AA SL Practice Set 3 Paper 1 Solution

Section A

1. (a) The equation of the axis of symmetry:

$x = -\frac{-20}{2(2)}$ (A1) for substitution

$x = 5$ A1 N2

[2]

(b) (i) 2 A1 N1

(ii) 5 A1 N1

(iii) $k = 2(5)^2 - 20(5) + 60$ (M1)(A1) for substitution

$k = 10$ A1 N2

[5]

2. (a) The common difference

$= 95 - 100$ (M1) for valid approach

$= -5$ A1 N2

[2]

(b) The fifteenth term

$= 100 + (15-1)(-5)$ (A1) for substitution

$= 30$ A1 N2

[2]

(c) The sum of the first fifteen terms

$= \frac{15}{2}\left[2(100) + (15-1)(-5)\right]$ (A1) for substitution

$= 975$ A1 N2

[2]

3. (a) The gradient of L_1 is 2. A1 N1

The y-intercept of L_1 is -20. A1 N1

[2]

(b) The gradient of L_2 is $-\frac{1}{2}$. (A1) for correct value

The equation of L_2:

$y-(-20)=-\frac{1}{2}(x-0)$ A1

$y+20=-\frac{1}{2}x$

$2y+40=-x$

$x+2y+40=0$ A1 N2

[3]

4. (a) (i) 4 A1 N1

(ii) $\frac{1}{3}$ A1 N1

(iii) -1 A1 N1

[3]

(b) $\log_{27} x+\frac{8}{3}=\log_4 256+\log_{125} 5+\log_\pi \frac{1}{\pi}$

$\log_{27} x+\frac{8}{3}=4+\frac{1}{3}-1$ (M1) for substitution

$\log_{27} x=\frac{2}{3}$

$x=27^{\frac{2}{3}}$ (A1) for correct approach

$x=(3^3)^{\frac{2}{3}}$

$x=3^2$

$x=9$ A1 N3

[3]

5. $\left(1-\frac{3}{4}x\right)^n(1+2nx)^3$

$=\left(1+\binom{n}{1}\left(-\frac{3}{4}x\right)+\cdots\right)\left(1+\binom{3}{1}(2nx)+\cdots\right)$ (M1) for valid expansion

$=\left(1+(n)\left(-\frac{3}{4}x\right)+\cdots\right)(1+(3)(2nx)+\cdots)$ (A1) for correct approach

$=\left(1-\frac{3}{4}nx+\cdots\right)(1+6nx+\cdots)$ A2

The coefficient of x

$=(1)(6n)+\left(-\frac{3}{4}n\right)(1)$ (A1) for correct approach

$=\frac{21}{4}n$

$\therefore \frac{21}{4}n=\frac{105}{4}$ (M1) for setting equation

$n=5$ A1 N5

[7]

6. $-3\sqrt{3}\le f(x)\le 3\sqrt{3}$

$-3\sqrt{3}\le 6\sin 2x\le 3\sqrt{3}$

$-\frac{\sqrt{3}}{2}\le \sin 2x\le \frac{\sqrt{3}}{2}$ A1

$\therefore \sin\left(-\frac{\pi}{3}\right)\le \sin 2x\le \sin\frac{\pi}{3}$,

$\sin\left(\pi-\frac{\pi}{3}\right)\le \sin 2x\le \sin\left(\pi+\frac{\pi}{3}\right)$ or (A2) for correct ranges

$\sin\left(2\pi-\frac{\pi}{3}\right)\le \sin 2x\le \sin\left(2\pi+\frac{\pi}{3}\right)$

$-\frac{\pi}{3}\le 2x\le\frac{\pi}{3}$, $\frac{2\pi}{3}\le 2x\le\frac{4\pi}{3}$ or $\frac{5\pi}{3}\le 2x\le\frac{7\pi}{3}$ A1

$-\frac{\pi}{6}\le x\le\frac{\pi}{6}$, $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ or $\frac{5\pi}{6}\le x\le\frac{7\pi}{6}$ (M1) for valid approach

$\therefore 0\le x\le\frac{\pi}{6}$, $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ or $\frac{5\pi}{6}\le x\le\frac{7\pi}{6}$ A3 N4

[8]

Section B

7. (a) (i) The number of girls

$= 35 + 45 - 50$ (M1) for valid approach

$= 30$ A1 N2

(ii) 5 A1 N1

[3]

(b) (i) $\frac{9}{10}$ A1 N1

(ii) The required probability

$= \frac{30}{50} \div \frac{9}{10}$ (A1) for substitution

$= \frac{2}{3}$ A1 N2

[3]

