Applications and Interpretation Higher Level for IBDP Mathematics
Practice Paper Set 1 – Paper 2 (120 Minutes)

Question – Answer Book

Instructions

1. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

2. A graphic display calculator is needed.

3. You are suggested to prepare a formula booklet of Applications and Interpretation for IBDP Mathematics when attempting the questions.

4. Supplementary answer sheets and graph papers will be supplied on request.

5. Unless otherwise specified, **ALL** working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

7. The diagrams in this paper are **NOT** necessarily drawn to scale.

8. Information to be read before you start the exam:

	Marker's Use Only	**Examiner's Use Only**	
Question Number	**Marks**	**Marks**	**Maximum Mark**
1			**17**
2			**17**
3			**19**
4			**12**
5			**15**
6			**17**
7			**13**
Overall			
Paper 2 Total			**110**

1. The equation of the straight line L_1 is given by $3x+y-10=0$. The coordinates of the point P are $(3,1)$.

(a) Show that P lies on L_1.

[1]

(b) Write down the y-intercept of L_1.

[1]

The coordinates of the point Q are $(11,-3)$. M is the mid-point of PQ.

(c) Find

(i) the coordinates of M;

(ii) the gradient of PQ;

(iii) the distance between P and Q.

[6]

The straight line L_2 passes through P and Q.

(d) Show that L_1 and L_2 are not perpendicular.

[2]

The straight line L_3 passes through P and is perpendicular to L_1.

(e) Show that the equation of L_3 is $x-3y=0$.

[4]

L_1 and L_3 intersect with the y-axis at R and S respectively.

(f) Find the area of the triangle PRS.

[3]

2. The relationship between the body temperature and the pulse rate of the students from a sports team is investigated. Six students from the group A of the team are first medically examined and their body temperature and their pulse rates are recorded in the table below.

Student	A	B	C	D	E	F
Body Temperature ($x^\circ\text{C}$)	35.8	36.2	36.4	36.7	37.4	37.1
Pulse Rate (y beats per minute)	80	81	87	117	100	93

(a) The relationship between the variables is modelled by the regression equation $y = ax + b$.

(i) Write down the value of a and of b.

(ii) Hence, estimate the pulse rate of a student whose body temperature is 37°C.

[4]

(b) (i) Write down the correlation coefficient.

(ii) State which **two** of the following describe the correlation between the variables.

[3]

positive	strong	zero
negative	weak	moderate

A similar investigation has been completed last year. The pulse rates of 100 students were recorded and the data was presented as follows:

Pulse Rate (y beats per minute)	Frequency
$75 \le y < 85$	16
$85 \le y < 95$	23
$95 \le y < 105$	32
$105 \le y < 115$	12
$115 \le y < 125$	17

Someone claims that the distribution of the data is expected to be evenly distributed. Hence, a χ^2 goodness of fit test is conducted at a 5% significance level.

(c) (i) Write down the null hypothesis of the test.

(ii) Find the p-value.

(iii) Hence, state the conclusion of the test with a reason.

[5]

Another five students from the Group B of the team are also medically examined and their pulse rates are recorded in the table below.

Student	G	H	I	J	K
Pulse Rate (y beats per minute)	95	99	117	87	110

The team manager wants to know whether the mean pulse rates μ_A and μ_B of the students from the Group A and the Group B respectively are different. A t-test is conducted at a 1% significance level.

(d) (i) Write down the alternative hypothesis of the test.

(ii) Find the p-value.

(iii) Hence, state the conclusion of the test with a reason.

[5]

3. The function f is given by $f(x)=\frac{4}{3}x^3+5x^2-6x+2$, $x\in\mathbb{R}$.

(a) Write down the y-intercept of the graph of f.

[1]

(b) Find $f(3)$.

[2]

(c) Find $f'(x)$.

[2]

(d) Solve the equation $f'(x)=0$.

[3]

(e) Write down the equations of the horizontal tangents of the graph of f.

[2]

(f) Write down the range of values of w such that the equation $f(x)=w$ has

(i) three solutions;

(ii) only one solution.

[4]

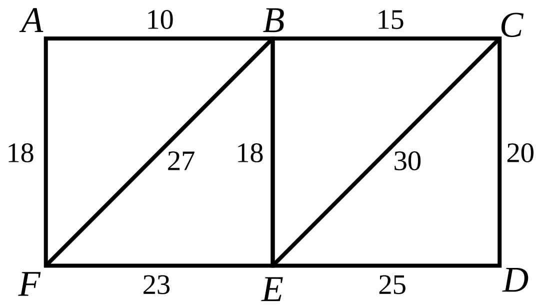
(g) Find the gradient of the tangent at $x=3$.

