Chapter 20 Solution

Exercise 82

1. (a) The expected number

$= (10)(0.25)$ (A1) for substitution

$= 2.5$ A1 N2

[2]

(b) The variance

$= (10)(0.25)(1-0.25)$ (A1) for substitution

$= 1.875$ A1 N2

[2]

(c) The required probability

$= \binom{10}{3}(0.25)^3(1-0.25)^{10-3}$ (A1) for substitution

$= 0.250282287$

$= 0.250$ A1 N2

[2]

2. (a) $\mathrm{E}(X) = 18$

$0.6n = 18$ (A1) for substitution

$n = 30$ A1 N2

[2]

(b) $\mathrm{Var}(X)$

$= (30)(0.6)(1-0.6)$ (A1) for substitution

$= 7.2$ A1 N2

[2]

(c) $\mathrm{P}(X = 19)$

$= \binom{30}{19}(0.6)^{19}(1-0.6)^{30-19}$ (A1) for substitution

$= 0.139618638$

$= 0.140$ A1 N2

[2]

3.	(a)	$20p(1-p)=3.2$	(A1) for substitution	
		$-20p^2+20p-3.2=0$	(M1) for quadratic equation	
		By considering the graph of $y=-20p^2+20p-3.2$,		
		$p=0.8$ or $p=0.2$ (*Rejected*).	A1 N3	
				[3]
	(b)	The expected number		
		$=(20)(0.8)$	(A1) for substitution	
		$=16$	A1 N2	
				[2]
	(c)	The required probability		
		$=\binom{20}{17}(0.8)^{17}(1-0.8)^{20-17}$	(A1) for substitution	
		$=0.205364143$		
		$=0.205$	A1 N2	
				[2]
4.	(a)	$\begin{cases} np=1800 \\ np(1-p)=990 \end{cases}$	(A1) for correct equations	
		$1800(1-p)=990$	(M1) for substitution	
		$1-p=0.55$		
		$p=0.45$	A1 N3	
				[3]
	(b)	$0.45n=1800$	(A1) for substitution	
		$n=4000$	A1 N2	
				[2]
	(c)	2.3511×10^{-11}	A2 N2	
				[2]

Exercise 83

1. (a) $E(X) = (80)(0.06)$ (A1) for substitution

$E(X) = 4.8$ A1 N2

[2]

(b) $P(X = 10)$

$= \binom{80}{10}(0.06)^{10}(1-0.06)^{80-10}$ (A1) for substitution

$= 0.0130924797$

$= 0.0131$ A1 N2

[2]

(c) $P(X \geq 15)$

$= 1 - P(X \leq 14)$ (M1) for valid approach

$= 1 - 0.9999251314$ (A1) for correct value

$= 0.0000748686$

$= 0.0000749$ A1 N3

[3]

2. (a) $E(X) = (135)(0.12)$ (A1) for substitution

$E(X) = 16.2$ A1 N2

[2]

(b) $P(X = 20)$

$= \binom{135}{20}(0.12)^{20}(1-0.12)^{135-20}$ (A1) for substitution

$= 0.0597993427$

$= 0.0598$ A1 N2

[2]

(c) $P(X > 16)$

$= 1 - P(X \leq 16)$ (M1) for valid approach

$= 1 - 0.5449524887$ (A1) for correct value

$= 0.4550475113$

$= 0.455$ A1 N3

[3]

3. (a) $\mathrm{E}(X) = (50)(0.02)$ (A1) for substitution

$\mathrm{E}(X) = 1$ A1 N2

[2]

(b) $\mathrm{P}(X = 9)$

$= \binom{50}{9}(0.02)^9(1-0.02)^{50-9}$ (A1) for substitution

$= 0.000000560302$

$= 0.000000560$ A1 N2

[2]

(c) $\mathrm{P}(X \le 2)$

$= 0.9215722517$ (M1) for valid approach

$= 0.922$ A1 N2

[2]

4. (a) The mean number of heads

$= (9)(0.69)$ (A1) for substitution

$= 6.21$ A1 N2

[2]

