AI HL Practice Set 3 Paper 2 Solution

1. (a) $a = 5.6$ A1
$b = 34.8$ A1
[2]

(b) The estimated hardness
$= 5.6(6.3) + 34.8$ (A1) for substitution
$= 70.08$ A1
[2]

(c) The required probability
$= \frac{120 - 56}{120}$ (M1) for valid approach
$= \frac{8}{15}$ A1
[2]

(d) (i) Let X be the number of selected ingots of the hardness at least 65, where
$X \sim \text{B}\left(10, \frac{8}{15}\right)$.
The required probability
$= \text{P}(X = 5)$ (M1) for valid approach
$= 0.2406733955$
$= 0.241$ A1

(ii) The required probability
$= \text{P}(X < 4)$ (M1) for valid approach
$= 0.1226252054$
$= 0.123$ A1

(iii) $\frac{16}{3}$ A1
[5]

(d) (i) $H_1: \mu_1 \neq \mu_2$ A1

(ii) $p\text{-value} = 0.0741679182$ (A1) for correct value
$p\text{-value} = 0.0742$ A1

(iii) The null hypothesis is not rejected. A1
As $p\text{-value} > 0.05$. R1
[5]

2. (a) $P(0)=116$

$\therefore a+b\times c^{0}=116$ — (M1) for setting equation

$a+b=116$ — A1

[2]

(b) $P(1)=172$

$\therefore a+b\times c^{-1}=172$ — (M1) for setting equation

$a+\frac{b}{c}=172$ — A1

[2]

(c) (i) $\log_c 81=4$

$\therefore c^4=81$ — M1

$c^4=3^4$ — A1

$c=3$ — AG

(ii) The system is $\begin{cases} a+b=116 \\ a+\frac{1}{3}b=172 \end{cases}$. — (M1) for valid approach

Solving, we have $a=200$ and $b=-84$. — A2

[5]

(d) The number of elephants

$=200-84\times 3^{-3}$ — (M1) for substitution

$=196.8888889$

$=197$ — A1

[2]

(e) 200 — A1

[1]

(f) $200-84\times 3^{-t}>195$ — (M1) for setting inequality

$5-84\times 3^{-t}>0$

By considering the graph of $y=5-84\times 3^{-t}$,

$t=2.5681297$.

Thus, the number of years needed is 2.57 years. — A1

[2]

(g) By considering the graphs of $y = 200 - 84 \times 3^{-t}$, $y = 170$, $y = 180$ and $y = 190$, y reaches 170, 180 and 190 at $t_1 = 0.9372$, $t_2 = 1.3062702$ and $t_3 = 1.9372$ respectively. M1A1

$\because 2(t_2 - t_1)$

$= 2(1.3062702 - 0.9372)$

$= 0.7381404$

$\neq t_3 - t_2$ R1

Thus, the claim is disagreed. A1

[4]

3. (a) (i) $(4, 8)$ A2

(ii) $\{y : 4 \le y \le 8,\ y \in \mathbb{R}\}$ A2

[4]

(b) $f'(x)$

$= -0.25(2x) + 2(1) + 0$ (A1) for correct derivatives

$= -0.5x + 2$ A1

[2]

(c) $f'(x) = -1$ M1

$\therefore -0.5x + 2 = -1$ A1

$-0.5x = -3$

$x = 6$ A1

$f(6)$

$= -0.25(6)^2 + 2(6) + 4$ M1

$= 7$

Thus, the coordinates of P are $(6, 7)$. AG

[4]

(d) The equation of the tangent:

$y - 7 = -1(x - 6)$ (A1) for substitution

$y - 7 = -x + 6$

$x + y - 13 = 0$ A1

[2]

(e) (i) 4 A1

(ii) 5.75 A1

[2]

(f) The estimate of $\int_0^8 f(x)\mathrm{d}x$

$= \frac{1}{2}(1)\left[4 + 4 + 2\begin{pmatrix}5.75 + 7 + 7.75 \\ +8 + 7.75 + 7 + 5.75\end{pmatrix}\right]$ (A2) for substitution

$= 53$ A1

[3]

(g) Underestimate A1

[1]

4. (a) The period of W_2

$= \frac{2\pi}{2\pi}$ (M1) for valid approach

$= 1\text{ s}$ A1

[2]

(b) $W_1 + W_2$

$= 11\cos(2\pi t - 0.1) + 13\cos(2\pi t - 0.3)$

$= \text{Re}(11e^{(2\pi t - 0.1)\text{i}}) + \text{Re}(13e^{(2\pi t - 0.3)\text{i}})$ (M1) for valid approach

