Analysis and Approaches Standard Level for IBDP Mathematics
Practice Paper Set 1 – Paper 1 (90 Minutes)

Question – Answer Book

Instructions

1. This paper consists of **TWO** sections: A and B.

2. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

3. No calculator is allowed.

4. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.

5. Supplementary answer sheets and graph papers will be supplied on request.

6. Unless otherwise specified, **ALL** working must be clearly shown.

7. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

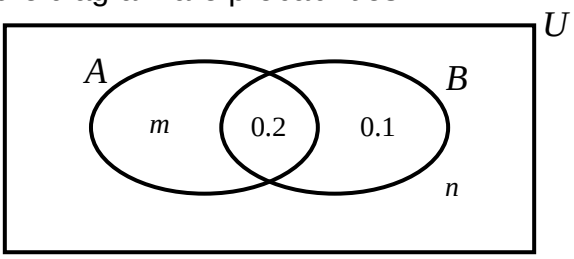
8. The diagrams in this paper are **NOT** necessarily drawn to scale.

9. Information to be read before you start the exam:

	Marker's Use Only	Examiner's Use Only	
Question Number	**Marks**	**Marks**	**Maximum Mark**
Section A			
1			**6**
2			**6**
3			**5**
4			**6**
5			**8**
6			**8**
Section A Total			**39**
Section B			
7			**15**
8			**14**
9			**12**
Section B Total			**41**
Overall			
Paper 1 Total			**80**

Section A (39 marks)

1. The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.6$. The values in the diagram are probabilities.



(a) Find m.

[2]

(b) Find n.

[2]

(c) Find $\mathrm{P}(B')$.

[2]

2. There are 15 items in a data set. The sum of the items is 300.

(a) Find the mean.

[2]

The variance of this data set is 9. Each value in the set is multiplied by -2.

(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance.

(iii) Hence, write down the value of the new standard deviation.

[4]

3. A straight line L_1 passes through the points $(8, 0)$ and $(24, 32)$.

(a) Find the equation of L_1, giving the answer in general form.

[3]

(b) The equation of another straight line, L_2, is given as $x - ay + 2021 = 0$, $a \in \mathbb{R}$. If L_1 and L_2 are perpendicular, find a.

[2]

4. (a) Show that $(2n+1)^2+(2n+3)^2+(2n+5)^2=3(4n^2+12n+11)+2$, where $n\in\mathbb{Z}$.

[3]

(b) Hence, or otherwise, prove that the sum of the squares of any three consecutive odd numbers is greater than a multiple of 3 by 2.

[3]

5. Let $f(x) = px^3 + qx^2 - 2x + 1$. At $x = 1$, the slope of the normal of the curve of f is $-\frac{1}{15}$. It is given that $f^{-1}(41) = 2$, find the value of p and of q.

[8]

6. The equation $kx^2 + (8+k)x - 1 = 0$ has no real roots. Find the possible values of k.

[8]

Section B (41 marks)

7. A rectangular box has length $4x$ cm, width $2x$ cm and height y cm, where x, $y > 0$. It is given that the sum of the length and the height of the rectangular box is 20 cm.

(a) Write down an expression for y in terms of x.

[1]

(b) Express V in terms of x, where V cm^3 is the volume of the rectangular box.

[2]

(c) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$.

[2]

(d) Find the value of x when V attains its maximum, justifying the answer.

[7]

(e) Hence, find the maximum volume.

[2]

(f) Write down the height of the rectangular box when V attains its maximum.

[1]

8. Let $f(x) = \cos^4 x$, $x \in \mathbb{R}$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Find the number of solutions to this equation.

[5]

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where a, p, $q \in \mathbb{Z}$.

[2]

(c) Let $g(x) = 2\sin x$ for $0 \le x \le \pi$. Find the total area of the regions bounded by the graph of $y = f(x)g(x)$ and the x-axis.

[7]

9. The graph of f is given by $f(t) = a\sin b(t-c) + d$, $a > 0$, $t \geq 0$.

When $t = 2$, there is a maximum value of 37, at P. When $t = 11$, there is a minimum value of -5. The graph of f is strictly decreasing at $2 < t < 11$.

(a) (i) Show that $a = 21$.

(ii) Find the exact value of b.

(iii) Find the value of d.

(iv) Write down a possible value of c.

[7]

The graph of f is then transformed to the graph of g by a horizontal stretch of scale factor 3, followed by a translation of $\begin{pmatrix} 17 \\ 8 \end{pmatrix}$. Let P' be the image of P.

(b) Find the coordinates of P'.

[2]

The graph of g is then transformed to the graph of h by a translation of $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$.

(c) Give a full geometric description of the transformation that maps the graph of h to the graph of f.

[3]

END OF PAPER