(b) The required probability

$= \binom{9}{6}(0.69)^6(1-0.69)^{9-6}$ (A1) for substitution

$= 0.2700591597$

$= 0.270$ A1 N2

[2]

(c) The required probability

$= 0.005271637$ (M1) for valid approach

$= 0.00527$ A1 N2

[2]

Exercise 84

1. (a) The required probability

$=\binom{120}{3}p^3(1-p)^{120-3}$ (A1) for substitution

$=\binom{120}{3}p^3(1-p)^{117}$ A1 N2

[2]

(b) $\binom{120}{3}p^3(1-p)^{117}=0.16$ (M1) for setting equation

$\binom{120}{3}p^3(1-p)^{117}-0.16=0$

By considering the graph of

$y=\binom{120}{3}p^3(1-p)^{117}-0.16$, $p=0.0148695$

or $p=0.0388023$.

$\therefore p=0.0149$ or $p=0.0388$ A2 N3

[3]

2. (a) The required probability

$=\binom{5}{4}p^4(1-p)^{5-4}$ (A1) for substitution

$=5p^4(1-p)$ A1 N2

[2]

(b) $5p^4(1-p)=0.3$ (M1) for setting equation

$5p^4(1-p)-0.3=0$

By considering the graph of $y=5p^4(1-p)-0.3$,

$p=0.6381051$ or $p=0.9140419$.

$\therefore p=0.638$ or $p=0.914$ A2 N3

[3]

3. (a) The required probability

$= \binom{10}{9} q^9 (1-q)^{10-9} + \binom{10}{10} q^{10} (1-q)^{10-10}$ (A1) for substitution

$= 10q^9(1-q) + q^{10}$ A1 N2

[2]

(b) $10q^9(1-q) + q^{10} = 0.09$ (M1) for setting equation

$10q^9(1-q) + q^{10} - 0.09 = 0$

By considering the graph of

$y = 10q^9(1-q) + q^{10} - 0.09$, $q = 0.6539559$.

$\therefore q = 0.654$ A2 N3

[3]

4. (a) The required probability

$= \binom{100}{0} q^0 (1-q)^{100-0} + \binom{100}{1} q^1 (1-q)^{100-1}$ (A1) for substitution

$= (1-q)^{100} + 100q(1-q)^{99}$ A1 N2

[2]

(b) $(1-q)^{100} + 100q(1-q)^{99} = 0.03$ (M1) for setting equation

$(1-q)^{100} + 100q(1-q)^{99} - 0.03 = 0$

By considering the graph of

$y = (1-q)^{100} + 100q(1-q)^{99} - 0.03$, $q = 0.0524073$.

$\therefore q = 0.0524$ A2 N3

[3]

Exercise 85

1.	(a)	The required probability	
		$= 0.56 \times 0.12 + (1 - 0.56) \times 0.76$	(M1)(A1) for substitution
		$= 0.56 \times 0.12 + 0.44 \times 0.76$	
		$= 0.4016$	A1 N3
			[3]
	(b)	The required probability	
		$= \dfrac{0.44 \times 0.76}{0.4016}$	(M1)(A1) for substitution
		$= 0.8326693227$	
		$= 0.833$	A1 N3
			[3]
	(c)	$X \sim B(6, 0.5984)$	(R1) for binomial distribution
		$\mathrm{P}(X = 4)$	
		$= \binom{6}{4}(0.5984)^4(1 - 0.5984)^{6-4}$	(A1) for substitution
		$= 0.3102022951$	
		$= 0.310$	A1 N3
			[3]
	(d)	The probability that Joyce did not stay at home for	
		all n days is 0.4016^n.	(M1) for valid approach
		$1 - 0.4016^n > 0.84$	(M1)(A1) for correct inequality
		$0.16 - 0.4016^n > 0$	
		By considering the graph of $y = 0.16 - 0.4016^n$,	
		$n > 2.0087516$.	(A1) for correct value
		$\therefore n = 3$	A1 N5
			[5]

