AI SL Practice Set 1 Paper 2 Solution

1. (a) $3x + y - 10$

$= 3(3) + 1 - 10$ A1

$= 0$

Thus, P lies on L_1. AG N0

[1]

(b) 10 A1 N1

[1]

(c) (i) The coordinates of M

$= \left(\frac{3+11}{2}, \frac{1+(-3)}{2}\right)$ (A1) for substitution

$= (7, -1)$ A1 N2

(ii) The gradient of PQ

$= \frac{-3-1}{11-3}$ (A1) for substitution

$= -\frac{1}{2}$ A1 N2

(iii) The distance between P and Q

$= \sqrt{(11-3)^2 + (-3-1)^2}$ (A1) for substitution

$= 8.94427191$

$= 8.94$ A1 N2

[6]

(d) The gradient of L_1

$= -\frac{3}{1}$

$= -3$ A1

$\because -3 \times -\frac{1}{2}$ M1

$= \frac{3}{2}$

$\neq -1$

Thus, L_1 and L_2 are not perpendicular. AG N0

[2]

(e) The gradient of L_3

$= \frac{-1}{-3}$ M1

$= \frac{1}{3}$ A1

The equation of L_3:

$y - 1 = \frac{1}{3}(x - 3)$ A1

$3y - 3 = x - 3$ A1

$x - 3y = 0$ AG N0

[4]

(f) The coordinates of S are $(0, 0)$. (A1) for correct value

The area of the triangle PRS

$= \frac{(10-0)(3-0)}{2}$ (M1) for valid approach

$= 15$ A1 N3

[3]

2. (a) The required probability

$= \mathrm{P}(W < 400)$ — (M1) for valid approach

$= 0.7791219069$

$= 0.779$ — A1 N2

[2]

(b) The expected number

$= (800)(0.7791219069)$ — (A1) for substitution

$= 623.2975255$

$= 623$ — A1 N2

[2]

(c) The required probability

$= \mathrm{P}(W < 385 \mid W < 400)$ — (M1) for valid approach

$= \dfrac{\mathrm{P}(W < 385 \cap W < 400)}{\mathrm{P}(W < 400)}$

$= \dfrac{\mathrm{P}(W < 385)}{\mathrm{P}(W < 400)}$ — (A1) for correct approach

$= 0.4495589773$

$= 0.450$ — A1 N3

[3]

(d) (i) 390 — A1 N1

(ii) 30% — A1 N1

(iii) $\mathrm{P}(W > k) = 0.2$ — (M1) for valid approach

$\mathrm{P}(W < k) = 0.8$

$k = 400.941076$

$k = 401$ — A1 N2

[4]

(e) The expected daily income

$= 800((4)(50\%) + (4.5)(30\%) + (5)(20\%))$ — (A2) for correct approach

$= \$3480$ — A1 N3

[3]

3.	(a)	(i)	$a = 14.02298851$			
			$a = 14.0$	A1	N1	
			$b = -420.2413793$			
			$b = -420$	A1	N1	
		(ii)	The estimated pulse rate			
			$= 14.02298851(37) - 420.2413793$	(A1) for substitution		
			$= 98.60919557$ beats per minute			
			$= 98.6$ beats per minute	A1	N2	
						[4]
	(b)	(i)	$r = 0.592701087$			
			$r = 0.593$	A1	N1	
		(ii)	Moderate, Positive	A2	N2	
						[3]
	(c)	(i)	H_0: The number of students in each range of pulse rates are evenly distributed.	A1	N1	
		(ii)	p-value $= 0.0166229271$	(A1) for correct value		
			p-value $= 0.0166$	A1	N2	
		(iii)	The null hypothesis is rejected.	A1		
			As p-value < 0.05.	R1	N2	
						[5]
	(d)	(i)	$H_1: \mu_A \neq \mu_B$	A1	N1	
		(ii)	p-value $= 0.3065878383$	(A1) for correct value		
			p-value $= 0.307$	A1	N2	
		(iii)	The null hypothesis is not rejected.	A1		
			As p-value > 0.01.	R1	N2	
						[5]

4.	(a)	(i)	$y=20-4x$	A1	N1	
		(ii)	$0<x<5$	A1	N1	
						[2]
	(b)	$V=(4x)(2x)(20-4x)$		(M1) for valid approach		
		$V=8x^2(20-4x)$				
		$V=160x^2-32x^3$		A1	N2	
						[2]
	(c)	(i)	By considering the graph of $V=160x^2-32x^3$, the coordinates of the maximum point are $(3.3333342, 592.59259)$.	(M1) for valid approach		
			Thus, the maximum volume is 593 cm^3.	A1	N2	
		(ii)	3.33	A1	N1	
		(iii)	$y=20-4(3.3333342)$	(M1) for substitution		
			$y=6.6666632$			
			$y=6.67$	A1	N2	
						[5]
	(d)	$A=2(4x)(2x)+2(4x)(20-4x)+2(2x)(20-4x)$		(M1) for valid approach		
		$A=16x^2+160x-32x^2+80x-16x^2$				
		$A=240x-32x^2$		A1	N2	
						[2]
	(e)	The x-coordinate of the vertex of the graph of $y=240x-32x^2$				
		$=-\frac{240}{2(-32)}$		A1		
		$=3.75$				
		$\neq 3.3333342$				
		Therefore, the total surface area of the box does not attain its maximum when its volume attains its maximum.		R1		
		Thus, the claim is incorrect.		AG	N0	
						[2]

5. (a) 2 A1 N1

[1]

(b) $f(3) = \frac{4}{3}(3)^3 + 5(3)^2 - 6(3) + 2$ (M1) for substitution

$f(3) = 65$ A1 N2

[2]

(c) $f'(x) = \frac{4}{3}(3x^2) + 5(2x) - 6(1) + 0$ (A1) for correct derivatives

$f'(x) = 4x^2 + 10x - 6$ A1 N2

[2]

(d) $4x^2 + 10x - 6 = 0$

$2(x+3)(2x-1) = 0$ (M1) for valid approach

$x = -3$ or $x = \frac{1}{2}$ A2 N3

[3]

(e) $y = 29$, $y = \frac{5}{12}$ A2 N2

[2]

(f) (i) $\frac{5}{12} < w < 29$ A2 N2

(ii) $w < \frac{5}{12}$ or $w > 29$ A2 N2

[4]

(g) The gradient of the tangent

$= f'(3)$

$= 4(3)^2 + 10(3) - 6$ (A1) for substitution

$= 60$ A1 N2

[2]

(h) The equation of the normal:

$y - 65 = \frac{-1}{60}(x-3)$ M1A1

$-60y + 3900 = x - 3$ A1

$x + 60y - 3903 = 0$ AG N0

[3]