$= \text{Re}(11e^{(2\pi t - 0.1)\text{i}} + 13e^{(2\pi t - 0.3)\text{i}})$ (A1) for correct approach

$= \text{Re}(e^{2\pi t\text{i}}(11e^{-0.1\text{i}} + 13e^{-0.3\text{i}}))$

$\therefore z + w = 11e^{-0.1\text{i}} + 13e^{-0.3\text{i}}$ A1

[3]

(c) (i) $z = 11e^{-0.1\text{i}}$

$z = 11(\cos(-0.1) + \text{i}\sin(-0.1))$ A1

(ii) $w = 13e^{-0.3\text{i}}$

$w = 13(\cos(-0.3) + \text{i}\sin(-0.3))$ A1

[2]

(d) (i) $z + w$

$= 11(\cos(-0.1) + \text{i}\sin(-0.1))$

$+13(\cos(-0.3) + \text{i}\sin(-0.3))$

$= (11\cos(-0.1) + 13\cos(-0.3))$

$+\text{i}(11\sin(-0.1) + 13\sin(-0.3))$ (M1) for valid approach

$z + w = 23.36442018 - 4.93993027\text{i}$ (A1) for correct values

L

$= \sqrt{23.36442018^2 + (-4.93993027)^2}$ M1

$= 23.88093468$

$= 23.9$ A1

(ii) α

$= \tan^{-1}\frac{-4.93993027}{23.36442018}$ M1

$= -0.2083610278$

$= -0.208$ A1

[6]

(e) $W_1 + W_2$

$= \text{Re}(e^{2\pi t\text{i}}(z+w))$

$= \text{Re}(e^{2\pi t\text{i}} \cdot 23.88093468e^{-0.2083610278\text{i}})$ (M1) for substitution

$= \text{Re}(23.88093468e^{2\pi t\text{i}-0.2083610278\text{i}})$ (A1) for correct approach

$= 23.88093468\cos(2\pi t - 0.2083610278)$

$= 23.9\cos(2\pi t - 0.208)$ A1

[3]

5. (a) Eulerian trail does not exist. A1

As there are more than two vertices of odd degrees. A1

[2]

(b)
$$\mathbf{M}=\begin{pmatrix}0&1&0&0&0&1&1\\1&0&1&0&0&0&1\\0&1&0&1&0&0&1\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\\1&0&0&0&1&0&1\\1&1&1&1&1&1&0\end{pmatrix}$$
A2

[2]

(c)
$$\mathbf{M}^4=\begin{pmatrix}24&18&23&18&23&18&32\\18&24&18&23&18&23&32\\23&18&24&18&23&18&32\\18&23&18&24&18&23&32\\23&18&23&18&24&18&32\\18&23&18&23&18&24&32\\32&32&32&32&32&32&60\end{pmatrix}$$
(M1) for valid approach

Thus, the total number of walks of length 4 from D to A is 18. A1

[2]

(d) For any three edges correct A1

For all edges correct A1

1. Choose AF of weight 50
2. Choose BC of weight 52
3. Choose AG of weight 53
4. Choose DE of weight 54
5. Choose CG of weight 58
6. Choose EF of weight 59

Thus, the minimum spanning tree is a tree containing AF, BC, AG, DE, CG and EF. A1

[3]

(e) 326 A1

[1]

(f) For all edges correct A2

1. Choose ED of weight 54
2. Choose DC of weight 61
3. Choose CB of weight 52
4. Choose BA of weight 63
5. Choose AF of weight 50
6. Choose FG of weight 57
7. Choose GE of weight 61

Thus, an upper bound of the total weight of a cycle that passes through all seven vertices is 398. AG

[2]

(g) For any three edges correct A1

For all edges correct A1

1. Choose AF of weight 50
2. Choose BC of weight 52
3. Choose AG of weight 53
4. Choose CG of weight 58
5. Choose CD of weight 61

Therefore, the weight of a minimum spanning tree after deleting the vertex E is 274. A1

The required lower bound

$= 274 + 54 + 59$

$= 387$ A1

[4]

6. (a) (i) $\overrightarrow{\mathrm{BD}}$

$$=\begin{pmatrix}0\\-\pi\\0\end{pmatrix}-\begin{pmatrix}\pi\\0\\\pi\end{pmatrix}$$

$$=\begin{pmatrix}-\pi\\-\pi\\-\pi\end{pmatrix}$$ A1

The vector equation of BD:

$$\mathbf{r}=\begin{pmatrix}\pi\\0\\\pi\end{pmatrix}+t\begin{pmatrix}-\pi\\-\pi\\-\pi\end{pmatrix}$$ A1

