Formula List of Analysis and Approaches Standard Level for IBDP Mathematics

	Intensive Notes by Topics here		IBDP Maths Info. & Exam Tricks here
	Exam & GDC Skills Video List here		More Mock Papers & Marking Service here

1 Standard Form

✓ Standard Form:

A number in the form $(\pm)a\times 10^k$, where $1\leq a<10$ and k is an integer

2 Quadratic Functions

✓ General form $y=ax^2+bx+c$, where $a\neq 0$:

$a>0$	The graph opens upward
$a<0$	The graph opens downward
c	y-intercept
$h=-\frac{b}{2a}$	x-coordinate of the vertex
$k=ah^2+bh+c$	y-coordinate of the vertex
	Extreme value of y
$x=h$	Equation of the axis of symmetry

✓ Other forms:

1. $y=a(x-h)^2+k$: Vertex form
2. $y=a(x-p)(x-q)$: Factored form with x-intercepts p and q

✓ Solving the quadratic equation $ax^2+bx+c=0$, where $a\neq 0$:

1. Factorization by cross method
2. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$: Quadratic Formula
3. Method of completing the square

✓ The discriminant $\Delta = b^2 - 4ac$ of $ax^2 + bx + c = 0$:

$\Delta > 0$	The quadratic equation has two distinct real roots
$\Delta = 0$	The quadratic equation has one double real root
$\Delta < 0$	The quadratic equation has no real root

✓ The x-intercepts of the quadratic function $y = ax^2 + bx + c$ are the roots of the corresponding quadratic equation $ax^2 + bx + c = 0$

3 Functions

✓ The function $y = f(x)$:

1. $f(a)$: Functional value when $x = a$
2. Set of values of x: Domain
3. Set of values of y: Range

✓ $f \circ g(x) = f(g(x))$: Composite function when $g(x)$ is substituted into $f(x)$

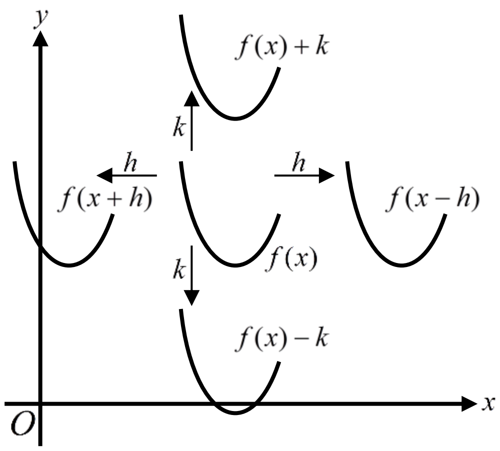
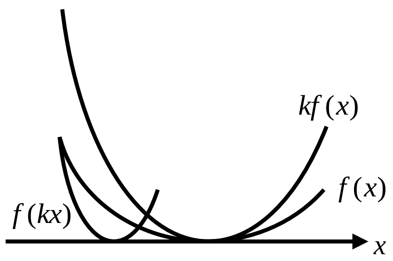
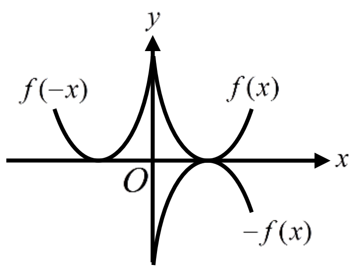
✓ Steps of finding the inverse function $y = f^{-1}(x)$ of $f(x)$:

1. Start from expressing y in terms of x
2. Interchange x and y
3. Make y the subject in terms of x

✓ Properties of $y = f^{-1}(x)$:

1. $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
2. The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ about $y = x$

✓ Summary of transformations:

	$f(x) \rightarrow f(x)+k$: Translate upward by k units
	$f(x) \rightarrow f(x)-k$: Translate downward by k units
	$f(x) \rightarrow f(x+h)$: Translate to the left by h units
	$f(x) \rightarrow f(x-h)$: Translate to the right by h units
	$f(x) \rightarrow kf(x)$: Vertical stretch of scale factor k
	$f(x) \rightarrow f(kx)$: Horizontal compression of scale factor k
	$f(x) \rightarrow -f(x)$: Reflection about the x-axis
	$f(x) \rightarrow f(-x)$: Reflection about the y-axis

