Chapter 14 Solution

Exercise 62

1. $\lim_{x\to 0}\dfrac{\sin\frac{\pi}{4}x}{\ln(x+1)}=\lim_{x\to 0}\dfrac{\left(\cos\frac{\pi}{4}x\right)\left(\frac{\pi}{4}\right)}{\left(\frac{1}{x+1}\right)(1)}\left(\because \frac{0}{0}\right)$ M1A2

$\lim_{x\to 0}\dfrac{\sin\frac{\pi}{4}x}{\ln(x+1)}=\lim_{x\to 0}\dfrac{\pi}{4}(x+1)\cos\dfrac{\pi}{4}x$

$\lim_{x\to 0}\dfrac{\sin\frac{\pi}{4}x}{\ln(x+1)}=\dfrac{\pi}{4}(0+1)\cos\dfrac{\pi}{4}(0)$ M1

$\lim_{x\to 0}\dfrac{\sin\frac{\pi}{4}x}{\ln(x+1)}=\dfrac{\pi}{4}$ A1

[5]

2. $\lim_{x\to 0}\dfrac{3^x-1}{\sin 2x}=\lim_{x\to 0}\dfrac{3^x\ln 3-0}{(\cos 2x)(2)}\left(\because \frac{0}{0}\right)$ M1A2

$\lim_{x\to 0}\dfrac{3^x-1}{\sin 2x}=\lim_{x\to 0}\dfrac{3^x\ln 3}{2\cos 2x}$

$\lim_{x\to 0}\dfrac{3^x-1}{\sin 2x}=\dfrac{3^0\ln 3}{2\cos 2(0)}$ M1

$\lim_{x\to 0}\dfrac{3^x-1}{\sin 2x}=\dfrac{\ln 3}{2}$ A1

[5]

3. $\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\lim_{x\to0}\frac{2x}{(-\sin 2x)(2)-(-\sin x)}\left(\because \frac{0}{0}\right)$ M1A2

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\lim_{x\to0}\frac{2x}{-2\sin 2x+\sin x}$

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\lim_{x\to0}\frac{2(1)}{-2(\cos 2x)(2)+\cos x}\left(\because \frac{0}{0}\right)$ A2

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\lim_{x\to0}\frac{2}{-4\cos 2x+\cos x}$

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\frac{2}{-4\cos 2(0)+\cos 0}$ M1

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=\frac{2}{-4+1}$

$\lim_{x\to0}\frac{x^2}{\cos 2x-\cos x}=-\frac{2}{3}$ A1

[7]

4. $\lim_{x\to0}\frac{1-\sec x}{\sin 2x+\cos 2x-1-2x}$

$=\lim_{x\to0}\frac{0-\sec x\tan x}{(\cos 2x)(2)+(-\sin 2x)(2)-0-2(1)}\left(\because \frac{0}{0}\right)$ M1A2

$=\lim_{x\to0}\frac{-\sec x\tan x}{2\cos 2x-2\sin 2x-2}$

$=\lim_{x\to0}\frac{(-\sec x\tan x)(\tan x)+(-\sec x)(\sec^2 x)}{2(-\sin 2x)(2)-2(\cos 2x)(2)-0}\left(\because \frac{0}{0}\right)$ A2

$=\lim_{x\to0}\frac{-\sec x\tan^2 x-\sec^3 x}{-4\sin 2x-4\cos 2x}$

$=\frac{-\sec 0\tan^2 0-\sec^3 0}{-4\sin 2(0)-4\cos 2(0)}$ M1

$=\frac{0-1}{0-4}$

$=\frac{1}{4}$ A1

[7]

Exercise 63

1.	$f(0)=e^{2(0)}\tan 0=0$	(A1) for correct value
	$f'(x)=2e^{2x}\tan x+e^{2x}\sec^2 x$	
	$f'(x)=e^{2x}(2\tan x+\sec^2 x)$	
	$f'(0)=e^{2(0)}(2\tan 0+\sec^2 0)=1$	(A1) for correct value
	$f''(x)=2e^{2x}(2\tan x+\sec^2 x)$ $+e^{2x}(2\sec^2 x+2\sec x(\sec x\tan x))$	(M1) for valid approach
	$f''(x)=e^{2x}(4\tan x+4\sec^2 x+2\sec^2 x\tan x)$	
	$f''(0)=e^{2(0)}(4\tan 0+4\sec^2 0+2\sec^2 0\tan 0)=4$	(A1) for correct value
	$f(x)=f(0)+xf'(0)+\dfrac{x^2}{2!}f''(0)+\cdots$	
	$f(x)=0+x(1)+\dfrac{x^2}{2}(4)+\cdots$	M1
	$f(x)=x+2x^2+\cdots$	A1
		[6]