[2]

(h) Hence, show that the equation of the normal at $x=3$ is $x+60y-3903=0$.

[3]

4. Consider the following weighted graph:



(a) Write down

(i) the degree of B;

(ii) the number of vertices of odd degree;

(iii) the number of vertices of even degree.

[3]

Kruskal's algorithm is used to find the minimum spanning tree for this graph.

(b) State the edge of the smallest weight.

[1]

(c) By using the algorithm, find the minimum spanning tree.

[3]

(d) Write down the weight of the minimum spanning tree.

[1]

(e) Use the Chinese postman algorithm to find a possible route of minimum weight that passes through all edges, starting and finishing at C.

[3]

(f) Write down the corresponding weight of the route.

[1]

5. Let $\mathbf{M} = \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix}$.

(a) (i) Find $\mathbf{M}^2$.

(ii) Find $\mathbf{M}^3$.

(iii) By using the above results, write down $\mathbf{M}^{30}$.

[5]

Let $s(n) = \mathbf{M} + \mathbf{M}^2 + \mathbf{M}^3 + \cdots + \mathbf{M}^n$, where $n \geq 1$.

(b) (i) Write down $s(2)$.

(ii) Write down $s(3)$.

(iii) By using the above results, find $s(30)$.

[6]

Let $r(n) = \mathbf{M} + \mathbf{M}^2 + \mathbf{M}^4 + \mathbf{M}^8 + \cdots + \mathbf{M}^{2^{n-1}}$, where $n \geq 1$.

(c) Find $r(10)$.

[4]

6. In an experiment, a metal ball moves with velocity $v\,\text{cms}^{-1}$ and displacement $x\,\text{cm}$ with respect to the starting point O. By considering the rate of change of its velocity, the relationship between the variables can be modelled by the differential equation $\frac{\text{d}^2x}{\text{d}t^2} = 25x$.

(a) By using $v = \frac{\text{d}x}{\text{d}t}$, express the differential equation in a coupled system.

[1]

Euler's method with a step length of 0.2 is used to approximate the displacement of the particle at $t = 1$. It is given that initially the particle is at rest with displacement one centimetre.

(b) Find, when $t = 0.2$, the approximate value of

(i) v;

(ii) x.

[4]

(c) Write down the approximate value of the displacement at

(i) $t = 0.4$;

(ii) $t = 1$;

(iii) $t = 2.6$.

[3]

The system can be expressed by a matrix equation $\dot{\mathbf{X}} = \mathbf{MX}$, where $\mathbf{M}$ is a 2×2 matrix, and $\dot{\mathbf{X}} = \begin{pmatrix} \frac{\text{d}v}{\text{d}t} \\ \frac{\text{d}x}{\text{d}t} \end{pmatrix}$ and $\mathbf{X} = \begin{pmatrix} v \\ x \end{pmatrix}$ are two 2×1 matrices. Let λ_1 and λ_2 be the eigenvalues of $\mathbf{M}$, where $\lambda_1 < \lambda_2$.

(d) Find $\det(\mathbf{M} - \lambda\mathbf{I})$, giving the answer in terms of λ.

[2]

(e) Hence, write down the values of λ_1 and λ_2.

[2]

Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the eigenvectors of $\mathbf{M}$ corresponding to λ_1 and λ_2 respectively.

(f) Write down $\mathbf{v}_1$ and $\mathbf{v}_2$.

[2]

(g) Hence, show that the particular solution of x is $x = 0.5e^{-5t} + 0.5e^{5t}$.

[3]

7. In a game, two balls A and B are moving in a three-dimensional space, which can be modelled by a three-dimensional coordinate plane. At the start of the game, A is at $(5, 5, 0)$ and its velocity vector is $\begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$.

(a) Write down the vector equation for the displacement of A, giving the answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$,

[2]

After p seconds, A is at $(-85, 95, 0)$.

(b) Find p.

[2]

At the start of the game, B is at $(0, 0, -50)$. After 5 seconds, it is at $(-50, 50, 0)$.

(c) Find the velocity vector of B.

[2]

(d) Write down the vector equation for the displacement of B, giving the answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$,

[2]

(e) Find the shortest distance between A and B.

[4]

(f) Hence, write down the time when A and B are closest to each other.

[1]

END OF PAPER