CD} = 70\mathbf{i} - 210\mathbf{j} + 105\mathbf{k}$ A1

The volume of the pyramid ABCD

$= \frac{1}{3}(49)(\sqrt{70^2 + (-210)^2 + 105^2})$ M1A1

$= \frac{1}{3}(49)(35)(\sqrt{2^2 + (-6)^2 + 3^2})$

$= \frac{1}{3}(49)(35)(7)$ A1

$= \frac{12005}{3}$ AG

[4]

**2.** (a) $L_1:\begin{cases} x=8+2t \\ y=8 \\ z=7 \end{cases}$, $L_2:\begin{cases} x=6+s \\ y=8+2\sqrt{3}+\sqrt{3}s \\ z=7 \end{cases}$ M1

$8=8+2\sqrt{3}+\sqrt{3}s$

$-2\sqrt{3}=\sqrt{3}s$

$s=-2$ A1

$x=6+(-2)$ M1

$x=4$

Thus, the coordinates of C are $(4, 8, 7)$. AG

[3]

(b) $(\mathbf{i}+\sqrt{3}\mathbf{j})\cdot\mathbf{j}=|\mathbf{i}+\sqrt{3}\mathbf{j}||\mathbf{j}|\cos\theta$ (M1) for valid approach

$(1)(0)+(\sqrt{3})(1)=(\sqrt{1^2+(\sqrt{3})^2})(1)\cos\theta$ (A1) for correct approach

$\sqrt{3}=2\cos\theta$

$\cos\theta=\dfrac{\sqrt{3}}{2}$

$\theta=30^\circ$ A1

[3]

(c) The coordinates of A and B are $(6, 8, 7)$ and $(5, 8+\sqrt{3}, 7)$ respectively. (A1) for correct values

$\overrightarrow{\text{CA}}=(6\mathbf{i}+8\mathbf{j}+7\mathbf{k})-(4\mathbf{i}+8\mathbf{j}+7\mathbf{k})$

$\overrightarrow{\text{CA}}=2\mathbf{i}$ A1

$\overrightarrow{\text{CB}}=(5\mathbf{i}+(8+\sqrt{3})\mathbf{j}+7\mathbf{k})-(4\mathbf{i}+8\mathbf{j}+7\mathbf{k})$

$\overrightarrow{\text{CB}}=\mathbf{i}+\sqrt{3}\mathbf{j}$ A1

$\overrightarrow{\text{CA}}\cdot\overrightarrow{\text{CB}}=|\overrightarrow{\text{CA}}||\overrightarrow{\text{CB}}|\cos\text{A}\hat{\text{C}}\text{B}$ (M1) for valid approach

$2\mathbf{i}\cdot(\mathbf{i}+\sqrt{3}\mathbf{j})=|2\mathbf{i}||\mathbf{i}+\sqrt{3}\mathbf{j}|\cos\text{A}\hat{\text{C}}\text{B}$ (A1) for substitution

$(2)(1)+(0)(\sqrt{3})=(2)(\sqrt{1^2+(\sqrt{3})^2})\cos\text{A}\hat{\text{C}}\text{B}$

$2=4\cos\text{A}\hat{\text{C}}\text{B}$

$\cos\text{A}\hat{\text{C}}\text{B}=\dfrac{1}{2}$

$\text{A}\hat{\text{C}}\text{B}=60^\circ$ A1

[6]

(d) $\text{CA} = \text{CB} = 2$

The area of the triangle ABC

$= \frac{1}{2}(\text{CA})(\text{CB}) \sin \text{A}\hat{\text{C}}\text{B}$ (M1) for valid approach

$= \frac{1}{2}(2)(2) \sin 60^\circ$ (A1) for substitution

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$ A1

[3]

(e) Let $h$ be the height of the prism ABCFED.

The triangle ABC is an equilateral triangle.