2. $f(0)=(1+0)^{\frac{2}{3}}=1$ (A1) for correct value

$f'(x)=\frac{2}{3}(1+x)^{-\frac{1}{3}}(1)$

$f'(x)=\frac{2}{3}(1+x)^{-\frac{1}{3}}$

$f'(0)=\frac{2}{3}(1+0)^{-\frac{1}{3}}=\frac{2}{3}$ (A1) for correct value

$f''(x)=\frac{2}{3}\left(-\frac{1}{3}\right)(1+x)^{-\frac{4}{3}}(1)$

$f''(x)=-\frac{2}{9}(1+x)^{-\frac{4}{3}}$

$f''(0)=-\frac{2}{9}(1+0)^{-\frac{4}{3}}=-\frac{2}{9}$ (A1) for correct value

$f^{(3)}(x)=-\frac{2}{9}\left(-\frac{4}{3}\right)(1+x)^{-\frac{7}{3}}(1)$

$f^{(3)}(x)=\frac{8}{27}(1+x)^{-\frac{7}{3}}$

$f^{(3)}(0)=\frac{8}{27}(1+0)^{-\frac{7}{3}}=\frac{8}{27}$ (A1) for correct value

$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f^{(3)}(0)+\cdots$

$f(x)=1+x\left(\frac{2}{3}\right)+\frac{x^2}{2}\left(-\frac{2}{9}\right)+\frac{x^3}{6}\left(\frac{8}{27}\right)+\cdots$ M1

$f(x)=1+\frac{2}{3}x-\frac{1}{9}x^2+\frac{4}{81}x^3+\cdots$ A1

[6]

3. $f(0)=3(0)^2-\pi^2(0)+\frac{\pi^2}{2}-\pi\arccos\pi(0)=0$ (A1) for correct approach

$f'(x)=6x-\pi^2+0-\pi\left(-\frac{\pi}{\sqrt{1-(\pi x)^2}}\right)$ (M1) for valid approach

$f'(x)=6x-\pi^2+\frac{\pi^2}{\sqrt{1-\pi^2x^2}}$

$f'(0)=6(0)-\pi^2+\frac{\pi^2}{\sqrt{1-\pi^2(0)^2}}=0$ (A1) for correct approach

$f''(x)=6-0-\frac{\pi^2}{2}(1-\pi^2x^2)^{-\frac{3}{2}}(-2\pi^2x)$ (M1) for valid approach

$f''(x)=6+\pi^4x(1-\pi^2x^2)^{-\frac{3}{2}}$

$f''(0)=6+\pi^4(0)(1-\pi^2 0^2)^{-\frac{3}{2}}=6$ (A1) for correct value

The first non-zero term

$=\frac{x^2}{2!}(6)$ M1

$=3x^2$ A1

[7]

4. $f(0)=0+\cos 0-1=0$ (A1) for correct approach

$f'(x)=3x^2-(\sin(x^3))(3x^2)-0$ (M1) for valid approach

$f'(x)=3x^2-3x^2\sin(x^3)$

$f'(0)=3(0)^2-3(0)^2\sin 0=0$ (A1) for correct approach

$f''(x)=6x-((6x)(\sin(x^3))+3x^2(\cos(x^3))(3x^2))$

$f''(x)=6x-6x\sin(x^3)-9x^4\cos(x^3)$

$f''(0)=6(0)-6(0)\sin 0-9(0)^4\cos 0=0$ (A1) for correct approach

$f^{(3)}(x)=6-((6)(\sin(x^3))+6x(\cos(x^3))(3x^2))$

$-((36x^3)(\cos(x^3))+9x^4(-\sin(x^3))(3x^2))$

$f^{(3)}(x)=6+(27x^6-6)\sin(x^3)-54x^3\cos(x^3)$

$f^{(3)}(0)=6+(27(0)^6-6)\sin 0-54(0)^3\cos 0=6$ (A1) for correct value

The first non-zero term

$=\frac{x^3}{3!}(6)$ M1

$=x^3$ A1

[7]

