Applications and Interpretation Higher Level for IBDP Mathematics
Practice Paper Set 1 – Paper 1 (120 Minutes)

Question – Answer Book

Instructions

1. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

2. A graphic display calculator is needed.

3. You are suggested to prepare a formula booklet of Applications and Interpretation for IBDP Mathematics when attempting the questions.

4. Supplementary answer sheets and graph papers will be supplied on request.

5. Unless otherwise specified, **ALL** working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

7. The diagrams in this paper are **NOT** necessarily drawn to scale.

8. Information to be read before you start the exam:

	Marker's Use Only	**Examiner's Use Only**	
Question Number	**Marks**	**Marks**	**Maximum Mark**
1			**5**
2			**5**
3			**6**
4			**5**
5			**6**
6			**6**
7			**7**
8			**6**
9			**6**
10			**5**
11			**8**
12			**7**
13			**6**
14			**7**
15			**7**
16			**5**
17			**7**
18			**6**
Overall			
Paper 1 Total			**110**

1. In a football match, eight players take penalty kicks one by one. The table below shows the ball speed of each penalty kick:

Player	Ball Speed	Player	Ball Speed
Abraham	$80\ \text{kmh}^{-1}$	Essien	$40\ \text{kmh}^{-1}$
Berg	$76\ \text{kmh}^{-1}$	Flores	$116\ \text{kmh}^{-1}$
Clyne	$100\ \text{kmh}^{-1}$	Gana	$90\ \text{kmh}^{-1}$
Denayer	$66\ \text{kmh}^{-1}$	Harry	$76\ \text{kmh}^{-1}$

(a) Find the mean ball speed.

[2]

(b) Write down

(i) the median speed;

(ii) the standard deviation of the speeds;

(iii) the range of the speeds.

[3]

2. The number of seats in a theatre is investigated. The number of seats in the first row of the theatre u_1 is 100. The number of seats in each subsequent row forms an arithmetic sequence. The number of seats in the tenth row u_{10} is 181.

(a) Find the value of d, the common difference.

[2]

(b) Hence, write down the number of seats in the thirteenth row.

[1]

There are 15 rows in the theatre.

(c) Find the **total** number of seats in the theatre.

[2]

3. A triangular park ABC is built such that $\mathrm{AB} = 28\ \mathrm{m}$, $\mathrm{AC} = 32\ \mathrm{m}$ and $\mathrm{BC} = 41\ \mathrm{m}$.

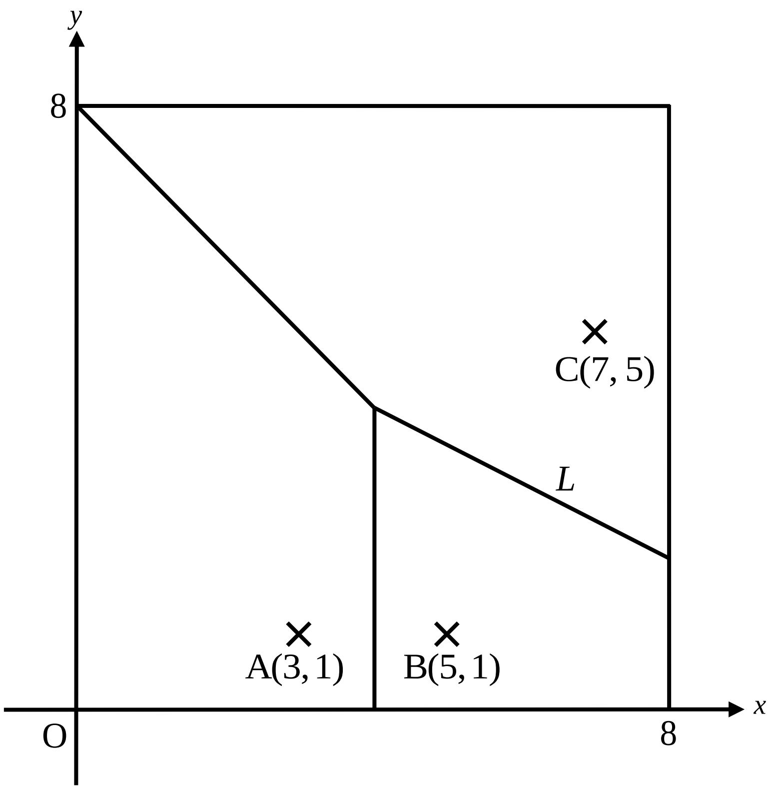
(a) Find $\mathrm{A\hat{B}C}$.

