

Chapter 1 Solution

Exercise 1

1. (a) $|2x+1| \leq 4x-1$
 $-(4x-1) \leq 2x+1 \leq 4x-1$ M1
 $-4x+1 \leq 2x+1$ and $2x+1 \leq 4x-1$
 $0 \leq 6x$ and $2 \leq 2x$
 $x \geq 0$ and $x \geq 1$ A1
 $\therefore x \geq 1$ A1
- (b) $|2x+1| > |4x-1|$
 $|2x+1|^2 > |4x-1|^2$
 $4x^2 + 4x + 1 > 16x^2 - 8x + 1$ M1
 $12x^2 - 12x < 0$ (A1) for correct inequality
 $12x(x-1) < 0$
 $\therefore 0 < x < 1$ A2
- [3]
- [4]

2. (a) $\frac{|x|+1}{3} < 2|x|-5$

$$\frac{1}{3}|x| + \frac{1}{3} < 2|x|-5 \quad \text{M1}$$

$$\frac{16}{3} < \frac{5}{3}|x|$$

$$|x| > \frac{16}{5}$$

$$\therefore x > \frac{16}{5} \text{ or } x < -\frac{16}{5} \quad \text{A2}$$

[3]

(b) $\left| \frac{x+1}{3} \right| < |2x-5|$

$$\left| \frac{x+1}{3} \right|^2 < |2x-5|^2$$

$$\frac{x^2+2x+1}{9} > 4x^2-20x+25 \quad \text{M1}$$

$$x^2+2x+1 > 36x^2-180x+225$$

$$35x^2-182x+224 < 0$$

(A1) for correct inequality

$$7(x-2)(5x-16) < 0$$

$$\therefore 2 < x < \frac{16}{5} \quad \text{A2}$$

[4]

3. $f(|x|) > 5$

$$2|x| + \frac{3}{|x|} > 5 \quad \text{M1}$$

$$2|x|^2 + 3 > 5|x|$$

$$2|x|^2 - 5|x| + 3 > 0 \quad \text{(A1) for correct inequality}$$

$$(|x|-1)(2|x|-3) > 0$$

$$\therefore |x| < 1 \text{ or } |x| > \frac{3}{2} \quad \text{A1}$$

$$\therefore 0 < x < 1 \text{ or } x > \frac{3}{2} \quad \text{A2}$$

[5]

4. $f(|x|) \geq |x| + 2$

$$\frac{|x|^3 - 14|x| + 8}{|x| + 4} \geq |x| + 2 \quad \text{M1}$$

$$|x|^3 - 14|x| + 8 \geq (|x| + 2)(|x| + 4)$$

$$|x|^3 - 14|x| + 8 \geq |x|^2 + 6|x| + 8$$

$$|x|^3 - |x|^2 - 20|x| \geq 0 \quad \text{(A1) for correct inequality}$$

$$|x|(|x|^2 - |x| - 20) \geq 0$$

$$|x|^2 - |x| - 20 \geq 0 \quad \text{M1}$$

$$(|x| + 4)(|x| - 5) \geq 0$$

$$\therefore |x| < -4 \text{ (Rejected) or } |x| > 5 \quad \text{A1}$$

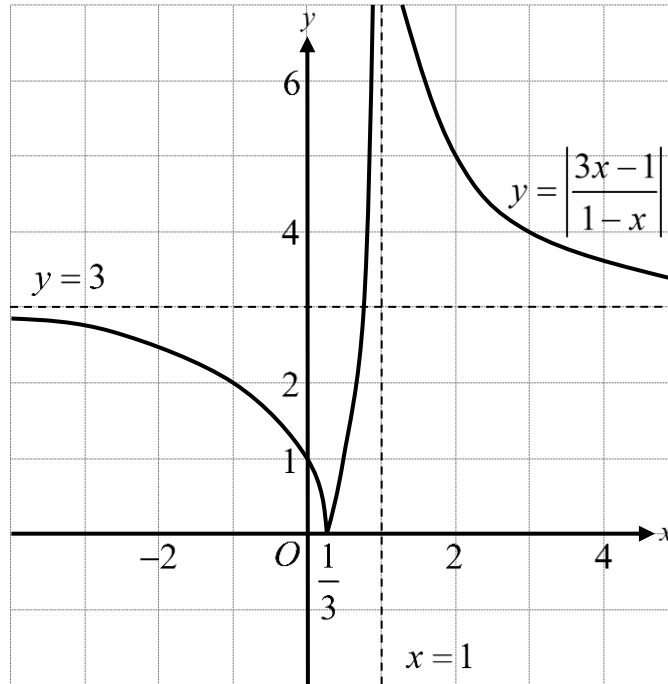
$$x > 5 \text{ (Rejected) or } x < -5 \quad \text{A1}$$

