Chapter 5 Solution

Exercise 11

1. (a) $9 = a - b^{-0}$ (M1) for substitution

$9 = a - 1$

$a = 10$ A1 N2

[2]

(b) $\frac{262}{27} = 10 - b^{-3}$ (M1) for substitution

$-\frac{8}{27} + b^{-3} = 0$

By considering the graph of $y = -\frac{8}{27} + b^{-3}$,

$b = 1.5$. A1 N2

[2]

(c) $y = 10$ A2 N2

[2]

2. (a) $11 = p \times q^{0} + 7$ (M1) for substitution

$11 = p + 7$

$p = 4$ A1 N2

[2]

(b) $8 = 4 \times q^{2} + 7$ (M1) for substitution

$1 = 4q^{2}$

$q^{2} = \frac{1}{4}$

$q = -\sqrt{\frac{1}{4}}$ or $q = \sqrt{\frac{1}{4}}$

$q = -\frac{1}{2}$ (*Rejected*) or $q = \frac{1}{2}$ A1 N2

[2]

(c) $\{y : y > 7\}$ A2 N2

[2]

3. (a) $2=3\times 2^{-p(0)}+q$ (M1) for substitution

$2=3+q$

$q=-1$ A1 N2

$47=3\times 2^{-p(-2)}-1$ (M1) for substitution

$48-3\times 2^{2p}=0$

By considering the graph of $y=48-3\times 2^{2p}$,

$p=2$. A1 N2

[4]

(b) $0=3\times 2^{-2x}-1$ (M1) for substitution

By considering the graph of $y=3\times 2^{-2x}-1$,

$x=0.7924813$.

Thus, the x-intercept is $x=0.792$. A1 N2

[2]

(c) 0 A1 N1

[1]

4. (a) $6=2\times a^0+b$ (M1) for substitution

$6=2+b$

$b=4$ A1 N2

$166=2\times a^4+4$ (M1) for substitution

$162-2\times a^4=0$

By considering the graph of $y=162-2\times a^4$,

$a=3$. A1 N2

[4]

(b) Increases A1 N1

[1]

(c) 1 A1 N1

[1]

Exercise 12

1. (a) \$18000 A1 N1

[1]

(b) The price of the car

$= 2000 + 16000e^{-\frac{6}{8}}$ (M1) for substitution

$= 9557.864844$

$= \$9560$ A1 N2

[2]

(c) $2000 + 16000e^{-\frac{t}{8}} < 7000$ (M1) for valid approach

$16000e^{-\frac{t}{8}} - 5000 < 0$

By considering the graph of $y = 16000e^{-\frac{t}{8}} - 5000$,

$t = 9.3052065$.

Thus, the time taken is 9.31 years. A1 N2

[2]

(d) \$2000 A1 N1

[1]

2. (a) 270 A1 N1

[1]

(b) The increase in the number of bacteria

$= \left(115 + 155e^{\frac{7}{25}}\right) - 270$ (M1) for substitution

$= 50.08512091$

$= 50$ A1 N2

[2]

(c) $1200 = 115 + 155e^{\frac{t}{25}}$ (M1) for valid approach

$1085 - 155e^{\frac{t}{25}} = 0$

By considering the graph of $y = 1085 - 155e^{\frac{t}{25}}$,

$t = 48.647754$.

Thus, the time taken is 48.6 days. A1 N2

[2]

3. (a) The initial price of the computer system. A1 N1

[1]

(b) $840 = 90 + A \times 0.7^0$ (M1) for substitution

$840 = 90 + A$

$A = 750$ A1 N2

[2]

(c) $420 = 90 + 750 \times 0.7^t$ (M1) for valid approach

$330 - 750 \times 0.7^t = 0$

By considering the graph of $y = 330 - 750 \times 0.7^t$,

$t = 2.3017612$.

Thus, the time required is 2.30 years. A1 N2

[2]

(d) EUR 90 A1 N1

[1]

4. (a) The initial amount of electric charge stored. A1 N1

[1]

(b) $4 = p - q^0$ (M1) for substitution

$4 = p - 1$

$p = 5$ A1 N2

$4.488 = 5 - q^3$ (M1) for valid approach

$0.512 - q^3 = 0$

By considering the graph of $y = 0.512 - q^3$,

$q = 0.8$. A1 N2

[4]

(c) $4.3 = 5 - 0.8^t$ (M1) for valid approach

$0.7 - 0.8^t = 0$

By considering the graph of $y = 0.7 - 0.8^t$,

$t = 1.5984103$.

Thus, the time required to reach the charge is
1.60 hours. A1 N2

[2]

Exercise 13

1. (a) The required magnitude

$= 3\log_{10}(2\times5\times10^3)$ (M1) for correct formula

$= 12$ A1 N2

[2]

(b) $0.9 = 3\log_{10}(2E)$ (M1) for substitution

$0.3 = \log_{10}(2E)$

$\log_{10}(2E) - 0.3 = 0$

By considering the graph of $y = \log_{10}(2E) - 0.3$,

$E = 0.997631157$.

Thus, the amount of energy released is 0.998 units. A1 N2

[2]

2. $139.8 = 120 + 9.9\log_{10} I$ (M1) for substitution

$9.9\log_{10} I - 19.8 = 0$

By considering the graph of $y = 9.9\log_{10} I - 19.8$, $I = 100$. (A1) for correct working

$169.5 = 120 + 9.9\log_{10} I$ (M1) for substitution

$9.9\log_{10} I - 49.5 = 0$

By considering the graph of $y = 9.9\log_{10} I - 49.5$,

$I = 100000$. (A1) for correct working

The required ratio

$= 100000 : 100$

$= 1000 : 1$ A1 N5

[5]

3. (a) The y-intercept

$= 2(0) + 3\log_{10}(0+2)$ (M1) for substitution

$= 0.903089987$

$= 0.903$ A1 N2

[2]

(b) 1 A1 N1

[1]

(c) $2x + 3\log_{10}(x+2) = x^2$ (M1) for substitution

$2x + 3\log_{10}(x+2) - x^2 = 0$

By considering the graph of

$y = 2x + 3\log_{10}(x+2) - x^2$,

$x = -0.3004$ or $x = 2.7399173$.

Thus, $x = -0.300$ or $x = 2.74$. A2 N3

[3]

4. $f(0) = 0 + \log_{10}(0+10) + 2$ (M1) for substitution

$f(0) = 3$

Thus, the coordinates of B are $(0, 3)$. (A1) for correct working

$f(x) = 0$ (M1) for substitution

$x + \log_{10}(x+10) + 2 = 0$

By considering the graph of $y = x + \log_{10}(x+10) + 2$,

$x = -2.854059$.

Thus, the coordinates of A are $(-2.854059, 0)$. (A1) for correct working

The required area

$= \dfrac{(2.854059)(3)}{2}$ (M1) for valid approach

$= 4.2810885$

$= 4.28$ A1 N6

[6]