[3]

(b) Hence, find the area of the park.

[3]

4. The diagram below shows the Voronoi diagram of three restaurants for take-away meals, A, B and C, in a town bounded by the coordinate axes, the lines $x=8$ and $y=8$, where 1 unit represents $1\,\mathrm{km}$.



The straight line L is the boundary separating the Voronoi cells of B and C. It is given that $(4, 4)$ is a point on L.

(a) (i) Find the gradient of L.

(ii) Hence, find the equation of L, giving the answer in slope-intercept form.

[4]

Kimberly would like to find a restaurant closest to her office to minimize the delivery time of her meal during lunchtime. The position of her office is at $(7, 2.5)$.

(b) State the reason that she is indifferent from choosing the restaurant B and the restaurant C.

[1]

5. A fair eight-faced die with numbered faces $1, 2, 3, 4, 5, 6, 7$ and 8 is tossed for thirteen times.

(a) Find the expected number of multiples of 3 landed. [2]

(b) Find the variance of the number of multiples of 3 landed. [2]

(c) Find the probability that the number of multiples of 3 landed is 8. [2]

6. A closed rectangular box has length $4x$ cm, width $2x$ cm and height y cm, where x, $y > 0$. It is given that the sum of the length and the height of the rectangular box is 20 cm.

(a) Write down

(i) an expression for y in terms of x;

(ii) the possible range of values of x.

[2]

(b) Express V in terms of x, where V cm^3 is the volume of the rectangular box.

[2]

(c) Using the graphic display calculator to find its maximum volume.

[2]

7. Mitsuhide is going to purchase a boat. He is suggested a plan to repay the loan of $\$950000$:

A total of 120 equal monthly payments have to be paid at the end of each month, with a nominal annual interest rate of 3.3%, compounded monthly.

(a) Find the amount of monthly payment. [3]

(b) Find the total amount to be paid. [2]

(c) Hence, find the amount of interest paid. [2]

8. The graph of a quadratic function has y-intercept 150 and one of its x-intercept is -5. The x-coordinate of the vertex of the graph is 5. The equation of the quadratic function is in the form $y = ax^2 + bx + c$.

(a) Write down the value of c. [1]

(b) Write down the second x-intercept of the function. [1]

(c) Find the value of a and of b. [4]

9. In a supermarket, the weights of apples are normally distributed with mean 140 g and standard deviation 9 g, and the weights of oranges are normally distributed with mean 200 g and standard deviation 14 g. Three apples are randomly chosen. Let X be the total weight of the selected apples.

(a) Write down

(i) the mean of X ;

(ii) the variance of X .

[2]

Three apples and seven oranges are randomly chosen. Let Y be the total weight of the selected fruits.

(b) Write down

(i) the mean of Y ;

(ii) the standard deviation of Y .

[2]

(c) Hence, find $\mathrm{P}(Y \geq 1770)$.

[2]

10. The weight of a plate of area $A\,\text{cm}^2$ is $W\,\text{g}$. It is given that W varies directly as $\sqrt[3]{A}$. When $A = 512$, $W = 96$.

(a) Express W in terms of A.

[2]

(b) Write down the area of a plate of weight 60 grams.

[1]

The graph of W is transformed to the new graph of $W = 7 + 24\sqrt[3]{A}$ by two transformations.

(c) Describe geometrically for the two transformations.