[5]

Exercise 2

1. (a) For correct asymptotes A1
 For correct intercepts A1
 For correct shape A2

[4]



(b) $\frac{|3x-1|}{|1-x|} = 5$

$\frac{3x-1}{1-x} = 5$ or $\frac{3x-1}{1-x} = -5$ M1

$3x-1 = 5(1-x)$ or $3x-1 = -5(1-x)$

$3x-1 = 5-5x$ or $3x-1 = -5+5x$

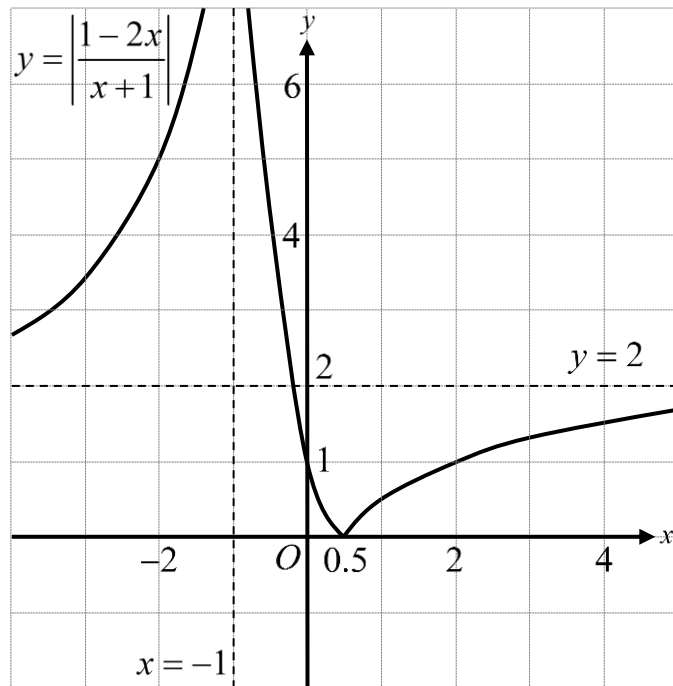
$8x = 6$ or $-2x = -4$

$x = \frac{3}{4}$ or $x = 2$ A2

[3]

2. (a) For correct asymptotes A1
 For correct intercepts A1
 For correct shape A2

[4]



- (b) 0 and 2 A2

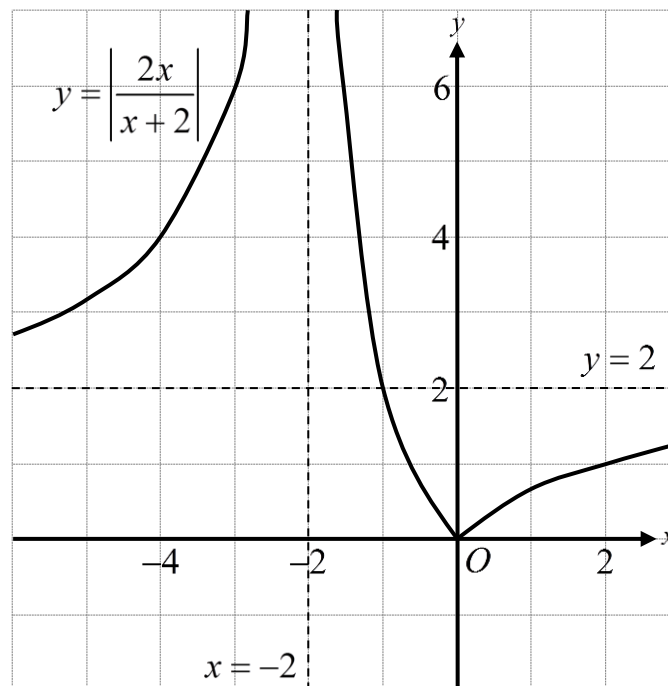
[2]