Exercise 64

1. (a) $f(0)=e^{3(0)}=1$ (A1) for correct value

$f'(x)=3e^{3x}$

$f'(0)=3e^{3(0)}=3$ (A1) for correct value

$f''(x)=3(3e^{3x})$

$f''(x)=9e^{3x}$

$f''(0)=9e^{3(0)}=9$ (A1) for correct value

$f^{(3)}(x)=9(3e^{3x})$

$f^{(3)}(x)=27e^{3x}$

$f^{(3)}(0)=27e^{3(0)}=27$ (A1) for correct value

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f^{(3)}(0)+\cdots$$

$$f(x)=1+x(3)+\frac{x^2}{2}(9)+\frac{x^3}{6}(27)+\cdots$$ M1

$$f(x)=1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3+\cdots$$ A1

[6]

(b) $3x=0.03$

$x=0.01$ (A1) for correct value

$\therefore e^{0.03}\approx 1+3(0.01)+\frac{9}{2}(0.01)^2+\frac{9}{2}(0.01)^3$ M1

$e^{0.03}\approx 1.0304545$ A1

[3]

2. (a) $f(0) = \ln \tan\left(0 + \frac{\pi}{4}\right) = 0$ (A1) for correct value

$$f'(x) = \left(\frac{1}{\tan\left(x + \frac{\pi}{4}\right)}\right)\left(\sec^2\left(x + \frac{\pi}{4}\right)\right)$$

$$f'(x) = \frac{1}{\sin\left(x + \frac{\pi}{4}\right)\cos\left(x + \frac{\pi}{4}\right)}$$

$$f'(x) = \frac{2}{\sin\left(2x + \frac{\pi}{2}\right)}$$

$$f'(0) = \frac{2}{\sin\left(2(0) + \frac{\pi}{2}\right)} = 2$$ (A1) for correct value

$$f''(x) = 2(-1)\left(\frac{1}{\sin^2\left(2x + \frac{\pi}{2}\right)}\right)\left(\cos\left(2x + \frac{\pi}{2}\right)\right)(2)$$ (M1) for valid approach

$$f''(x) = -\frac{4\cos\left(2x + \frac{\pi}{2}\right)}{\sin^2\left(2x + \frac{\pi}{2}\right)}$$

$$f''(0) = -\frac{4\cos\left(2(0) + \frac{\pi}{2}\right)}{\sin^2\left(2(0) + \frac{\pi}{2}\right)} = 0$$ (A1) for correct value

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$$

$$f(x) = 0 + x(2) + \frac{x^2}{2}(0) + \cdots$$ M1

$$f(x) = 2x + \cdots$$ A1

[6]

(b) $\ln \tan\left(\frac{\pi}{12}+\frac{\pi}{4}\right) \approx 2\left(\frac{\pi}{12}\right)$ M1

$\ln \tan\frac{\pi}{3} \approx \frac{\pi}{6}$

$\ln\sqrt{3} \approx \frac{\pi}{6}$ A1

$4\ln\sqrt{3} \approx 4\left(\frac{\pi}{6}\right)$

$\ln(\sqrt{3})^4 \approx \frac{2\pi}{3}$

$\ln 9 \approx \frac{2\pi}{3}$ A1

[3]

3. (a) $f(0)=\frac{1}{5}\ln(1+0^2)=0$ (A1) for correct value

$f'(x)=\frac{1}{5}\left(\frac{1}{1+x^2}\right)(2x)$

$f'(x)=\frac{2x}{5(1+x^2)}$

$f'(0)=\frac{2(0)}{5(1+0^2)}=0$ (A1) for correct value

$f''(x)=\frac{2}{5}\left(\frac{(1+x^2)(1)-(x)(2x)}{(1+x^2)^2}\right)$ (M1) for valid approach