$\therefore 2\sqrt{3} + 3(2h) = 2(30 + \sqrt{3})$ M1A1

$2\sqrt{3} + 6h = 60 + 2\sqrt{3}$

$6h = 60$

$h = 10$ A1

The volume of the prism ABCFED

$= (\sqrt{3})(10)$

$= 10\sqrt{3}$ A1

[4]

**3.** (a) $L_1 : \begin{cases} x = 14 - 5t \\ y = 18 - 6t \\ z = 8 - 2t \end{cases}$ (M1) for valid approach

$(14 - 5t) + 6 = (8 - 2t) + 6$ (M1) for setting equation

$20 - 5t = 14 - 2t$

$6 = 3t$

$t = 2$ A1

$\therefore \begin{cases} x = 14 - 5(2) = 4 \\ y = 18 - 6(2) = 6 \\ z = 8 - 2(2) = 4 \end{cases}$ (M1) for substitution

Thus, the coordinates of P are $(4, 6, 4)$. A1

[5]

(b) $a + 6 = 3 + 6$ (M1) for setting equation

$a = 3$ A1

[2]

(c) $\overrightarrow{RQ} = -4\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

$\therefore \overrightarrow{OQ} - \overrightarrow{OR} = -4\mathbf{i} - 4\mathbf{j} - \mathbf{k}$ (M1) for valid approach

$((14 - 5t)\mathbf{i} + (18 - 6t)\mathbf{j} + (8 - 2t)\mathbf{k}) - (3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$

$= -4\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

$14 - 5t - 3 = -4$

$-5t = -15$

$t = 3$ (A1) for correct value

$\therefore \begin{cases} x = 14 - 5(3) = -1 \\ y = 18 - 6(3) = 0 \\ z = 8 - 2(3) = 2 \end{cases}$ (M1) for substitution

Thus, the coordinates of Q are $(-1, 0, 2)$. A1

[4]

(d) $\overrightarrow{PQ}=(-\mathbf{i}+2\mathbf{k})-(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$

$\overrightarrow{PQ}=-5\mathbf{i}-6\mathbf{j}-2\mathbf{k}$ (A1) for correct values

$\overrightarrow{PR}=(3\mathbf{i}+4\mathbf{j}+3\mathbf{k})-(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$

$\overrightarrow{PR}=-\mathbf{i}-2\mathbf{j}-\mathbf{k}$ (A1) for correct values

$$\begin{cases} x+6=-4 \Rightarrow x=-10 \\ \dfrac{y+14}{2}=-4 \Rightarrow y=-22 \\ z+6=-4 \Rightarrow z=-10 \end{cases}$$

$\overrightarrow{PS}=(-10\mathbf{i}-22\mathbf{j}-10\mathbf{k})-(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$

$\overrightarrow{PS}=-14\mathbf{i}-28\mathbf{j}-14\mathbf{k}$ (A1) for correct values

$$\begin{cases} x=14-5(8)=-26 \\ y=18-6(8)=-30 \\ z=8-2(8)=-8 \end{cases}$$

$\overrightarrow{PT}=(-26\mathbf{i}-30\mathbf{j}-8\mathbf{k})-(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$

$\overrightarrow{PT}=-30\mathbf{i}-36\mathbf{j}-12\mathbf{k}$ (A1) for correct values

The area of the quadrilateral QRST

$=$ The area of PST $-$ The area of PQR (M1) for valid approach

$=\frac{1}{2}\left|\overrightarrow{PS}\times\overrightarrow{PT}\right|-\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

$=\frac{1}{2}\left|(-14\mathbf{i}-28\mathbf{j}-14\mathbf{k})\times(-30\mathbf{i}-36\mathbf{j}-12\mathbf{k})\right|$

(A1) for substitution

$-\frac{1}{2}\left|(-5\mathbf{i}-6\mathbf{j}-2\mathbf{k})\times(-\mathbf{i}-2\mathbf{j}-\mathbf{k})\right|$

$=\frac{1}{2}(14)(6)\left|(-\mathbf{i}-2\mathbf{j}-\mathbf{k})\times(-5\mathbf{i}-6\mathbf{j}-2\mathbf{k})\right|$

$-\frac{1}{2}\left|(-5\mathbf{i}-6\mathbf{j}-2\mathbf{k})\times(-\mathbf{i}-2\mathbf{j}-\mathbf{k})\right|$

