AA SL Practice Set 6 Paper 2 Solution

Section A

1. (a) $\frac{\sin A\hat{C}B}{AB} = \frac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\frac{\sin A\hat{C}B}{11} = \frac{\sin 95^\circ}{15}$ (A1) for substitution

$\sin A\hat{C}B = \frac{11\sin 95^\circ}{15}$

$A\hat{C}B = 46.93191642^\circ$

$A\hat{C}B = 46.9^\circ$ A1

[3]

(b) $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ (M1) for cosine rule

$BC^2 = 11^2 + 15^2$ (A1) for substitution

$-2(11)(15)\cos(180^\circ - 95^\circ - 46.93191642^\circ)$

$BC = 9.284290829 \text{ cm}$

$BC = 9.28 \text{ cm}$ A1

[3]

2. (a) $r = 0.2368587247$

$r = 0.237$ A1

[1]

(b) $a = 0.0900098912$

$a = 0.0900$ A1

$b = 14.30365974$

$b = 14.3$ A1

[2]

(c) The estimated number of objects remembered

$= 0.0900098912(30) + 14.30365974$ (A1) for substitution

$= 17.00395648$

$= 17.0$ A1

[2]

3. The common ratio r

$= u_4 \div u_3$

$= \frac{1}{27} \div \frac{1}{9}$

$= \frac{1}{3}$ (A1) for correct value

The first term u_1

$= u_3 \div r^2$

$= \frac{1}{9} \div \left(\frac{1}{3}\right)^2$

$= 1$ (A1) for correct value

$S_n < \frac{1499}{1001}$

$\frac{1\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} < \frac{1499}{1001}$ (M1) for setting inequality

$\frac{3\left(1-\left(\frac{1}{3}\right)^n\right)}{2} - \frac{1499}{1001} < 0$

By considering the graph of $y = \frac{3\left(1-\left(\frac{1}{3}\right)^n\right)}{2} - \frac{1499}{1001}$,

$n < 5.8236461$. (M1) for valid approach

Thus, the greatest value of n is 5. A1

[5]

4. (a) By considering the graph of $f(x)=\ln(4x-x^2)$,
the coordinates of the local maximum are
$(2, 1.3862944)$. (M1) for valid approach
Thus, $\mathrm{P}:(2, 1.39)$ A1

[2]

(b) By considering the graph of $f(x)=\ln(4x-x^2)$,
the x-intercepts are 0.2679492 and 3.7320508. (M1)(A1) for correct values
The required area
$=\int_{0.2679492}^{3.7320508}\ln(4x-x^2)\mathrm{d}x$ (M1) for definite integral
$=3.6074599$
$=3.61$ A1

[4]

5. $(1+px)^{12}$
$=1+C_1^{12}(px)^1+C_2^{12}(px)^2+C_3^{12}(px)^3+C_4^{12}(px)^4+\cdots$ (M1) for valid approach
Term in x in the expansion $(5x)^{-3}(1+px)^{12}$
$=(5x)^{-3}C_4^{12}(px)^4$ (A1) for correct term
$=\dfrac{495p^4x^4}{125x^3}$ (A1) for correct approach
$=\dfrac{99p^4}{25}x$
$\therefore \dfrac{99p^4}{25}=99$ (A1) for correct equation
$p^4=25$
$p=25^{\frac{1}{4}}$ or $p=-25^{\frac{1}{4}}$
$p=\sqrt{5}$ or $p=-\sqrt{5}$ A1A1

[6]

6. The area affected at 01:00 on 14 February 2023

$= 0.45e^{-k(0)}$

$= 0.45$ (A1) for correct value

$0.45(1-45\%) = 0.45e^{-k(16-1)}$ (A1) for correct equation

$0.2475 = 0.45e^{-15k}$ (M1) for valid approach

$0.55 = e^{-15k}$

$\ln 0.55 = -15k$ (M1) for valid approach

$k = -\frac{1}{15}\ln 0.55$ (A1) for correct value

The area affected at 10:00 on 15 February 2023

$= 0.45e^{-\left(-\frac{1}{15}\ln 0.55\right)(10-1+24)}$ (A1) for substitution

$= 0.1207842844 \text{ km}^2$

$= 0.121 \text{ km}^2$ A1

[7]

Section B

7. (a) $Q\hat{P}R = \alpha$

$\therefore O\hat{P}Q = \frac{\alpha}{2}$

$\because OP = OQ$

$\therefore O\hat{Q}P = \frac{\alpha}{2}$ A1

$P\hat{O}Q = \pi - \frac{\alpha}{2} - \frac{\alpha}{2}$

$P\hat{O}Q = \pi - \alpha$ A1

Similarly, $P\hat{O}R = \pi - \alpha$

$Q\hat{O}R = 2\pi - (\pi - \alpha) - (\pi - \alpha)$ M1

$Q\hat{O}R = 2\pi - \pi + \alpha - \pi + \alpha$

$Q\hat{O}R = 2\alpha$ AG

[3]

(b) (i) The area of the triangle QOR

$= \frac{1}{2}(OQ)(OR)\sin Q\hat{O}R$

$= \frac{1}{2}(r)(r)\sin 2\alpha$ (A1) for substitution

$= \frac{1}{2}r^2 \sin 2\alpha$ A1

(ii) The area of the shared region

$= \frac{1}{2}(OQ)^2(Q\hat{O}R) - \frac{1}{2}(OQ)(OR)\sin Q\hat{O}R$ M1

$= \frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2 \sin 2\alpha$ A1

$= \frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2(2\sin\alpha\cos\alpha)$ A1