(ii) $$\begin{cases}x=\pi-\pi t\\y=-\pi t\\z=\pi-\pi t\end{cases}$$ A1

$$\overrightarrow{\mathrm{CE}}=\begin{pmatrix}\pi-\pi t\\-\pi t\\\pi-\pi t\end{pmatrix}-\begin{pmatrix}\pi\\0\\0\end{pmatrix}$$

$$\overrightarrow{\mathrm{CE}}=\begin{pmatrix}-\pi t\\-\pi t\\\pi-\pi t\end{pmatrix}$$ A1

(iii) $\vec{CE} \cdot \vec{BD} = 0$

$\therefore (-\pi t)(-\pi) + (-\pi t)(-\pi)$
$+(\pi - \pi t)(-\pi) = 0$ M1

$\pi^2 t + \pi^2 t - \pi^2 + \pi^2 t = 0$

$3\pi^2 t = \pi^2$

$t = \frac{1}{3}$ A1

$$\therefore \begin{cases} x = \pi - \pi\left(\frac{1}{3}\right) = \frac{2}{3}\pi \\ y = -\pi\left(\frac{1}{3}\right) = -\frac{1}{3}\pi \\ z = \pi - \pi\left(\frac{1}{3}\right) = \frac{2}{3}\pi \end{cases}$$ M1

Therefore, the coordinates of E are
$\left(\frac{2}{3}\pi, -\frac{1}{3}\pi, \frac{2}{3}\pi\right)$. AG

[7]

(b) (i) $\vec{BA} = \begin{pmatrix} -\pi \\ 0 \\ 0 \end{pmatrix}$ A1

(ii) $\mathbf{w}$

$= \vec{BA} \times \vec{BD}$

$= \begin{pmatrix} -\pi \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -\pi \\ -\pi \\ -\pi \end{pmatrix}$

$= \begin{pmatrix} (0)(-\pi) - (0)(-\pi) \\ (0)(-\pi) - (-\pi)(-\pi) \\ (-\pi)(-\pi) - (0)(-\pi) \end{pmatrix}$ (A1) for substitution

$= \begin{pmatrix} 0 \\ -\pi^2 \\ \pi^2 \end{pmatrix}$ A1

(iii) $\begin{pmatrix}0\\-\pi^2\\\pi^2\end{pmatrix}\cdot\begin{pmatrix}1\\1\\2\end{pmatrix}=\left|\begin{pmatrix}0\\-\pi^2\\\pi^2\end{pmatrix}\right|\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\cos\theta$ (M1) for valid approach

$(0)(1)+(-\pi^2)(1)+(\pi^2)(2)$

$=(\sqrt{0^2+(-\pi^2)^2+(\pi^2)^2})(\sqrt{1^2+1^2+2^2})\cos\theta$ A1

$\pi^2=\sqrt{12\pi^4}\cos\theta$ (A1) for correct approach

$\cos\theta=\frac{1}{\sqrt{12}}$

$\theta=1.277953555\text{ rad}$

Therefore, the required acute angle is 1.28 rad. A1

[7]

7. (a) $\begin{cases} \dfrac{dv}{dt} = 7v - 10x \\ \dfrac{dx}{dt} = v \end{cases}$ A1

[1]

(b) $\det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} 7-\lambda & -10 \\ 1 & 0-\lambda \end{vmatrix}$ (M1) for valid approach

$= (7-\lambda)(-\lambda) - (-10)(1)$

$= -7\lambda + \lambda^2 + 10$

$= \lambda^2 - 7\lambda + 10$ A1

[2]

(c) $\lambda_1 = 2$, $\lambda_2 = 5$ A2

[2]

(d) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ A2

[2]

(e) $\mathbf{X} = Ae^{2t}\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} + Be^{5t}\begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} 3 \\ 0 \end{pmatrix} = Ae^{2(0)}\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} + Be^{5(0)}\begin{pmatrix} 1 \\ \frac{1}{5} \end{pmatrix}$ (M1) for substitution

$\begin{cases} 3 = A + B \\ 0 = \dfrac{1}{2}A + \dfrac{1}{5}B \end{cases}$

By solving this system, $A = -2$ and $B = 5$. (A1) for correct values

$\therefore v = -2e^{2t} + 5e^{5t}$ and $x = -e^{2t} + e^{5t}$. A2

[5]