3. (a) $f^{-1}(f(x)) = x$
 $\therefore \frac{2f(x)}{af(x)+2} = x$ M1
 $\frac{2\left(\frac{2x}{2-x}\right)}{a\left(\frac{2x}{2-x}\right)+2} = x$
 $\frac{2(2x)}{a(2x)+2(2-x)} = x$ M1
 $\frac{4x}{2ax+4-2x} = x$
 $4x = 2ax^2 + 4x - 2x^2$
 $2x^2 = 2ax^2$
 $a = 1$ A1

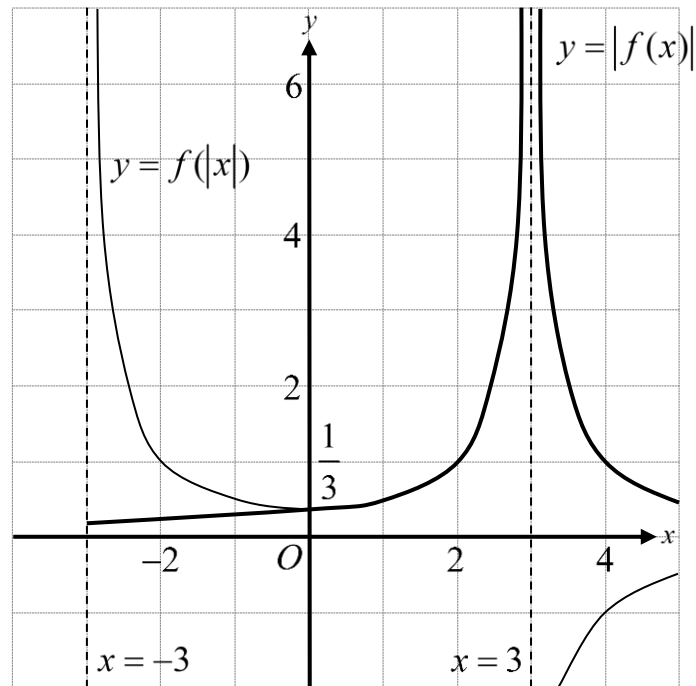
- (b) For correct asymptotes A1
 For correct intercept A1
 For correct shape A2

[3]

[4]



4. (a) (i) For correct asymptotes A1
 For correct intercept A1
 For correct shape A1



- (ii) For correct asymptotes A1
 For correct intercept A1
 For correct shape A1

- (b) 1 A1

[6]

[1]

Exercise 3

1. (a) $(f \circ g)(x) = \frac{4g(x)+1}{\sqrt{g(x)}}$
 $(f \circ g)(x) = \frac{4(4x^2)+1}{\sqrt{4x^2}}$ M1
 $(f \circ g)(x) = \frac{16x^2+1}{2x}$ A1
 [2]
- (b) $(f \circ g)(-x) = \frac{16(-x)^2+1}{2(-x)}$ M1
 $(f \circ g)(-x) = -\frac{16x^2+1}{2x}$ A1
 $(f \circ g)(-x) = -(f \circ g)(x)$
 Thus, $(f \circ g)(x)$ is an odd function. AG
 [2]
- (c) $\{y: y \geq 4\}$ A1
 [1]
2. (a) $f(-x) = (-x)^4 - (-x)^2$ M1
 $f(-x) = x^4 - x^2$ A1
 $f(-x) = f(x)$
 Thus, $f(x)$ is an even function. AG
 [2]
- (b) As $f(x)$ is an even function, the minimum point of f has x -coordinate $\frac{1}{\sqrt{2}}$ for $x > 0$.
 Thus, the two minimum points of $f(x)$ have the same y -coordinate. (R1) for correct argument
 $f\left(-\frac{1}{\sqrt{2}}\right) = \left(-\frac{1}{\sqrt{2}}\right)^4 - \left(-\frac{1}{\sqrt{2}}\right)^2$ M1
 $f\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{4} - \frac{1}{2}$
 $f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{4}$
 Thus, the range of $f(x)$ is $\left\{y: y \geq -\frac{1}{4}\right\}$. A1
 [3]