$=\frac{83}{2}\left|(-\mathbf{i}-2\mathbf{j}-\mathbf{k})\times(-5\mathbf{i}-6\mathbf{j}-2\mathbf{k})\right|$ (M1) for simplification

$$=\frac{83}{2}\left|\begin{pmatrix}(-2)(-2)-(-1)(-6)\\(-1)(-5)-(-1)(-2)\\(-1)(-6)-(-2)(-5)\end{pmatrix}\right|$$

$=\frac{83}{2}\left|-2\mathbf{i}+3\mathbf{j}-4\mathbf{k}\right|$

$= \frac{83}{2}\sqrt{(-2)^2 + 3^2 + 4^2}$

$= \frac{83\sqrt{29}}{2}$ A1

[8]

(e) $\frac{1}{3}\left(\frac{83\sqrt{29}}{2}\right)(\text{UQ}) = 166\sqrt{29}$ M1

$\frac{1}{6}\text{UQ} = 2$

$\text{UQ} = 12$

Thus, the shortest distance between U and QRST is 12. A1

[2]

**4.** (a) $\overrightarrow{BD} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$ A1

The vector equation of BD:

$\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$

$\begin{cases} x = 3 - 3t \\ y = -3t \\ z = 3 - 3t \end{cases}$ A1

$\overrightarrow{AE} = \begin{pmatrix} 3-3t \\ -3t \\ 3-3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{AE} = \begin{pmatrix} -3t \\ -3t \\ 3-3t \end{pmatrix}$ A1

$\overrightarrow{AE} \cdot \overrightarrow{BD} = 0$

$\therefore (-3t)(-3) + (-3t)(-3) + (3-3t)(-3) = 0$ M1

$9t + 9t - 9 + 9t = 0$

$27t = 9$

$t = \frac{1}{3}$ A1

$\therefore \begin{cases} x = 3 - 3\left(\frac{1}{3}\right) = 2 \\ y = -3\left(\frac{1}{3}\right) = -1 \\ z = 3 - 3\left(\frac{1}{3}\right) = 2 \end{cases}$ M1

Therefore, the coordinates of E are $(2, -1, 2)$. AG

[6]

(b) $\overrightarrow{BA} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

$\overrightarrow{BA} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$ (A1) for correct values

$\overrightarrow{BC} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ (A1) for correct values

$\mathbf{n}_1 = \overrightarrow{BA} \times \overrightarrow{BD}$ (M1) for valid approach

$\mathbf{n}_1 = -3\mathbf{k} \times (-3\mathbf{i} - 3\mathbf{j} - 3\mathbf{k})$

$\mathbf{n}_1 = \begin{pmatrix} (0)(-3) - (-3)(-3) \\ (-3)(-3) - (0)(-3) \\ (0)(-3) - (0)(-3) \end{pmatrix}$

$\mathbf{n}_1 = -9\mathbf{i} + 9\mathbf{j}$ A1

$\mathbf{n}_2 = \overrightarrow{BC} \times \overrightarrow{BD}$ (M1) for valid approach

$\mathbf{n}_2 = -3\mathbf{i} \times (-3\mathbf{i} - 3\mathbf{j} - 3\mathbf{k})$

$\mathbf{n}_2 = \begin{pmatrix} (0)(-3) - (0)(-3) \\ (0)(-3) - (-3)(-3) \\ (-3)(-3) - (0)(-3) \end{pmatrix}$

$\mathbf{n}_2 = -9\mathbf{j} + 9\mathbf{k}$ A1

$\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) for valid approach

$(-9\mathbf{i} + 9\mathbf{j}) \cdot (-9\mathbf{j} + 9\mathbf{k}) = |-9\mathbf{i} + 9\mathbf{j}||-9\mathbf{j} + 9\mathbf{k}|\cos\theta$

$(-9)(0) + (9)(-9) + (0)(9)$

$= (\sqrt{(-9)^2 + 9^2})(\sqrt{(-9)^2 + 9^2})\cos\theta$ A1

$-81 = 162\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$ A1

[9]

