AI HL Practice Set 4 Paper 2 Solution

1. (a) The gradient of L_1

$=\frac{40-0}{0-30}$ (A1) for substitution

$=-\frac{4}{3}$ A1

[2]

(b) The equation of L_1:

$y-40=-\frac{4}{3}(x-0)$ (A1) for substitution

$3y-120=-4x$

$4x+3y-120=0$ A1

[2]

(c) The gradient of L_2

$=-1\div-\frac{4}{3}$

$=\frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y=\frac{3}{4}x$ A1

[2]

(d) $4x+3\left(\frac{3}{4}x\right)-120=0$ (M1) for substitution

$6.25x=120$

$x=19.2$

$y=\frac{3}{4}(19.2)$ (M1) for substitution

$y=14.4$

Thus, the coordinates of C are $(19.2, 14.4)$. A1

[3]

(e) The area of the triangle OBC

$=\frac{(40-0)(19.2-0)}{2}$ (M1) for valid approach

$=384$ A1

[2]

(f) $\text{BC} = \sqrt{(0-19.2)^2 + (40-14.4)^2}$ (A1) for substitution

$\text{BC} = 32$ (A1) for correct value

$\text{OC} = \sqrt{(19.2-0)^2 + (14.4-0)^2}$

$\text{OC} = 24$ (A1) for correct value

The perimeter of the triangle OBC

$= 24 + 32 + 40$

$= 96$ A1

[4]

(g) $\frac{3}{4}k$ A1

[1]

(h) $\frac{(\text{BC})(\text{CD})}{2} = 624$ (A1) for correct equation

$32\text{CD} = 1248$

$\text{CD} = 39$ (A1) for correct value

$\therefore \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} = 39$ (A1) for correct equation

$\sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39 = 0$

By considering the graph of

$y = \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39$, $k = -12$ or

$k = 50.4$ (*Rejected*).

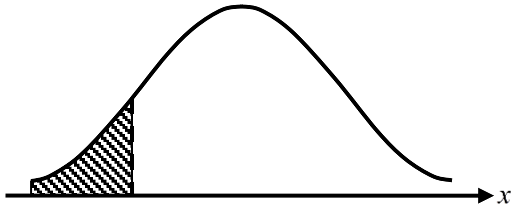
$\therefore k = -12$ A1

[4]

2. (a) For vertical line clearly to the left of the mean A1

For shading to the left of the vertical line A1

[2]



(b) (i) Let X be the volume of a randomly selected milk soda.

The required probability

$= P(X < 490)$ (M1) for valid approach

$= 0.105649839$

$= 0.106$ A1

(ii) The required probability

$= P(X > 483 \mid X < 490)$ (M1) for valid approach

$= \dfrac{P(X > 483 \cap X < 490)}{P(X < 490)}$

$= \dfrac{P(483 < X < 490)}{P(X < 490)}$ (A1) for correct approach

$= 0.8410480651$

$= 0.841$ A1

[5]

(c) The required probability

$= 2 \times P(X < 490) \times (1 - P(X < 490))$ (M1) for valid approach

$= 2 \times 0.105649839 \times (1 - 0.105649839)$ (A1) for substitution

$= 0.188975901$

$= 0.189$ A1

[3]

(d) (i) 0.327 A2

(ii) 0.0803 A2

(iii) –$1.29 A2

[6]

3.	(a)	(i)	$(6.67, 50.8)$	A2
		(ii)	$2 < x < 6.67$	A2
				[4]
	(b)	(i)	$f'(x) = -3x^2 + 13(2x) - 40(1) + 0$	(A1) for correct derivatives
			$f'(x) = -3x^2 + 26x - 40$	A1
		(ii)	15	A1
		(iii)	The equation of the tangent:	
			$y - f(5) = 15(x - 5)$	M1A1
			$y - 36 = 15x - 75$	A1
			$15x - y - 39 = 0$	AG
				[6]
	(c)	(i)	9	A1
		(ii)	$\int_2^9 f(x)\mathrm{d}x$	A1
		(iii)	$\int_2^9 f(x)\mathrm{d}x = \frac{2401}{12}$	A2
				[4]

(d) The estimate of $\int_2^9 f(x)\mathrm{d}x$

$= \frac{1}{2}(1.75)\begin{bmatrix} f(2) + f(9) \\ +2(f(3.75) + f(5.5) + f(7.25)) \end{bmatrix}$ (A2) for substitution

$= \frac{1}{2}(1.75)\left[0 + 0 + 2\begin{pmatrix} 16.078125 \\ +42.875 + 48.234375 \end{pmatrix}\right]$ (A1) for correct approach

$= 187.578125$

$= 188$ A1

[4]

(e) Underestimate A1

[1]

4. (a) The required distance

$= \sqrt{(12-0)^2 + (5-0)^2}$ (A1) for substitution

$= 13$ A1

[2]

(b) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ A2

[2]

(c) The velocity vector

$= \frac{1}{2}\left(2\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 12 \\ 5 \end{pmatrix}\right)$ (M1) for valid approach

$= \frac{1}{2}\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 2.5 \end{pmatrix}$ A1

[2]

(d) $\begin{pmatrix} 8 \\ 10 \end{pmatrix} + x\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5.5 \\ 17.5 \end{pmatrix}$ (M1) for valid approach

$8 - x = 5.5$

$x = 2.5$ A1

[2]

