Chapter 3 Solution

Exercise 12

1. (a) $a+b+c=998$ A1
$9a+3b+c=982$ A1
$36a+6b+c=928$ A1
[3]

(b) $a=-2$, $b=0$ and $c=1000$
For any one correct answer A1
For all correct answers A1
[2]

2. (a) $x+y+z=8400$ A1
$x+z=y-6288$ A1
[2]

(b) $42x+84y+21z=655872$ A1

By solving the system $\begin{cases} x+y+z=8400 \\ x-y+z=-6288 \\ 42x+84y+21z=655872 \end{cases}$,

$x=800$, $y=7344$ and $z=256$.
For any one correct answer A1
For all correct answers A1
[3]

3. (a) $\begin{cases} 10a+12b+13c=150 \\ 14a+8b+19c=178 \\ 22a+23b+7c=230 \end{cases}$ M1A1

By solving the system $\begin{cases} 10a+12b+13c=150 \\ 14a+8b+19c=178 \\ 22a+23b+7c=230 \end{cases}$,

$a=5$, $b=4$ and $c=4$.

For any one correct answer A1

For all correct answers A1

[4]

(b) The total price

$=5(30)+4(0)+4(35)$ M1

$=\$290$ A1

[2]

4. (a) $\begin{cases} 30x+16y=152 \\ 23x+15y+8z=114 \\ 11x+17y+18z=60 \end{cases}$ M1A1

By solving the system $\begin{cases} 30x+16y=152 \\ 23x+15y+8z=114 \\ 11x+17y+18z=60 \end{cases}$,

$x=4$, $y=2$ and $z=-1$.

For any one correct answer A1

For all correct answers A1

[4]

(b) A team drops 1 point for losing a game. A1

[1]

Exercise 13

1. (a) $\begin{cases} x+3y-2z=3 \\ 2x+y-z=1 \\ -x+2y+az=2 \end{cases}$

$\rightarrow \begin{cases} x+3y-2z=3 \\ -5y+3z=-5 \quad (R_2-2R_1 \ \& \ R_3+R_1) \\ 5y+(a-2)z=5 \end{cases}$ M1A1

$\rightarrow \begin{cases} x+3y-2z=3 \\ -5y+3z=-5 \ (R_3+R_2) \\ (a+1)z=0 \end{cases}$ A1

The system has an infinite number of solutions.

$\therefore 0=a+1$

$a=-1$ A1

[4]

(b) The system has a unique solution when $a \neq -1$. R1

$\therefore z=0$

$-5y+3(0)=-5$

$y=1$

$x+3(1)-2(0)=3$

$x=0$

Thus, $x=0$, $y=1$ and $z=0$. A2

[3]

2. (a) $\begin{cases} x+y+z=3 \\ 2x+y-3z=-2 \\ x-2y+az=-21 \end{cases}$

$\rightarrow \begin{cases} x+y+z=3 \\ -y-5z=-8 \\ -3y+(a-1)z=-24 \end{cases}$ $(R_2-2R_1 \,\&\, R_3-R_1)$ M1A1

$\rightarrow \begin{cases} x+y+z=3 \\ -y-5z=-8 \, (R_3-3R_2) \\ (a+14)z=0 \end{cases}$ A1

The system has an infinite number of solutions.

$\therefore 0=a+14$

$a=-14$ A1

[4]

(b) The system has a unique solution when $a=6$. R1

$\therefore z=0$

$-y-5(0)=-8$

$y=8$

$x+8+0=3$

$x=-5$

Thus, $x=-5$, $y=8$ and $z=0$. A2

[3]

3. (a) (i) $\begin{cases} 2x - y + z = 1 \\ -x + y + az = 0 \\ x - 2y - 2z = b \end{cases}$

$\rightarrow \begin{cases} 2x - y + z = 1 \\ 0.5y + (a + 0.5)z = 0.5 \\ -1.5y - 2.5z = b - 0.5 \end{cases}$ M1A1

$(R_2 + 0.5R_1 \; \& \; R_3 - 0.5R_1)$

$\rightarrow \begin{cases} 2x - y + z = 1 \\ 0.5y + (a + 0.5)z = 0.5 \, (R_3 + 3R_2) \\ (3a - 1)z = b + 1 \end{cases}$ A1

The system has no solutions when
$3a - 1 = 0$ and $b + 1 \neq 0$.

$a = \frac{1}{3}$ and $b \neq -1$ A1

(ii) The system has a unique solution when
$3a - 1 \neq 0$.

$a \neq \frac{1}{3}$ A1

(iii) The system has an infinite number of solutions when $3a - 1 = 0$ and $b + 1 = 0$.

$a = \frac{1}{3}$ and $b = -1$ A1

[6]

(b) $(3(0) - 1)z = 0 + 1$

$z = -1$

$0.5y + (0 + 0.5)(-1) = 0.5$

$y = 2$

$2x - 2 + (-1) = 1$

$x = 2$

Thus, $x = 2$, $y = 2$ and $z = -1$. A2

[2]

4. (a) (i) $\begin{cases} 8x+3y+z=2 \\ 4x-4y+6z=1 \\ 4x+y+az=b \end{cases}$

$\rightarrow \begin{cases} 8x+3y+z=2 \\ -5.5y+5.5z=0 \\ 5y+(a-6)z=b-1 \end{cases}$ M1A1

$(R_2-0.5R_1 \ \& \ R_3-R_2)$

$\rightarrow \begin{cases} 8x+3y+z=2 \\ -5.5y+5.5z=0 \, (R_3+\frac{10}{11}R_2) \\ (a-1)z=b-1 \end{cases}$ A1

The system has no solutions when
$a-1=0$ and $b-1\neq 0$.
$a=1$ and $b\neq 1$ A1

(ii) The system has a unique solution when
$a-1\neq 0$.
$a\neq 1$ A1

(iii) The system has an infinite number of solutions when $a-1=0$ and $b-1=0$.
$a=1$ and $b=1$ A1

[6]

(b) The system has an infinite number of solutions.
$-5.5y+5.5t=0$
$-y+t=0$
$y=t$ A1
$8x+3t+t=2$
$x=0.25-0.5t$
Thus, $x=0.25-0.5t$, $y=t$ and $z=t$, where $t\in\mathbb{R}$. A1

[2]