(c) The area of OABC

$= (OA)(OC)$

$= (3)(3)$

$= 9$ (A1) for correct value

$\therefore \frac{1}{3}(9)(OF) = 15$ (M1) for setting equation

$3OF = 15$

$OF = 5$ A1

Thus, the possible coordinates of F are $(0, 5, 0)$ and $(0, -5, 0)$. A1

The possible values of DF

$= 5-(-3)$ or $= (-3)-(-5)$

$= 8$ or $2$ A2

[6]

## Exercise 31

**1.** Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be the normal vectors of $2x-9y+4z-1=0$ and $3x+14y+3z-3=0$ respectively.

$\mathbf{n}_1 = \begin{pmatrix} 2 \\ -9 \\ 4 \end{pmatrix}$

$\mathbf{n}_2 = \begin{pmatrix} 3 \\ 14 \\ 3 \end{pmatrix}$ (A1) for correct values

Let $\theta$ be the acute angle between the planes.

$\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) for valid approach

$(2)(3)+(-9)(14)+(4)(3)$

$=(\sqrt{2^2+(-9)^2+4^2})(\sqrt{3^2+14^2+3^2})\cos\theta$ (A1) for substitution

$-108=(\sqrt{101})(\sqrt{214})\cos\theta$

$\cos\theta = -\frac{108}{(\sqrt{101})(\sqrt{214})}$ A1

$\theta = 137.2741785^\circ$

$\theta = 137^\circ$ A1

[5]

**2.** By using row operations, the system $\left(\begin{array}{ccc|c} 5 & 3 & -2 & 7 \\ 4 & 2 & -4 & 5 \end{array}\right)$ is

reduced to $\left(\begin{array}{ccc|c} 1 & 0 & -4 & \frac{1}{2} \\ 0 & 1 & 6 & \frac{3}{2} \end{array}\right)$. (M1) for valid approach

$y + 6z = \frac{3}{2}$

$y = \frac{3}{2} - 6z$ A1

$x - 4z = \frac{1}{2}$

$x = \frac{1}{2} + 4z$ A1

Let $z = t$.

$x = \frac{1}{2} + 4t$

$y = \frac{3}{2} - 6t$

Thus, the vector equation of the line of intersection is

$\mathbf{r} = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ -6 \\ 1 \end{pmatrix}$. A2

[5]

**3.** By using row operations, the system $\left(\begin{array}{ccc|c} 4 & 2 & -1 & 5 \\ 3 & 1 & -2 & 2 \\ 1 & 1 & 1 & 3 \end{array}\right)$ is

reduced to $\left(\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} & \frac{7}{2} \\ 0 & 0 & 0 & 0 \end{array}\right)$. (M1) for valid approach

$y+\frac{5}{2}z=\frac{7}{2}$

$y=\frac{7}{2}-\frac{5}{2}z$ A1

$x-\frac{3}{2}z=-\frac{1}{2}$

$x=-\frac{1}{2}+\frac{3}{2}z$ A1

Let $z=t$.

$x=-\frac{1}{2}+\frac{3}{2}t$, $y=\frac{7}{2}-\frac{5}{2}t$

Thus, the vector equation of the line of intersection is

$\mathbf{r}=\begin{pmatrix} -\frac{1}{2} \\ \frac{7}{2} \\ 0 \end{pmatrix}+t\begin{pmatrix} \frac{3}{2} \\ -\frac{5}{2} \\ 1 \end{pmatrix}$. A2

[5]

**4.** (a) By using row operations, the system

$\left(\begin{array}{ccc|c} 2 & -2 & -3 & 9 \\ 1 & -4 & -4 & 9 \\ 2 & 1 & 2 & -3 \end{array}\right)$ is reduced to $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right)$. (M1) for valid approach

Thus, the coordinates of A are $(1,1,-3)$. A3

[4]

(b) $\mathbf{r}=\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}+t\begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$ A2

[2]