(e) $\cos\theta = \frac{(4)(-1) + (5)(3)}{(\sqrt{4^2 + 5^2})(\sqrt{(-1)^2 + 3^2})}$ M1A1

$\cos\theta = 0.5432512782$

$\theta = 0.9964914966 \text{ rad}$

Thus, the required angle is 0.996 rad. A1

[3]

(f) $10 + 3t = 31$ (M1) for valid approach

$3t = 21$

$t = 7$ (A1) for correct value

The amount of time needed

$= 7 + 2$

$= 9 \text{ s}$ A1

[3]

5. (a) $0 \le y < 6$ A1

[1]

(b) $\det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} -6-\lambda & 0 \\ -1 & 5-\lambda \end{vmatrix}$ (M1) for valid approach

$= (-6-\lambda)(5-\lambda) - (0)(-1)$

$= -30 + 6\lambda - 5\lambda + \lambda^2$

$= \lambda^2 + \lambda - 30$ A1

[2]

(c) $\lambda_1 = -6$, $\lambda_2 = 5$ A2

[2]

(d) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ A2

[2]

(e) (i) $\mathbf{X} = Ae^{-6t}\begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} + Be^{5t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} 22 \\ 5 \end{pmatrix} = Ae^{-6(0)}\begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} + Be^{5(0)}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (M1) for substitution

$\begin{cases} 22 = A \\ 5 = \frac{1}{11}A + B \end{cases}$

By solving this system, $A = 22$ and $B = 3$. (A1) for correct values

$\therefore x = 22e^{-6t}$ A1

(ii) $y = 2e^{-6t} + 3e^{5t}$ A1

[5]

(f) (i) The population of brown bear will approach zero. A1

(ii) The population of giant panda will increase exponentially. A1

[2]

6. (a) V_2

$= V - V_1$

$= 29\sin(6\pi t - 0.31) - 23\sin(6\pi t - 0.17)$

$= \text{Im}(29e^{(6\pi t - 0.31)\text{i}}) - \text{Im}(23e^{(6\pi t - 0.17)\text{i}})$ (M1) for valid approach

$= \text{Im}(29e^{(6\pi t - 0.31)\text{i}} - 23e^{(6\pi t - 0.17)\text{i}})$ (A1) for correct approach

$= \text{Im}(e^{6\pi t\text{i}}(29e^{-0.31\text{i}} - 23e^{-0.17\text{i}}))$

$\therefore z - w = 29e^{-0.31\text{i}} - 23e^{-0.17\text{i}}$ A1

[3]

(b) (i) $z = 29e^{-0.31\text{i}}$

$z = 29(\cos(-0.31) + \text{i}\sin(-0.31))$ A1

(ii) $w = 23e^{-0.17\text{i}}$

$w = 23(\cos(-0.17) + \text{i}\sin(-0.17))$ A1

[2]

(c) (i) $z - w$

$= 29(\cos(-0.31) + \text{i}\sin(-0.31))$

$-23(\cos(-0.17) + \text{i}\sin(-0.17))$

$= (29\cos(-0.31) - 23\cos(-0.17))$

$+\text{i}(29\sin(-0.31) - 23\sin(-0.17))$ (M1) for valid approach

$= 4.949223888 - 4.955506428\text{i}$ (A1) for correct values

L

$= \sqrt{4.949223888^2 + (-4.955506428)^2}$ M1

$= 7.003703381$

$= 7.00$ A1

(ii) α

$= \tan^{-1}\dfrac{-4.955506428}{4.949223888}$ M1

$= -0.7860324602$

$= -0.786$ A1

[6]

(d) V_2

$= \text{Im}(e^{6\pi t\text{i}}(z - w))$

$= \text{Im}(e^{6\pi t\text{i}} \cdot 7.003703381e^{-0.7860324602\text{i}})$ (M1) for substitution

$= \text{Im}(7.003703381e^{6\pi t\text{i} - 0.7860324602\text{i}})$ (A1) for correct approach

$= 7.003703381\sin(6\pi t - 0.7860324602)$

$= 7.00\sin(6\pi t - 0.786)$ A1

[3]

7. (a) (i) 5 A1

(ii) 4 A1

(iii) 4 A1

(iv) $43 A1

(v) $61 A1

[5]

(b) For any four edges correct A1

For any eight edges correct A1

1. Choose HA of weight 12
2. Choose AB of weight 22
3. Choose BC of weight 14
4. Choose CD of weight 15
5. Choose DE of weight 16
6. Choose EF of weight 18
7. Choose FG of weight 24
8. Choose GH of weight 17
9. Choose HE of weight 20
10. Choose EA of weight 25
11. Choose AE of weight 25
12. Choose EB of weight 10

Thus, a possible route contains HA, AB, BC, CD, DE, EF, FG, GH, HE, EA, AE and EB. A1

[3]

(c) $218 A1

[1]

(d) (i) For any five edges correct A1

For any ten edges correct A1

1. Choose BC of weight 14
2. Choose CD of weight 15
3. Choose DE of weight 16
4. Choose EF of weight 18
5. Choose FG of weight 24
6. Choose GH of weight 17
7. Choose HA of weight 12
8. Choose AB of weight 22
9. Choose BE of weight 10
10. Choose EH of weight 20
11. Choose HA of weight 12
12. Choose AE of weight 25
13. Choose EB of weight 10

Thus, a possible route contains BC, CD, DE, EF, FG, GH, HA, AB, BE, EH, HA, AE and EB. A1

(ii) $215 A1

[4]