3. (a) $f(-x) = \frac{-x}{(-x)^2 + 0.19}$ M1
- $f(-x) = -\frac{x}{x^2 + 0.19}$ A1
- $f(-x) = -f(x)$
- Thus, $f(x)$ is an odd function. AG
- (b) $f(a) = a$
- $\frac{a}{a^2 + 0.19} = a$
- $a = a(a^2 + 0.19)$ (M1) for valid approach
- $a(a^2 + 0.19 - 1) = 0$
- $a(a^2 - 0.81) = 0$
- $a = 0$ or $a^2 = 0.81$
- $a = 0, a = -0.9$ or $a = 0.9$ A2
- (c) $y = 0$ A1
4. (a) $f(|-x|) = \frac{2 - 5|-x|}{2 - 9|-x|}$ M1
- $f(|-x|) = \frac{2 - 5|x|}{2 - 9|x|}$ A1
- $f(|-x|) = f(|x|)$
- Thus, $f(|x|)$ is an even function. AG
- (b) $y = \frac{5}{9}$ A1
- (c) $x = \frac{2}{9}, x = -\frac{2}{9}$ A2

[2]

[3]

[1]

[2]

[1]

[2]

Exercise 4

1. (a) $\{x : x \geq 7\}$ A2 [2]
- (b) $f(x) = 10 - 2(x - 7)$ (A1) for correct function
 $y = -2x + 24$
 $\Rightarrow x = -2y + 24$ (M1) for swapping variables
 $2y = 24 - x$
 $y = 12 - \frac{1}{2}x$
 $\therefore f^{-1}(x) = 12 - \frac{1}{2}x$ A1 [3]
2. (a) $\{x : x \leq 2\}$ A2 [2]
- (b) $f(x) = -(x^3 - 8)$ (A1) for correct function
 $y = -x^3 + 8$
 $\Rightarrow x = -y^3 + 8$ (M1) for swapping variables
 $y^3 = 8 - x$
 $y = \sqrt[3]{8 - x}$
 $\therefore f^{-1}(x) = \sqrt[3]{8 - x}$ A1 [3]
- (c) $\frac{1}{2}$ A1 [1]

3. (a) $\left\{x: -\frac{1}{2} \leq x \leq 4\right\}$

A2

[2]

(b) $f(x) = (2x+1)^2$

$$y = (2x+1)^2$$

$$\Rightarrow x = (2y+1)^2$$

(M1) for swapping variables

$$\sqrt{x} = 2y+1$$

$$\sqrt{x}-1 = 2y$$

$$y = \frac{\sqrt{x}-1}{2}$$

$$\therefore f^{-1}(x) = \frac{\sqrt{x}-1}{2}$$

A1

[2]

(c) $(f \circ g)(x) = (4x+7)^2$

$$g(x) = f^{-1}((4x+7)^2)$$

M1

$$g(x) = \frac{\sqrt{(4x+7)^2}-1}{2}$$

$$g(x) = \frac{4x+7-1}{2}$$

$$g(x) = \frac{4x+6}{2}$$

$$g(x) = 2x+3$$

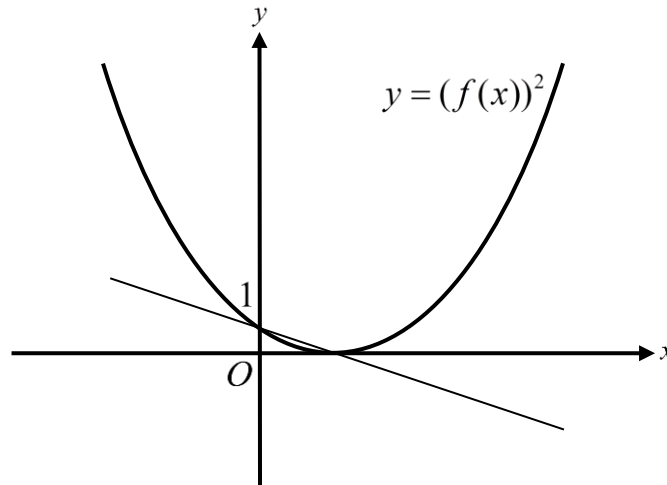
A1

[2]