$= r^2\alpha - r^2\sin\alpha\cos\alpha$

$= r^2(\alpha - \sin\alpha\cos\alpha)$ AG

[5]

(c) The area of the triangle QPR

$= \text{The area of the triangle QOR}$

$+ \text{The area of the triangle POQ}$ (M1) for valid approach

$+ \text{The area of the triangle POR}$

$$= \frac{1}{2}r^2 \sin 2\alpha + \frac{1}{2}(\text{OP})(\text{OQ}) \sin \text{P}\hat{\text{O}}\text{Q}$$

$$+ \frac{1}{2}(\text{OP})(\text{OR}) \sin \text{P}\hat{\text{O}}\text{R}$$

$$= \frac{1}{2}r^2 \sin 2\alpha + \frac{1}{2}(r)(r)\sin(\pi - \alpha)$$

(A1) for substitution

$$+ \frac{1}{2}(r)(r)\sin(\pi - \alpha)$$

$$= \frac{1}{2}r^2 \sin 2\alpha + r^2 \sin(\pi - \alpha)$$

$\therefore r^2(\alpha - \sin\alpha\cos\alpha) = \frac{1}{2}r^2 \sin 2\alpha + r^2 \sin(\pi - \alpha)$ (A1) for correct equation

$$\alpha - \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha + \sin(\pi - \alpha)$$

$$\alpha - \sin\alpha\cos\alpha - \frac{1}{2}\sin 2\alpha - \sin(\pi - \alpha) = 0$$

By considering the graph of

$$y = \alpha - \sin\alpha\cos\alpha - \frac{1}{2}\sin 2\alpha - \sin(\pi - \alpha),$$

$\alpha = 1.3703001$. (M1) for valid approach

$\therefore \alpha = 1.37$ A1

[5]

8. (a) The initial velocity

$= v(0)$

$= \frac{100}{20+0} - 4$ (M1) for substitution

$= 1 \text{ ms}^{-1}$ A1

[2]

(b) $v(t) = 0$ (M1) for setting equation

$\frac{100}{20+t} - 4 = 0$

By considering the graph of $y = \frac{100}{20+t} - 4$,

$t = 5$. A1

[2]

(c) The particle's acceleration

$= \frac{\mathrm{d}}{\mathrm{d}t}(v(t))\Big|_{t=6}$ (M1) for valid approach

$= -0.147929 \text{ ms}^{-2}$

$= -0.148 \text{ ms}^{-2}$ A1

[2]

(d) $s(t)$

$= \int v(t)\mathrm{d}t$ (M1) for valid approach

$= \int \left(\frac{100}{20+t} - 4 \right) \mathrm{d}t$

$= 100\ln(20+t) - 4t + C$ (A1) for correct approach

$\therefore 100\ln 20 = 100\ln(20+0) - 4(0) + C$ (M1) for substitution

$100\ln 20 = 100\ln 20 + C$

$C = 0$

$\therefore s(t) = 100\ln(20+t) - 4t$ A1

[4]

(e) The total distance travelled

$= \int_0^2 |v(t)|\mathrm{d}t$ (M1) for valid approach

$= \int_0^2 \left| \frac{100}{20+t} - 4 \right| \mathrm{d}t$ (A1) for substitution

$= 1.53101798 \text{ m}$

$= 1.53 \text{ m}$ A1

[3]

(f)	$\int_2^{2+a} \lvert v(t) \rvert \, dt = \int_0^2 \lvert v(t) \rvert \, dt$	(M1) for setting equation
	$\therefore \int_2^{2+a} \left\lvert \frac{100}{20+t} - 4 \right\rvert dt = 1.53101798$	
	$\int_2^{2+a} \left\lvert \frac{100}{20+t} - 4 \right\rvert dt - 1.53101798 = 0$	
	By considering the graph of	
	$y = \int_2^{2+a} \left\lvert \frac{100}{20+t} - 4 \right\rvert dt - 1.53101798$,	
	$a = 6.1829872$.	(M1) for valid approach
	$\therefore a = 6.18$	A1

[3]

9. (a) The required probability

$= P(W > 83)$ (M1) for valid approach

$= 0.0912112819$

$= 0.0912$ A1

[2]

(b) The required probability

$= P(W > 83) \cdot (1 - P(W > 83))^4$ (M1)(A1) for correct approach

$= 0.0622157395$

$= 0.0622$ A1

[3]

(c) $P(W < \alpha) + P(\alpha < W < \beta) + P(W > \beta) = 1$ (M1) for setting equation

$\therefore 3P(W > \beta) + 0.74 + P(W > \beta) = 1$ (A1) for correct equation

$4P(W > \beta) = 0.26$

$P(W > \beta) = 0.065$ (A1) for correct value

$\beta = 84.08461132$

Thus, the minimum weight of an overweight player is 84.1 kg. A1

[4]

(d) $P(W < \alpha) = 3P(W > 84.08461132)$

$\alpha = 69.84229583$ (A1) for correct value

The required probability

$= P(W > 77 \mid W > 69.84229583)$ (M1) for valid approach

$= \dfrac{P(W > 77 \cap W > 69.84229583)}{P(W > 69.84229583)}$

$= \dfrac{P(W > 77)}{P(W > 69.84229583)}$ (A1) for correct approach

$= \dfrac{0.3694414037}{0.805}$

$= 0.4589334207$

$= 0.459$ A1

[4]

(e) Let Y be the number of players selected from this nine-player group.

$Y \sim B(9, 0.4589334207)$ (R1) for binomial distribution

The required probability

$= P(Y = 3)$ (M1) for valid approach

$= 0.2037194978$

$= 0.204$ A1

[3]