[2]

11. The number of torches sold in a store each week follows a Poisson distribution with mean λ, where $\lambda > 20$, $\lambda \in \mathbb{Z}$. The probability that 25 torches are sold in a particular week is 0.0555460.

(a) Find λ.

[2]

(b) Hence, find the probability that

(i) at least 19 torches are sold on a particular week;

(ii) exactly 1 torch is sold on a particular day.

(iii) exactly 1 torch is sold for each of the four consecutive days.

[6]

12. The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = 8e^t \sin 3t$, $0 \le t \le \pi$.

(a) Find the maximum distance of the particle from O.

[2]

(b) (i) Find the time when the particle first goes back to O.

(ii) Find $s'(t)$.

(iii) Hence, write down the acceleration of the particle at the instant it first goes back to O.

[5]

13. Two surveys are conducted to measure the residents' satisfaction on the services provided by the community centre. A score from 0 to 10 is used in the surveys, where 0 represents absolute dissatisfaction and 10 represents absolute satisfaction. The table below shows the results of the surveys completed by 6 residents:

Resident	A	B	C	D	E	F
Scores from the first survey (x)	5	7	3	6	8	8
Scores from the second survey (y)	4	9	5	5	9	9

The manager of the community centre wants to investigate whether the mean scores of the second survey has improved. A paired t-test is conducted at a 5% significance level. Define $d = x - y$.

(a) (i) Write down the null hypothesis of the test.

(ii) Write down the alternative hypothesis of the test.

[2]

(b) Find the p-value.

[2]

(c) State the conclusion of the test with a reason.

[2]

14. Let $f(x)=9x+1$, $g(x)=2\sin\left(\dfrac{x}{3}\right)-6$. Let $h(x)=(g\circ f)(x)$.

(a) Find an expression for $h(x)$.

[3]

(b) Find the period of h.

[2]

(c) Write down the range of h.

[2]

15. A quadratic function is given by $f(x) = ax^2 + bx + c$. It is given that the complex roots of $f(x) = 0$ are $\frac{1}{2} + \frac{1}{4}\mathrm{i}$ and $\frac{1}{2} - \frac{1}{4}\mathrm{i}$.

(a) Write down the values of

(i) $\left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right) + \left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)$;

(ii) $\left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)\left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)$.

[2]

(b) Hence, find the expression of $f(x)$, giving the answer in terms of a.

[3]

The graph of $f(x)$ passes through $\left(1, \frac{5}{2}\right)$.

(c) Find the value of a.

[2]

16. The value $V(t)$ of a pendulum clock (in dollars) t years after 31st December, 1888 can be modelled by $V(t)=\begin{cases}\dfrac{1000000}{1+29e^{-2.175}}(t+15) & 0\le t<15\\ \dfrac{30000000}{1+29e^{-0.145t}} & t\ge 15\end{cases}$.

(a) Find the value of the pendulum clock at the end of 1899.

[2]

(b) Find t when $V(t)=10000000$.

[2]

(c) Interpret the condition on the value of the pendulum clock after a long period of time.

[1]

17. R is defined to be the region bounded by the lines $y = e^{0.25x} - 1.25$, $y = 8$, the x-axis and the y-axis

(a) (i) Show that $x = 4\ln(y + 1.25)$.

(ii) Hence, find the area of R.

[5]

A solid model is formed by rotating the region R through 2π about the y-axis.

(b) Find the volume of the solid model.

[2]

18. The width of photo frames, in centimetres, sold in a bazaar is studied. 11 photo frames are randomly selected and the corresponding widths are measured. It is given that the sample mean is 38 cm and the width of the 99% confidence interval for the population mean is 13.8 cm.

(a) Explain why the 90% confidence interval for the population mean is a subset of the 99% confidence interval for the population mean.

[1]

(b) Write down the 99% confidence interval for the population mean.

[1]

Let σ^2 be the known population variance.

(c) Find σ^2.

[4]

END OF PAPER