## Exercise 32

**1.** (a) $\mathbf{n}=(3\mathbf{i}-2\mathbf{j})\times(\mathbf{j}-3\mathbf{k})$ (M1) for valid approach

$$\mathbf{n}=\begin{pmatrix}(-2)(-3)-(0)(1)\\(0)(0)-(3)(-3)\\(3)(1)-(-2)(0)\end{pmatrix}$$

$\mathbf{n}=6\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ (A1) for correct values

The Cartesian equation of the plane $\pi$:

$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(6\mathbf{i}+9\mathbf{j}+3\mathbf{k})$

$=(-6\mathbf{i}+18\mathbf{k})\cdot(6\mathbf{i}+9\mathbf{j}+3\mathbf{k})$ M1A1

$6x+9y+3z=(-6)(6)+(0)(9)+(18)(3)$

$6x+9y+3z=18$

$2x+3y+z=6$ A1

[5]

(b) $\mathbf{r}=\begin{pmatrix}1\\3\\13\end{pmatrix}+t\begin{pmatrix}-10\\5\\-13\end{pmatrix}$

$$\begin{cases}x=1-10t\\y=3+5t\\z=13-13t\end{cases}$$

$\therefore 2(1-10t)+3(3+5t)+(13-13t)=6$ (M1) for substitution

$2-20t+9+15t+13-13t=6$

$-18t=-18$

$t=1$ (A1) for correct value

$$\therefore\begin{cases}x=1-10(1)=-9\\y=3+5(1)=8\\z=7-13(1)=-6\end{cases}$$

Thus, the coordinates of the point of intersection are $(-9,8,-6)$. A1

[3]

(c) $a=3$, $b=2$, $c=6$ A3

[3]

(d) The volume of the pyramid OABC

$= \frac{1}{3}\left(\frac{(\text{OA})(\text{OB})}{2}\right)(\text{OC})$ (M1) for valid approach

$= \frac{1}{3}\left(\frac{(3)(2)}{2}\right)(6)$ (A1) for substitution

$= 6$ A1

[3]

(e) (i) $\overrightarrow{\text{CA}} = 3\mathbf{i} - 6\mathbf{k}$ , $\overrightarrow{\text{CB}} = 2\mathbf{j} - 6\mathbf{k}$ A2

(ii) $\frac{1}{2}\left|\overrightarrow{\text{CA}} \times \overrightarrow{\text{CB}}\right| = \alpha\sqrt{14}$ (M1) for setting equation

$\frac{1}{2}\left|(3\mathbf{i} - 6\mathbf{k}) \times (2\mathbf{j} - 6\mathbf{k})\right| = \alpha\sqrt{14}$

$\frac{1}{2}\left|\begin{pmatrix} (0)(-6) - (-6)(2) \\ (-6)(0) - (3)(-6) \\ (3)(2) - (0)(0) \end{pmatrix}\right| = \alpha\sqrt{14}$

$\frac{1}{2}\left|12\mathbf{i} + 18\mathbf{j} + 6\mathbf{k}\right| = \alpha\sqrt{14}$ (A1) for correct values

$\frac{1}{2}(6\left|2\mathbf{i} + 3\mathbf{j} + \mathbf{k}\right|) = \alpha\sqrt{14}$

$3\sqrt{2^2 + 3^2 + 1^2} = \alpha\sqrt{14}$ M1

$3\sqrt{14} = \alpha\sqrt{14}$

$\therefore \alpha = 3$ A1

[6]

(f) Let $h$ be the required perpendicular distance.

$\frac{1}{3}(3\sqrt{14})h = 6$ (M1) for setting equation

$h = \frac{6}{\sqrt{14}}$

$h = \frac{3\sqrt{14}}{11}$

Thus, the required perpendicular distance is $\frac{3\sqrt{14}}{11}$. A1

[2]

**2.** (a) $\mathbf{n} = (3\mathbf{i}+\mathbf{j})\times(4\mathbf{i}+\mathbf{k})$ (M1) for valid approach

$$\mathbf{n} = \begin{pmatrix} (1)(1)-(0)(0) \\ (0)(4)-(3)(1) \\ (3)(0)-(1)(4) \end{pmatrix}$$