$f''(x)=\frac{2(1-x^2)}{5(1+x^2)^2}$

$f''(0)=\frac{2(1-0^2)}{5(1+0^2)^2}=\frac{2}{5}$ (A1) for correct value

$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\cdots$

$f(x)=0+x(0)+\frac{x^2}{2}\left(\frac{2}{5}\right)+\cdots$ M1

$f(x)=\frac{1}{5}x^2+\cdots$ A1

[6]

(b) $f(x)\approx\frac{1}{5}x^2$

$f'(x)\approx\frac{1}{5}(2x)$ M1

$f'(x)\approx\frac{2}{5}x$ A1

$\therefore f'(0.1)\approx\frac{2}{5}(0.1)$

$f'(0.1)\approx 0.04$ A1

[3]

4. (a) $f(0)=\cot\left(\frac{\pi}{2}(0+1)\right)=0$ (A1) for correct value

$f'(x)=-\text{cosec}^2\left(\frac{\pi}{2}(x+1)\right)\cdot\frac{\pi}{2}$

$f'(x)=-\frac{\pi}{2}\text{cosec}^2\left(\frac{\pi}{2}(x+1)\right)$

$f'(0)=-\frac{\pi}{2}\text{cosec}^2\left(\frac{\pi}{2}(0+1)\right)=-\frac{\pi}{2}$ (A1) for correct value

$f''(x)=-\frac{\pi}{2}\left(2\text{cosec}\left(\frac{\pi}{2}(x+1)\right)\right)$

(M1) for valid approach

$\cdot\left(-\text{cosec}\left(\frac{\pi}{2}(x+1)\right)\cot\left(\frac{\pi}{2}(x+1)\right)\right)\cdot\frac{\pi}{2}$

$f''(x)=\frac{\pi^2}{2}\text{cosec}^2\left(\frac{\pi}{2}(x+1)\right)\cot\left(\frac{\pi}{2}(x+1)\right)$

$f''(0)=\frac{\pi^2}{2}\text{cosec}^2\left(\frac{\pi}{2}(0+1)\right)\cot\left(\frac{\pi}{2}(0+1)\right)=0$ (A1) for correct value

$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\cdots$

$f(x)=0+x\left(-\frac{\pi}{2}\right)+\frac{x^2}{2}(0)+\cdots$ M1

$f(x)=-\frac{\pi}{2}x+\cdots$ A1

[6]

(b) $\int_1^{1.5} f(t)\text{d}t\approx\int_1^{1.5}-\frac{\pi}{2}t\text{d}t$ M1

$\int_1^{1.5} f(t)\text{d}t\approx-0.9817477042$

$\int_1^{1.5} f(t)\text{d}t\approx-0.982$ A1

[2]

Exercise 65

1. (a) (i)

$f'(x) = (2^{\tan x} \ln 2)(\sec^2 x)$ (M1) for valid approach

$f'(x) = \ln 2 \cdot 2^{\tan x} \sec^2 x$ A1

$f''(x) = \ln 2 \begin{pmatrix} (2^{\tan x} \ln 2)(\sec^2 x) \\ +(2^{\tan x})(2\sec x)(\sec x \tan x) \end{pmatrix}$ (M1) for valid approach

$f''(x) = \ln 2 \cdot 2^{\tan x} \sec^2 x(\ln 2 + 2\tan x)$ A1

(ii)

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \cdots$

$f(x) = 2^{\tan 0} + x(\ln 2 \cdot 2^{\tan 0} \sec^2 0)$

$+\frac{x^2}{2}(\ln 2 \cdot 2^{\tan 0} \sec^2 0(\ln 2 + 2\tan 0)) + \cdots$ M2

$f(x) = 1 + x(\ln 2) + \frac{x^2}{2}(\ln 2 \cdot (\ln 2 + 0)) + \cdots$ A1

$f(x) = 1 + (\ln 2)x + \frac{(\ln 2)^2}{2} x^2 + \cdots$ A1

[8]

(b)

