## Applications and Interpretation Standard Level for IBDP Mathematics Practice Paper Set 1 - Paper 2 (90 Minutes)

## Question - Answer Book

## Instructions

1. Attempt ALL questions. Write your answers in the spaces provided in this Question - Answer Book.
2. A graphic display calculator is needed.
3. You are suggested to prepare a formula booklet of Applications and Interpretation for IBDP Mathematics when attempting the questions.
4. Supplementary answer sheets and graph papers will be supplied on

|  | Marker's <br> Use Only | Examiner's <br> Use Only |  |
| :---: | :---: | :---: | :---: |
| Question <br> Number | Marks | Marks | Maximum <br> Mark |
| 1 |  |  | 17 |
| 2 |  |  | 14 |
| 3 |  |  | 17 |
| 4 |  |  | 13 |
| 5 | Overall |  |  |
| Paper 2 <br> Total |  |  | 19 |

5. Unless otherwise specified, ALL working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either EXACT or correct to 3 SIGNIFICANT FIGURES.
7. The diagrams in this paper are NOT necessarily drawn to scale.
8. Information to be read before you start the exam:

9. The equation of the straight line $L_{1}$ is given by $3 x+y-10=0$. The coordinates of the point P are $(3,1)$.
(a) Show that P lies on $L_{1}$.
(b) Write down the $y$-intercept of $L_{1}$.

The coordinates of the point Q are $(11,-3) . \mathrm{M}$ is the mid-point of PQ .
(c) Find
(i) the coordinates of M ;
(ii) the gradient of PQ ;
(iii) the distance between P and Q .

The straight line $L_{2}$ passes through P and Q .
(d) Show that $L_{1}$ and $L_{2}$ are not perpendicular.

The straight line $L_{3}$ passes through P and is perpendicular to $L_{1}$.
(e) Show that the equation of $L_{3}$ is $x-3 y=0$.
$L_{1}$ and $L_{3}$ intersect with the $y$-axis at R and S respectively.
(f) Find the area of the triangle PRS.
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2. A coffee shop provides 800 breakfast packages for customers every morning. The weights $W \mathrm{~g}$ of the packages are normally distributed with mean 390 g and standard deviation 13 g .
(a) Find the probability that the weight of a randomly chosen package is less than 400 g .
(b) Hence, find the expected number of packages less than 400 g in any morning.
(c) Given that the weight of a randomly chosen package is less than 400 g , find the probability that its weight is less than 385 g .

The manager of the coffee shop wants to set the prices for the breakfast packages of different ranges of weights, as shown in the following table.

| Weight $W \mathrm{~g}$ | $W<j$ | $j \leq W \leq k$ | $W>k$ |
| :---: | :---: | :---: | :---: |
| Price $\$ P$ | $\$ 4$ | $\$ 4.5$ | $\$ 5$ |

It is given that $50 \%$ and $20 \%$ of the breakfast packages provided in every morning cost $\$ 4$ and $\$ 5$ respectively.
(d) (i) Write down $j$.
(ii) Write down the percentage of the breakfast packages provided in every morning that costs $\$ 4.5$
(ii) Find $k$.
(e) Hence, find the expected daily income from selling the packages, assuming that all packages can be sold.
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3. The relationship between the body temperature and the pulse rate of the students from a sports team is investigated. Six students from the group A of the team are first medically examined and their body temperature and their pulse rates are recorded in the table below.

| Student | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Body Temperature $\left(x^{\circ} \mathrm{C}\right)$ | 35.8 | 36.2 | 36.4 | 36.7 | 37.4 | 37.1 |
| Pulse Rate $(y$ beats per minute $)$ | 80 | 81 | 87 | 117 | 100 | 93 |

(a) The relationship between the variables is modelled by the regression equation $y=a x+b$.
(i) Write down the value of $a$ and of $b$.
(ii) Hence, estimate the pulse rate of a student whose body temperature is $37^{\circ} \mathrm{C}$.
(b) (i) Write down the correlation coefficient.
(ii) State which two of the following describe the correlation between the variables.

| positive | strong | zero |
| :--- | :--- | :--- |
| negative | weak | moderate |

A similar investigation has been completed last year. The pulse rates of 100 students were recorded and the data was presented as follows:

| Pulse Rate $(y$ beats per minute $)$ | Frequency |
| :---: | :---: |
| $75 \leq y<85$ | 16 |
| $85 \leq y<95$ | 23 |
| $95 \leq y<105$ | 32 |
| $105 \leq y<115$ | 12 |
| $115 \leq y<125$ | 17 |

Someone claims that the distribution of the data is expected to be evenly distributed. Hence, a $\chi^{2}$ goodness of fit test is conducted at a $5 \%$ significance level.
(c) (i) Write down the null hypothesis of the test.
(ii) Find the $p$-value.
(iii) Hence, state the conclusion of the test with a reason.

Another five students from the Group B of the team are also medically examined and their pulse rates are recorded in the table below.

| Student | G | H | I | J | K |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Pulse Rate $(y$ beats per minute $)$ | 95 | 99 | 117 | 87 | 110 |

The team manager wants to know whether the mean pulse rates $\mu_{A}$ and $\mu_{B}$ of the students from the Group A and the Group B respectively are different. A $t$-test is conducted at a $1 \%$ significance level.
(d) (i) Write down the alternative hypothesis of the test.
(ii) Find the $p$-value.
(iii) Hence, state the conclusion of the test with a reason.
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4. A closed rectangular box has length $4 x \mathrm{~cm}$, width $2 x \mathrm{~cm}$ and height $y \mathrm{~cm}$, where $x, y>0$. It is given that the sum of the length and the height of the rectangular box is 20 cm .
(a) Write down
(i) an expression for $y$ in terms of $x$;
(ii) the possible range of values of $x$.
(b) Express $V$ in terms of $x$, where $V \mathrm{~cm}^{3}$ is the volume of the rectangular box.
(c) Using the graphic display calculator to find
(i) the maximum volume;
(ii) the value of $x$ when $V$ attains its maximum;
(iii) the value of $y$ when $V$ attains its maximum.
(d) Express $A$ in terms of $x$, where $A \mathrm{~cm}^{2}$ is the total surface area of the rectangular box.
(e) Someone claims that the total surface area of the box attains its maximum when its volume attains its maximum. Explain why the claim is incorrect.
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5. The function $f$ is given by $f(x)=\frac{4}{3} x^{3}+5 x^{2}-6 x+2, x \in \mathbb{R}$.
(a) Write down the $y$-intercept of the graph of $f$.
(b) Find $f(3)$.
(c) Find $f^{\prime}(x)$.
(d) Solve the equation $f^{\prime}(x)=0$.
(e) Write down the equations of the horizontal tangents of the graph of $f$.
(f) Write down the range of values of $w$ such that the equation $f(x)=w$ has
(i) three solutions;
(ii) only one solution.
(g) Find the gradient of the tangent at $x=3$.
(h) Hence, show that the equation of the normal at $x=3$ is $x+60 y-3903=0$.
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