✓ Properties of rational function $y = \dfrac{ax+b}{cx+d}$:

1. $y = \dfrac{1}{x}$: Reciprocal function

2. $y = \dfrac{a}{c}$: Horizontal asymptote

3. $x = -\dfrac{d}{c}$: Vertical asymptote

4 Exponential and Logarithmic Functions

✓ $y = a^x$: Exponential function of base $a \neq 1$

✓ Methods of solving an exponential equation $a^x = b$:

1. Change b into a^y such that $a^x = a^y \Rightarrow x = y$
2. Take logarithm for both sides

✓ $y = \log_a x$: Logarithmic function of base $a > 0$

✓ $y = \log x = \log_{10} x$: Common Logarithmic function

✓ $y = \ln x = \log_e x$: Natural Logarithmic function, where $e = 2.71828...$ is an exponential number

✓ Laws of logarithm, where a, b, c, p, q, $x > 0$:

1. $x = a^y \Leftrightarrow y = \log_a x$
2. $\log_a 1 = 0$
3. $\log_a a = 1$
4. $\log_a p + \log_a q = \log_a pq$
5. $\log_a p - \log_a q = \log_a \frac{p}{q}$
6. $\log_a p^n = n \log_a p$
7. $\log_b a = \frac{\log_c a}{\log_c b}$

✓ Properties of the graphs of $y = a^x$:

$a > 1$	$0 < a < 1$
y-intercept $= 1$	
y increases as x increases	y decreases as x increases
y tends to zero as x tends to negative infinity	y tends to zero as x tends to positive infinity
Horizontal asymptote: $y = 0$	

✓ Properties of the graphs of $y=\log_a x$:

$a>1$	$0<a<1$
x-intercept $=1$	
y increases as x increases	y decreases as x increases
x tends to zero as y tends to negative infinity	x tends to zero as y tends to positive infinity
Vertical asymptote: $x=0$	

5 Arithmetic Sequences

✓ Properties of an arithmetic sequence u_n:

1. u_1: First term
2. $d=u_2-u_1=u_n-u_{n-1}$: Common difference
3. $u_n=u_1+(n-1)d$: General term (nth term)
4. $S_n=\frac{n}{2}[2u_1+(n-1)d]=\frac{n}{2}[u_1+u_n]$: The sum of the first n terms

✓ $\sum_{r=1}^{n} u_r=u_1+u_2+u_3+\cdots+u_{n-1}+u_n$: Summation sign

6 Geometric Sequences

✓ Properties of a geometric sequence u_n:

1. u_1: First term
2. $r=u_2\div u_1=u_n\div u_{n-1}$: Common ratio
3. $u_n=u_1\times r^{n-1}$: General term (nth term)
4. $S_n=\frac{u_1(1-r^n)}{1-r}$: The sum of the first n terms
5. $S_\infty=\frac{u_1}{1-r}$: The sum to infinity, given that $-1<r<1$

7 Binomial Theorem

✓ Properties of the n factorial $n!$:

1. $n! = n\times(n-1)\times(n-2)\times\cdots\times 3\times 2\times 1$
2. $0! = 1$
3. $n! = n\times(n-1)!$

✓ Properties of the combination coefficient $\binom{n}{r}$:

1. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$
2. $\binom{n}{0} = \binom{n}{n} = 1$
3. $\binom{n}{1} = \binom{n}{n-1} = n$
4. $\binom{n}{r} = \binom{n}{n-r} = \dfrac{n(n-1)\cdots(n-r+1)}{r!}$

✓ The binomial theorem:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

$= \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$, where the $(r+1)$-th term $= \binom{n}{r}a^{n-r}b^r$

8 Proofs and Identities

✓ Identity of x: The equivalence of two expressions for all values of x

$\equiv$: Identity sign

9 Coordinate Geometry

✓ Consider the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a x - y plane:

1. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$: Slope of PQ
2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$: Distance between P and Q
3. $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$: The mid-point of PQ

✓ Forms of straight lines with slope m and y-intercept c:

1. $y = mx + c$: Slope-intercept form
2. $Ax + By + C = 0$: General form

✓ Ways to find the x-intercept and the y-intercept of a line:

1. Substitute $y = 0$ and make x the subject to find the x-intercept
2. Substitute $x = 0$ and make y the subject to find the y-intercept