(c) The required probability

$= \left(\frac{5}{50}\right)\left(\frac{4}{49}\right)$ (M1) for valid approach

$= \frac{2}{245}$ A1 N2

[2]

(d) (i) $\mathrm{P}(G \cap V) = \frac{30}{50}$ A1

$\because \mathrm{P}(G \cap V) \neq 0$ R1

Thus, G and V are not mutually exclusive. AG N0

(ii) $\mathrm{P}(G) = \frac{35}{50}$ A1

$\mathrm{P}(V) = \frac{45}{50}$

$\mathrm{P}(G) \cdot \mathrm{P}(V) = \frac{63}{100}$ A1

$\because \mathrm{P}(G) \cdot \mathrm{P}(V) \neq \mathrm{P}(G \cap V)$ R1

Thus, G and V are not independent. AG N0

[5]

8. (a) $f''(x) = k(2x) - 12(1) - 0$ (A1) for correct derivatives

$f''(x) = 2kx - 12$ A1 N2

[2]

(b) $f''(1.5) = 0$ M1

$\therefore 2k(1.5) - 12 = 0$ A1

$3k = 12$ A1

$k = 4$ AG N0

[3]

(c) $f'(4) = 4(4)^2 - 12(4) - 40$ (M1) for substitution

$f'(4) = -24$ A1

The slope of the normal

$= \frac{-1}{-24}$ (A1) for correct approach

$= \frac{1}{24}$

The equation of the normal:

$y - \frac{13}{6} = \frac{1}{24}(x - 4)$ M1A1

$y - \frac{13}{6} = \frac{1}{24}x - \frac{1}{6}$

$y = \frac{1}{24}x + 2$ A1 N3

[6]

(d) $f''(5) = 2(4)(5) - 12$ M1

$f''(5) = 28$ A1

$f''(5) > 0$ R1

Thus, the graph of f has a local minimum at $x = 5$. AG N0

[3]

9. (a) $g(x) - f(x) = 0$

$e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right) = 0$ (M1) for valid approach

$e^{\frac{1}{2}\sqrt{x}}\left(1 - \sin\left(\frac{\pi}{3}x\right)\right) = 0$

$1 - \sin\left(\frac{\pi}{3}x\right) = 0$

$\sin\left(\frac{\pi}{3}x\right) = 1$ A1

$\frac{\pi}{3}x = \frac{\pi}{2}$, $\frac{\pi}{3}x = \frac{5\pi}{2}$ or $\frac{\pi}{3}x = \frac{9\pi}{2}$ (A1) for correct values

$x = \frac{3}{2}$, $x = \frac{15}{2}$ or $x = \frac{27}{2}$ A3 N3

[6]

(b) (i) $\frac{\pi}{3}x_n = \frac{\pi}{2} + (n-1)(2\pi)$ A1

$x_n = \frac{3}{2} + 6(n-1)$

$x_{n+1} - x_n$

$= \left(\frac{3}{2} + 6((n+1)-1)\right) - \left(\frac{3}{2} + 6(n-1)\right)$ M1

$x_{n+1} - x_n = \left(\frac{3}{2} + 6n\right) - \left(\frac{3}{2} + 6n - 6\right)$

$x_{n+1} - x_n = 6$ A1

The differences between each pair of consecutive terms are equal to 6. Thus, $x_1, x_2, x_3, \ldots$ is an arithmetic sequence. AG N0

(ii) $x_n = \frac{3}{2} + 6n - 6$

$x_n = 6n - \frac{9}{2}$ A1 N1

[4]

(c) Note that $x_2 = \frac{15}{2}$ and $x_3 = \frac{27}{2}$.

$f(x) = 0$

$e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right) = 0$ M1

$\sin\left(\frac{\pi}{3}x\right) = 0$

$\frac{\pi}{3}x = 3\pi$ or $\frac{\pi}{3}x = 4\pi$

$x = 9$ or $x = 12$ (A1) for correct values

$\therefore R = \int_{\frac{15}{2}}^{9}\left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}}\sin\left(\frac{\pi}{3}x\right)\right)\mathrm{d}x + \int_{9}^{12} e^{\frac{1}{2}\sqrt{x}}\,\mathrm{d}x$ A2 N3

$+\int_{12}^{\frac{27}{2}}\left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}}\sin\left(\frac{\pi}{3}x\right)\right)\mathrm{d}x$

[4]