$e^{2x} - 1 = 1 + 2x + \frac{(2x)^2}{2!} + \cdots - 1$ (M1) for substitution

$e^{2x} - 1 = 2x + 2x^2 + \cdots$ (A1) for correct values

$\ln\left[(3e^{2x} - 2)(4e^{2x} - 3)\right]$

$= \ln(1 + 3(e^{2x} - 1))(1 + 4(e^{2x} - 1))$ (M1) for valid approach

$= \ln(1 + 3(e^{2x} - 1)) + \ln(1 + 4(e^{2x} - 1))$ (A1) for correct approach

$= 3(e^{2x} - 1) - \frac{(3(e^{2x} - 1))^2}{2} + \cdots$

$+4(e^{2x} - 1) - \frac{(4(e^{2x} - 1))^2}{2} + \cdots$ (M1) for valid approach

$= 7(e^{2x} - 1) - \frac{25(e^{2x} - 1)^2}{2} + \cdots$ (A1) for simplification

$= 7(2x + 2x^2 + \cdots) - \frac{25(2x + 2x^2 + \cdots)^2}{2} + \cdots$ (A1) for substitution

$= 14x + 14x^2 - \frac{25(4x^2 + \cdots)}{2} + \cdots$ (A1) for correct approach

$= 14x + 14x^2 - 50x^2 + \cdots$

$= 14x - 36x^2 + \cdots$ A1

[9]

(c) $\lim_{x\to 0}\frac{2+\ln\left[(3e^{2x}-2)(4e^{2x}-3)\right]}{f(x)}$

$=\lim_{x\to 0}\frac{2+14x-36x^2+\cdots}{1+(\ln 2)x+\frac{(\ln 2)^2}{2}x^2+\cdots}$ A1

$=\frac{2+0-0+\cdots}{1+0+0+\cdots}$ M1

$=2$ A1

[3]

2. (a) (i) $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$

$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$ (M1) for valid approach

$\frac{1}{1+x^2} = 1 - \frac{1}{3}(3x^2) + \frac{1}{5}(5x^4) + \cdots$ (A2) for correct approach

$\frac{1}{1+x^2} = 1 - x^2 + x^4 + \cdots$ A1

(ii) $g(x) = \frac{1}{1+(x^{1.5})^2}$

$g(x) = 1 - (x^{1.5})^2 + (x^{1.5})^4 + \cdots$ (A1) for substitution

$g(x) = 1 - x^3 + x^6 + \cdots$ A1

[6]

(b) $g(0.1) = 1 - 0.1^3 + 0.1^6 + \cdots$ M1

$g(0.1) > 1 - 0.1^3$

$g(0.1) > 0.999$ A1

$g(0.1) < 1 - 0.1^3 + 0.1^6$

$g(0.1) < 0.999001$ A1

$\therefore 0.999 < g(0.1) < 0.999001$ M1

$0.999 < \frac{1}{1+0.1^3} < 0.999001$

$0.999 < \frac{1}{1.001} < 0.999001$ AG

[4]

(c) $\lim_{x\to 0} \frac{g(x)-1}{e^{x^3}-1}$

$= \lim_{x\to 0} \frac{1 - x^3 + x^6 + \cdots - 1}{1 + x^3 + \frac{(x^3)^2}{2!} + \cdots - 1}$ M1A1

$= \lim_{x\to 0} \frac{-x^3 + x^6 + \cdots}{x^3 + \frac{1}{2}x^6 + \cdots}$ M1

$= \lim_{x\to 0} \frac{-1 + x^3 + \cdots}{1 + \frac{1}{2}x^3 + \cdots}$ M1A1

$= \frac{-1 + 0 + \cdots}{1 + 0 + \cdots}$ M1

$= -1$ AG

[6]

3. (a) $\sin x = x - \frac{x^3}{3!} + \cdots$

$\sin 2x = 2x - \frac{(2x)^3}{3!} + \cdots$ M1

$\sin 2x = 2x - \frac{4}{3}x^3 + \cdots$ A1

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \cdots$ M1

$\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 + \cdots$ A1

$\sin^2 4x = (2\sin 2x \cos 2x)^2$ A1

$\sin^2 4x = \left(2\left(2x - \frac{4}{3}x^3 + \cdots\right)\left(1 - 2x^2 + \frac{2}{3}x^4 + \cdots\right)\right)^2$ M1