10 Trigonometry

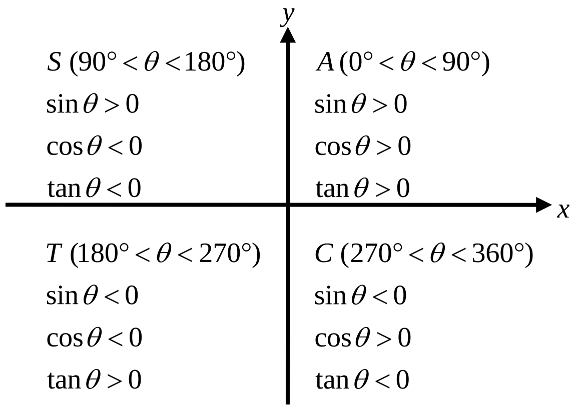
✓ Trigonometric identities:

1. $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$
2. $\sin^2\theta + \cos^2\theta \equiv 1$

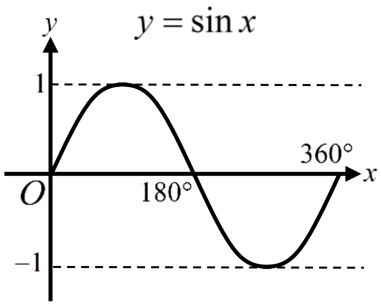
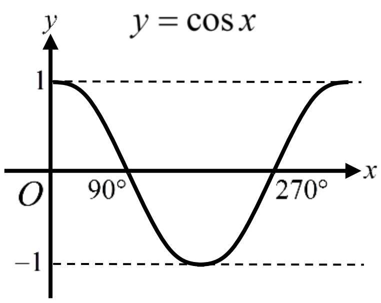
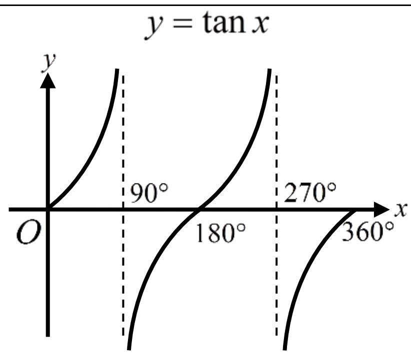
✓ Double angle formula:

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$

✓ ASTC diagram



✓ Properties of graphs of trigonometric functions:

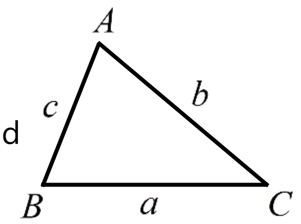
<table>
<tr><td>
</td><td>1. Amplitude $= 1$
2. Period $= 360°$
3. $-1 \le \sin x \le 1$</td></tr>
<tr><td>
</td><td>1. Amplitude $= 1$
2. Period $= 360°$
3. $-1 \le \cos x \le 1$</td></tr>
<tr><td>
</td><td>1. Period $= 180°$
2. $\tan x \in \mathbb{R}$
3. Vertical asymptotes: $x = 90°$, $x = 270°$</td></tr>
</table>

✓ Properties of a general trigonometric function $y = A\sin B(x-C)+D$:

1. $A = \frac{y_{max} - y_{min}}{2}$: Amplitude
2. $B = \frac{2\pi}{\text{Period}}$
3. $D = \frac{y_{max} + y_{min}}{2}$
4. C can be found by substitution of a point on the graph

11 2-D Trigonometry

✓ Consider a triangle ABC:



1. $\frac{\sin A}{a} = \frac{\sin B}{b}$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$: Sine rule

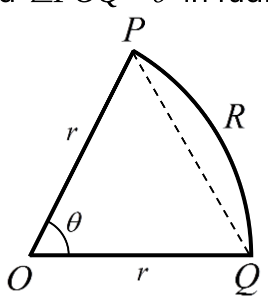
Note: The ambiguous case exists if two sides and an angle are known, and the angle is opposite to the shorter known side

2. $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$: Cosine rule
3. $\frac{1}{2}ab\sin C$: Area of the triangle ABC

✓ $\frac{x^\circ}{180^\circ} = \frac{y \text{ rad}}{\pi \text{ rad}}$: Method of conversions between degree and radian