2. (a) The required probability

$= 0.4 \times 0.2 + (1 - 0.4) \times 0.3$ (M1)(A1) for substitution

$= 0.4 \times 0.2 + 0.6 \times 0.3$

$= 0.26$ A1 N3

[3]

(b) The required probability

$= \frac{0.6 \times 0.3}{0.26}$ (M1)(A1) for substitution

$= 0.6923076923$

$= 0.692$ A1 N3

[3]

(c) $X \sim B(4, 0.26)$ (R1) for binomial distribution

$\mathrm{P}(X = 2)$

$= \binom{4}{2}(0.26)^2(1 - 0.26)^{4-2}$ (A1) for substitution

$= 0.22210656$

$= 0.222$ A1 N3

[3]

(d) $1 - 0.74^n - n(0.74)^{n-1}(0.26) > 0.75$ (M1)(A1) for correct inequality

$0.25 - 0.74^n - 0.26n(0.74)^{n-1} > 0$ (M1) for simplification

By considering the graph of

$y = 0.25 - 0.74^n - 0.26n(0.74)^{n-1}$, $n > 9.4689646$. (A1) for correct value

$\therefore n = 10$ A1 N5

[5]

3. (a) The required probability

$= 0.45 \times 0.13 + (1-0.45) \times 0.59$ (M1)(A1) for substitution

$= 0.45 \times 0.13 + 0.55 \times 0.59$

$= 0.383$ A1 N3

[3]

(b) The required probability

$= \dfrac{0.55 \times 0.59}{0.383}$ (M1)(A1) for substitution

$= 0.8472584856$

$= 0.847$ A1 N3

[3]

(c) $X \sim B(7, 0.383)$ (R1) for binomial distribution

$P(X = 3)$

$= \binom{7}{3}(0.383)^3(1-0.383)^{7-3}$ (A1) for substitution

$= 0.2849738583$

$= 0.285$ A1 N3

[3]

(d) The probability that Lydia caught a fish at most one day is $(1-0.383)^n + n(1-0.383)^{n-1}(0.383)$. (M1) for valid approach

$1-[(1-0.383)^n + n(1-0.383)^{n-1}(0.383)] > 0.93$ (M1)(A1) for correct inequality

$0.07 - 0.617^n - 0.383n \cdot 0.617^{n-1} > 0$

By considering the graph of

$y = 0.07 - 0.617^n - 0.383n \cdot 0.617^{n-1}$,

$n > 9.5074803$. (A1) for correct value

$\therefore n = 10$ A1 N5

[5]

4. (a) The required probability

$= p\times 0.3+(1-p)\times 0.48$ (M1)(A1) for substitution

$=0.3p+0.48-0.48p$

$=0.48-0.18p$ A1 N3

[3]

(b) $\dfrac{0.3p}{0.48-0.18p}$ A2 N2

[2]

(c) $X \sim B(8, 0.3702)$ (R1) for binomial distribution

$\mathrm{P}(X=6)$

$=\binom{8}{6}(0.3702)^6(1-0.3702)^{8-6}$ (A1) for substitution

$=0.0285878721$

$=0.0286$ A1 N3

[3]

(d) The probability that reaching the escape door for at most two trial

$=(1-0.3702)^n+n(1-0.3702)^{n-1}(0.3702)$

$+\binom{n}{2}(1-0.3702)^{n-2}(0.3702)^2$ (M1) for valid approach

$1-[0.6298^n+n(0.6298)^{n-1}(0.3702)$

$+\binom{n}{2}(0.6298)^{n-2}(0.3702)^2]>0.99$ (M1)(A1) for correct inequality

$0.01-0.6298^n-0.3702n(0.6298)^{n-1}$

$-\binom{n}{2}(0.6298)^{n-2}(0.3702)^2>0$

By considering the graph of

$y=0.01-0.6298^n-0.3702n(0.6298)^{n-1}$

$-\left(\dfrac{n(n-1)}{2}\right)(0.6298)^{n-2}(0.3702)^2$,

$n>19.237508$. (A1) for correct value

$\therefore n=20$ A1 N5

[5]