$\sin^2 4x = 4\left(2x - 4x^3 - \frac{4}{3}x^3 + \cdots\right)^2$

$\sin^2 4x = 4\left(2x - \frac{16}{3}x^3 + \cdots\right)^2$ A1

$\sin^2 4x = 4\left(4x^2 - \frac{64}{3}x^4 + \cdots\right)$

$\sin^2 4x = 16x^2 - \frac{256}{3}x^4 + \cdots$ AG

[7]

(b)	$\sin^2 4\left(\frac{\pi}{16}\right) > 16\left(\frac{\pi}{16}\right)^2 - \frac{256}{3}\left(\frac{\pi}{16}\right)^4$	M1A1
	$\sin^2 \frac{\pi}{4} > \frac{\pi^2}{16} - \frac{\pi^4}{768}$	A1
	$\sin^2 \frac{\pi}{4} > \frac{48\pi^2}{768} - \frac{\pi^4}{768}$	
	$\sin^2 \frac{\pi}{4} > \frac{\pi^2(48-\pi^2)}{768}$	A1
	$\sin^2 4\left(\frac{\pi}{16}\right) < 16\left(\frac{\pi}{16}\right)^2$	M1A1
	$\sin^2 \frac{\pi}{4} < \frac{\pi^2}{16}$	A1
	$\therefore \frac{\pi^2(48-\pi^2)}{768} < \sin^2 \frac{\pi}{4} < \frac{\pi^2}{16}$	A1
	$\sqrt{\frac{\pi^2(48-\pi^2)}{768}} < \sin \frac{\pi}{4} < \sqrt{\frac{\pi^2}{16}}$	M1
	$\frac{\pi}{16}\sqrt{\frac{48-\pi^2}{3}} < \sin \frac{\pi}{4} < \frac{\pi}{4}$	AG

[9]

4. (a) (i) $f'(x) = \sec x \tan x$ A1

$f''(x) = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$ (M1) for valid approach

$f''(x) = \sec x \tan^2 x + \sec^3 x$

$f''(x) = \sec x(\sec^2 x - 1) + \sec^3 x$

$f''(x) = \sec^3 x - \sec x + \sec^3 x$

$f''(x) = 2\sec^3 x - \sec x$ A1

$f^{(3)}(x) = 2(3)(\sec^2 x)(\sec x \tan x)$
$-\sec x \tan x$ (M1) for valid approach

$f^{(3)}(x) = \sec x \tan x(6\sec^2 x - 1)$ A1

(ii) $f(x) = f(0) + xf'(0)$
$+\dfrac{x^2}{2!} f''(0) + \dfrac{x^3}{3!} f^{(3)}(0) + \cdots$

$f(x) = \sec 0 + x(\sec 0 \tan 0)$
$+\dfrac{x^2}{2}(2\sec^3 0 - \sec 0)$ M2
$+\dfrac{x^3}{6}(\sec 0 \tan 0(6\sec^2 0 - 1)) + \cdots$

$f(x) = 1 + x(0) + \dfrac{x^2}{2}(1) + \dfrac{x^3}{6}(0) + \cdots$ A1

$f(x) = 1 + \dfrac{1}{2}x^2 + \cdots$ A1

[9]

(b) $\sin x = x - \dfrac{x^3}{3!} + \cdots$

$\tan x = \dfrac{\sin x}{\cos x}$

$\tan x = \sin x \sec x$ (A1) for correct approach

$\tan x = \left(x - \dfrac{x^3}{6} + \cdots\right)\left(1 + \dfrac{1}{2}x^2 + \cdots\right)$ M1A1

$\tan x = x + \dfrac{1}{2}x^3 - \dfrac{1}{6}x^3 + \cdots$ (M1) for valid approach

$\tan x = x + \dfrac{1}{3}x^3 + \cdots$ A1

[5]

(c) The approximate value of the area

$= \int_{0.5}^{1} y \tan y \, \mathrm{d}y$ (M1) for valid approach

$\approx \int_{0.5}^{1} y\left(y + \frac{1}{3}y^3 \right)\mathrm{d}y$ (A1) for substitution

$\approx \int_{0.5}^{1} \left(y^2 + \frac{1}{3}y^4 \right)\mathrm{d}y$ (M1) for valid approach

≈ 0.35625 A1

[4]