✓ Consider a sector $OPRQ$ with centre O, radius r and $\angle POQ = \theta$ in radian:

1. $r\theta$: Arc length PQ
2. $\frac{1}{2}r^2\theta$: Area of the sector $OPRQ$
3. $\frac{1}{2}r^2(\theta - \sin\theta)$: Area of the segment PRQ



12 Areas and Volumes

✓ For a cube of side length l:

1. $6l^2$: Total surface area
2. l^3: Volume

✓ For a cuboid of side lengths a, b and c:

1. $2(ab+bc+ac)$: Total surface area
2. abc: Volume

✓ For a prism of height h and cross-sectional area A:

1. Ah: Volume

✓ For a cylinder of height h and radius r:

1. $2\pi r^2 + 2\pi rh$: Total surface area
2. $2\pi rh$: Lateral surface area
3. $\pi r^2 h$: Volume

✓ For a pyramid of height h and base area A:

1. $\frac{1}{3}Ah$: Volume

✓ For a circular cone of height h and radius r:

1. $l = \sqrt{r^2 + h^2}$: Slant height
2. $\pi r^2 + \pi rl$: Total surface area
3. πrl: Curved surface area
4. $\frac{1}{3}\pi r^2 h$: Volume

✓ For a sphere of radius r:

1. $4\pi r^2$: Total surface area
2. $\frac{4}{3}\pi r^3$: Volume

✓ For a hemisphere of radius r:

1. $3\pi r^2$: Total surface area
2. $2\pi r^2$: Curved surface area
3. $\frac{2}{3}\pi r^3$: Volume

13 Differentiation

✓ Derivatives of a function $y = f(x)$:

1. $\frac{dy}{dx} = f'(x)$: First derivative
2. $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x)$: Second derivative
3. $\frac{d^n y}{dx^n} = f^{(n)}(x)$: n-th derivative

✓ Rules of differentiation:

$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$	$f(x) = p(x) + q(x) \Rightarrow f'(x) = p'(x) + q'(x)$
$f(x) = \sin x \Rightarrow f'(x) = \cos x$	$f(x) = cp(x) \Rightarrow f'(x) = cp'(x)$
$f(x) = \cos x \Rightarrow f'(x) = -\sin x$	$f(x) = p(q(x)) \Rightarrow f'(x) = p'(q(x)) \cdot q'(x)$
$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$	$f(x) = p(x)q(x)$ $\Rightarrow f'(x) = p'(x)q(x) + p(x)q'(x)$
$f(x) = e^x \Rightarrow f'(x) = e^x$	$f(x) = \frac{p(x)}{q(x)}$ $\Rightarrow f'(x) = \frac{p'(x)q(x) - p(x)q'(x)}{(q(x))^2}$
$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$	

✓ Relationships between graph properties and the derivatives:

1. $f'(x) > 0$ for $a \le x \le b$: $f(x)$ is increasing in the interval
2. $f'(x) < 0$ for $a \le x \le b$: $f(x)$ is decreasing in the interval
3. $f'(a) = 0$: $(a, f(a))$ is a stationary point of $f(x)$
4. $f'(a) = 0$ and $f'(x)$ changes from positive to negative at $x = a$: $(a, f(a))$ is a maximum point of $f(x)$
5. $f'(a) = 0$ and $f'(x)$ changes from negative to positive at $x = a$: $(a, f(a))$ is a minimum point of $f(x)$
6. $f''(a) = 0$ and $f''(x)$ changes sign at $x = a$: $(a, f(a))$ is a point of inflexion of $f(x)$

✓ Slopes of tangents and normals:

1. $f'(a)$: Slope of tangent at $x=a$
2. $\dfrac{-1}{f'(a)}$: Slope of normal at $x=a$

14 Applications of Differentiation

✓ Equations of tangents and normals:

1. $y-f(a)=f'(a)(x-a)$: Equation of tangent at $x=a$
2. $y-f(a)=\left(\dfrac{-1}{f'(a)}\right)(x-a)$: Equation of normal at $x=a$

✓ $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{\mathrm{d}N}{\mathrm{d}x}\cdot\dfrac{\mathrm{d}x}{\mathrm{d}t}$: Rate of change of N with respect to the time t

✓ Tests for optimization:

1. First derivative test
2. Second derivative test

✓ Applications in kinematics:

1. $s(t)$: Displacement with respect to the time t
2. $v(t)=s'(t)$: Velocity
3. $a(t)=v'(t)$: Acceleration

15 Integration

✓ Integrals of a function $y=f(x)$:

1. $\int f(x)\mathrm{d}x$: Indefinite integral of $f(x)$
2. $\int_a^b f(x)\mathrm{d}x$: Definite integral of $f(x)$ from a to b

✓ Rules of integration:

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$	$\int (p'(x) + q'(x))dx = p(x) + q(x) + C$
$\int \cos x dx = \sin x + C$	$\int cp'(x)dx = cp(x) + C$
$\int \sin x dx = -\cos x + C$	$\int_a^b f'(x)dx = [f(x)]_a^b = f(b) - f(a)$
$\int \frac{1}{\cos^2 x} dx = \tan x + C$	Integration by substitution
$\int e^x dx = e^x + C$	$\int \frac{1}{x} dx = \ln x + C$

16 Applications of Integration

✓ Areas on x-y plane, between $x = a$ and $x = b$:

1. $\int_a^b f(x)dx$: Area under the graph of $f(x)$ and above the x-axis
2. $-\int_a^b f(x)dx$: Area under the x-axis and above the graph of $f(x)$
3. $\int_a^b (f(x) - g(x))dx$: Area under the graph of $f(x)$ and above the graph of $g(x)$

✓ Applications in kinematics:

1. $a(t)$: Acceleration with respect to the time t
2. $v(t) = \int a(t)dt$: Velocity
3. $s(t) = \int v(t)dt$: Displacement
4. $d = \int_{t_1}^{t_2} |v(t)| dt$: Total distance travelled between t_1 and t_2

17 Statistics

✓ Relationship between frequencies and cumulative frequencies:

Data	Frequency	Data less than or equal to	Cumulative frequency
10	f_1	10	f_1
20	f_2	20	$f_1 + f_2$
30	f_3	30	$f_1 + f_2 + f_3$

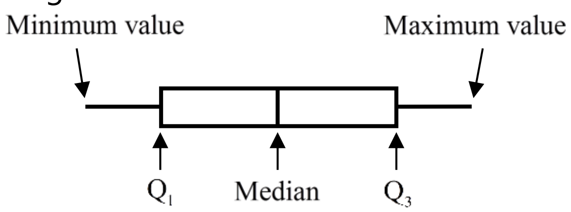
✓ Measures of central tendency for a data set $\{x_1, x_2, x_3, \cdots, x_n\}$ arranged in ascending order:

1. $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$: Mean
2. The datum or the average value of two data at the middle: Median
3. The datum appears the most: Mode

✓ Measures of dispersion for a data set $\{x_1, x_2, x_3, \cdots, x_n\}$ arranged in ascending order:

1. $x_n - x_1$: Range
2. Two subgroups A and B can be formed from the data set such that all data of the subgroup A are less than or equal to the median, while all data of the subgroup B are greater than or equal to the median
3. $Q_1 =$ The median of the subgroup A: Lower quartile
4. $Q_3 =$ The median of the subgroup B: Upper quartile
5. $Q_3 - Q_1$: Inter-quartile range (IQR)
6. $\sigma = \sqrt{\dfrac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$: Standard deviation

✓ Box-and-whisker diagram:



✓ A datum x is defined to be an outlier if $x < Q_1 - 1.5\text{IQR}$ or $x > Q_3 + 1.5\text{IQR}$

✓ Coding of data:

1. Only the mean, the median, the mode and the quartiles will change when each datum of the data set is added or subtracted by a value
2. All measures of central tendency and measures of dispersion will change when each datum of the data set is multiplied or divided by a value

18 Probability

✓ Terminologies:

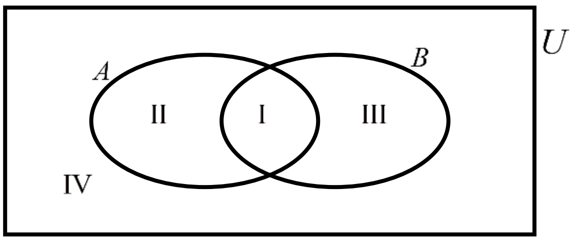
1. U : Universal set
2. A : Event
3. x : Outcome of an event
4. $n(U)$: Total number of elements
5. $n(A)$: Number of elements in A