$\mathbf{n} = \mathbf{i}-3\mathbf{j}-4\mathbf{k}$ (A1) for correct values

The Cartesian equation of the plane $\pi$:

$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(\mathbf{i}-3\mathbf{j}-4\mathbf{k}) = (-6\mathbf{i}+2\mathbf{j})\cdot(\mathbf{i}-3\mathbf{j}-4\mathbf{k})$ M1A1

$x-3y-4z = (-6)(1)+(2)(-3)+(0)(-4)$

$x-3y-4z = -12$ A1

[5]

(b) $x-3(0)-4(0) = -12$

$x = -12$

$\therefore \text{OA} = 12$ (A1) for correct value

$0-3y-4(0) = -12$

$y = 4$

$\therefore \text{OB} = 4$ (A1) for correct value

$0-3(0)-4z = -12$

$z = 3$

$\therefore \text{OC} = 3$ (A1) for correct value

The volume of the pyramid OABC

$= \frac{1}{3}\left(\frac{(\text{OA})(\text{OB})}{2}\right)(\text{OC})$ (M1) for valid approach

$= \frac{1}{3}\left(\frac{(12)(4)}{2}\right)(3)$ A1

$= 24$ A1

[6]

(c) The vector equation of the line $L$:

$\mathbf{r} = \begin{pmatrix} -12 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix}$ A2

[2]

(d) (i) (12, 0, 0) A1

(ii) The vector equation of the line $L'$:

$$\mathbf{r}=\begin{pmatrix}12\\0\\0\end{pmatrix}+s\begin{pmatrix}1\\-3\\-4\end{pmatrix}$$ (A1) for correct values

$$\begin{cases}x=12+s\\y=-3s\\z=-4s\end{cases}$$

$$x-12=-\frac{y}{3}=-\frac{z}{4}$$ A1

(iii) $$-\frac{\beta+12}{3}=-\frac{\beta}{4}$$ (M1) for setting equation

$$4(\beta+12)=3\beta$$

$$4\beta+48=3\beta$$

$$\beta=-48$$ A1

[5]

3. (a) By using row operations, the system

$$\left(\begin{array}{ccc|c} 3 & -1 & 2 & 12 \\ 3 & 1 & -2 & -12 \end{array}\right)$$ is reduced to

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -12 \end{array}\right).$$ M1

$y-2z=-12$

$y=-12+2z$ A1

$x=0$ A1

Let $z=t$.

$y=-12+2t$ A1

Thus, the vector equation of the line of intersection

is $\mathbf{r}=\begin{pmatrix} 0 \\ -12 \\ 0 \end{pmatrix}+t\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$. AG

[4]

(b) (i) $a=4$, $b=-12$, $c=6$, $\alpha=-4$ A4

(ii) Let O be the origin.

The volume of the pyramid A′ABC

$=\frac{1}{3}\left(\frac{(\text{A}'\text{A})(\text{OB})}{2}\right)(\text{OC})$ (M1) for valid approach

$=\frac{1}{3}\left(\frac{(4-(-4))(12)}{2}\right)(6)$ A1

$=96$ A1

[7]

(c) (i) $\overrightarrow{AC} = -4\mathbf{i} + 6\mathbf{k}$

$\overrightarrow{AC} \cdot (-\mathbf{i}) = \left|\overrightarrow{AC}\right| \left|-\mathbf{i}\right| \cos C\hat{A}A'$ (M1) for valid approach

$(-4\mathbf{i} + 6\mathbf{k}) \cdot (-\mathbf{i}) = (\sqrt{(-4)^2 + 6^2})(1) \cos C\hat{A}A'$ (A1) for substitution

$(-4)(-1) + (6)(0) = \sqrt{52} \cos C\hat{A}A'$

$\cos C\hat{A}A' = \dfrac{4}{\sqrt{52}}$

$C\hat{A}A' = 56.30993247^\circ$

$C\hat{A}A' = 56.3^\circ$ A1

(ii) $\because CA' = CA$

$\therefore C\hat{A}'A = 56.30993247^\circ$ (A1) for correct approach

$A\hat{C}A' + 56.30993247^\circ + 56.30993247^\circ = 180^\circ$

$A\hat{C}A' = 67.38013505^\circ$

$A\hat{C}A' = 67.4^\circ$ A1

[5]