4. (a) $\{x: -5 \leq x \leq 3\}$ A2 [2]
- (b) $f(x) = -(x-3)^2 + 5$
 $y = -(x-3)^2 + 5$
 $\Rightarrow x = -(y-3)^2 + 5$ (M1) for swapping variables
 $(y-3)^2 = 5-x$
 $y-3 = \sqrt{5-x}$ (*Rejected*) or $y-3 = -\sqrt{5-x}$ A1
 $y = -\sqrt{5-x} + 3$
 $\therefore f^{-1}(x) = -\sqrt{5-x} + 3$ A1 [3]
- (c) $(g^{-1} \circ f^{-1})(x) = 2x$
 $f^{-1}(x) = g(2x)$ M1
 $g(2x) = -\sqrt{5-x} + 3$
 $g\left(2\left(\frac{1}{2}x\right)\right) = -\sqrt{5-\frac{1}{2}x} + 3$ A1
 $\therefore g(x) = -\sqrt{5-\frac{1}{2}x} + 3$ A1 [3]

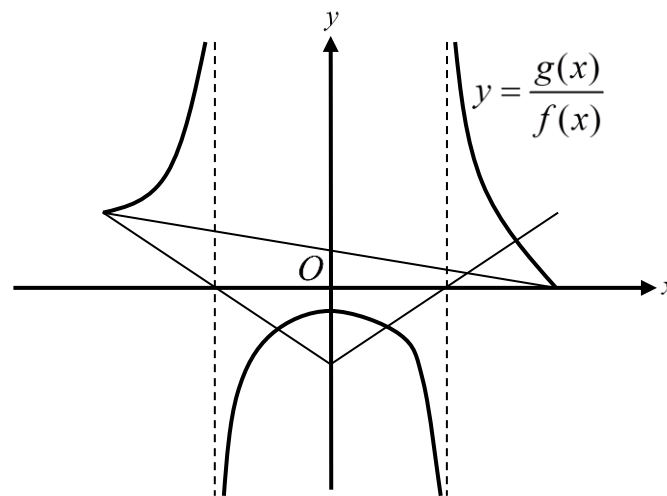
Exercise 5

1. For correct intercepts A2
 For correct concavity A1
 For correct minimum point A1



[4]

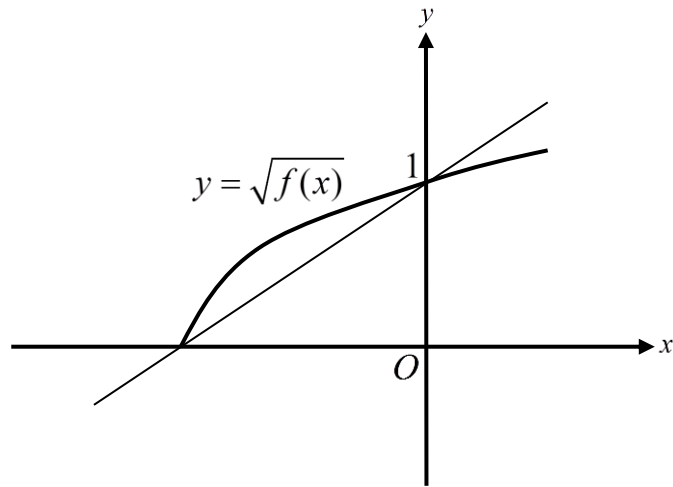
2. For correct intercepts A2
 For correct asymptotes A2
 For correct concavity A1



[5]

3. For correct intercepts A2
 For correct concavity A1
 For correct domain A1

[4]



4. For correct x -intercept A1
 For correct asymptote A1
 For correct concavity A2

[4]