✓ Formulae for probability:

1. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
2. $\mathrm{P}(A') = 1 - \mathrm{P}(A)$
3. $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
4. $\mathrm{P}(A) = \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$
5. $\mathrm{P}(A' \cap B') + \mathrm{P}(A \cup B) = 1$
6. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ and $\mathrm{P}(A \cap B) = 0$ if A and B are mutually exclusive
7. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$ and $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ if A and B are independent

✓ Venn diagram:

1. Region I: $A \cap B$
2. Region II: $A \cap B'$
3. Region III: $A' \cap B$
4. Region IV: $(A \cup B)'$



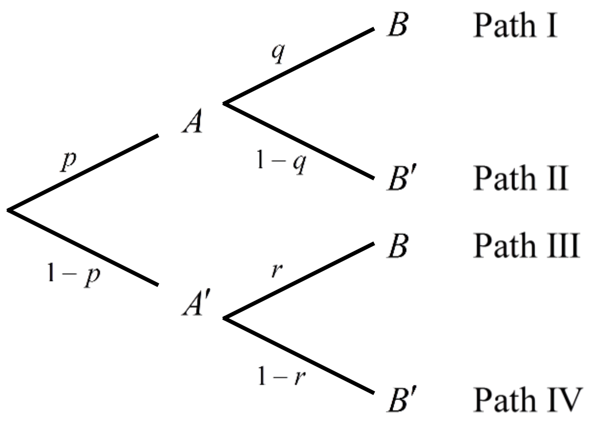
✓ Tree diagram:

1. Path I: $P(A \cap B) = pq$
2. Path I + Path III:

$= P(B)$

$= P(A \cap B) + P(A' \cap B)$

$= pq + (1-p)r$



19 Discrete Probability Distributions

✓ Properties of a discrete random variable X :

X	x_1	x_2	...	x_n
$P(X = x)$	$P(X = x_1)$	$P(X = x_2)$	...	$P(X = x_n)$

1. $P(X = x_1) + P(X = x_2) + \cdots + P(X = x_n) = 1$
2. $E(X) = x_1 P(X = x_1) + x_2 P(X = x_2) + \cdots + x_n P(X = x_n)$: Expected value of X
3. $E(X) = 0$ if a fair game is considered

20 Binomial Distribution

✓ Properties of a random variable $X \sim B(n, p)$ following binomial distribution:

1. Only two outcomes from every independent trial (Success and failure)
2. n : Number of trials
3. p : Probability of success
4. X : Number of successes in n trials

✓ Formulae for binomial distribution:

1. $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$ for $0 \le r \le n$, $r \in \mathbb{Z}$
2. $E(X) = np$: Expected value of X
3. $\mathrm{Var}(X) = np(1-p)$: Variance of X
4. $\sqrt{np(1-p)}$: Standard deviation of X
5. $P(X \le r) = P(X < r+1) = 1 - P(X \ge r+1)$

21 Normal Distribution

✓ Properties of a random variable $X \sim \mathrm{N}(\mu, \sigma^2)$ following normal distribution:

1. μ: Mean
2. σ: Standard deviation
3. The mean, the median and the mode are the same
4. The normal curve representing the distribution is a bell-shaped curve which is symmetric about the middle vertical line
5. $\mathrm{P}(X < \mu) = \mathrm{P}(X > \mu) = 0.5$
6. The total area under the curve is 1

✓ Standardization of a normal variable:

1. $Z \sim \mathrm{N}(0, 1^2)$: Standard normal distribution with mean 0 and standard deviation 1
2. $Z = \dfrac{X - \mu}{\sigma}$ for $X \sim \mathrm{N}(\mu, \sigma^2)$

22 Bivariate Analysis

✓ Correlations:

Positive	Strong	$0.75 < r < 1$
	Moderate	$0.5 < r < 0.75$
	Weak	$0 < r < 0.5$
No		$r = 0$
Negative	Weak	$-0.5 < r < 0$
	Moderate	$-0.75 < r < -0.5$
	Strong	$-1 < r < -0.75$

where r is the correlation coefficient

✓ Linear regression:

1. $y = ax + b$: Regression line of y on x
2. $x = ay + b$: Regression line of x on y

Notes