(d) The vector equation of $L$:

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$\begin{cases} x = 3s \\ y = -s \\ z = 2s \end{cases}$ (A1) for correct approach

$\therefore 3(3s) + (-s) - 2(2s) = -12$ (A1) for substitution

$4s = -12$

$s = -3$

$\begin{cases} x = 3(-3) = -9 \\ y = -(-3) = 3 \\ z = 2(-3) = -6 \end{cases}$ M1

Thus, the coordinates of Q are $(-9, 3, -6)$. A1

[4]

**4.** (a) The coordinates of $\text{A}$, $\text{B}$ and $\text{C}'$ are $(6, 0, 0)$, $(0, -4, 0)$ and $(0, 0, 3)$ respectively. A1

$\mathbf{n} = \overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}'}$ M1

$\mathbf{n} = (-6\mathbf{i} - 4\mathbf{j}) \times (-6\mathbf{i} + 3\mathbf{k})$ A1

$$\mathbf{n} = \begin{pmatrix} (-4)(3) - (0)(0) \\ (0)(-6) - (-6)(3) \\ (-6)(0) - (-4)(-6) \end{pmatrix}$$

$\mathbf{n} = -12\mathbf{i} + 18\mathbf{j} - 24\mathbf{k}$ A1

The Cartesian equation of the plane $\pi_2$ :

$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (-12\mathbf{i} + 18\mathbf{j} - 24\mathbf{k})$

$= 6\mathbf{i} \cdot (-12\mathbf{i} + 18\mathbf{j} - 24\mathbf{k})$ M1A1

$-12x + 18y - 24z = (6)(-12) + (0)(18) + (0)(-24)$

$-12x + 18y - 24z = -72$

$2x - 3y + 4z = 12$ AG

[6]

(b) The coordinates of $\text{C}$ are $(0, 0, -3)$. (A1) for correct values

The volume of the pyramid $\text{ABCC}'$

$= \dfrac{1}{3}\left(\dfrac{(\text{CC}')(\text{OA})}{2}\right)(\text{OB})$ (M1) for valid approach

$= \dfrac{1}{3}\left(\dfrac{(3 - (-3))(6)}{2}\right)(4)$ A1

$= 24$ A1

[4]

(c) $\mathbf{n}_1 = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$

$\mathbf{n}_2 = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ (A1) for correct values

Let $\theta$ be the obtuse angle between the planes.

$\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) for valid approach

$(2)(2) + (-3)(-3) + (-4)(4)$

$= (\sqrt{2^2 + (-3)^2 + (-4)^2})(\sqrt{2^2 + (-3)^2 + 4^2})\cos\theta$ (A1) for substitution

$-3 = (\sqrt{29})(\sqrt{29})\cos\theta$

$\cos\theta = -\dfrac{3}{29}$ A1

$\theta = 95.93777245^\circ$

$\theta = 95.9^\circ$ A1

[5]

(d) (i) $(0, -2, -1.5)$ A1

(ii) $\mathbf{n}_3 = \mathbf{n}_1 \times \mathbf{n}_2$ (M1) for valid approach

$\mathbf{n}_3 = (2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) \times (2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$

$$\mathbf{n}_3 = \begin{pmatrix} (-3)(4) - (-4)(-3) \\ (-4)(2) - (2)(4) \\ (2)(-3) - (-3)(2) \end{pmatrix}$$

$\mathbf{n}_3 = -24\mathbf{i} - 16\mathbf{j}$ A1

The vector equation of the line:

$$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ -1.5 \end{pmatrix} + t \begin{pmatrix} -24 \\ -16 \\ 0 \end{pmatrix}$$ A1

$$\begin{cases} x = -24t \\ y = -2 - 16t \\ z = -1.5 \end{cases}$$

$\dfrac{x}{-24} = \dfrac{y+2}{-16}, \; z = -1.5$ A1

[5]