Exercise 3.1

(a) The coordinates of the mid-point

$= \left(\frac{0+7}{2}, \frac{0+24}{2}, \frac{0+0}{2} \right)$ — $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$ (M1)

$= (3.5, 12, 0)$ — (A1)

(b) (i) $(7, 24, h)$ — (A1)

(ii) $\sqrt{(7-0)^2+(24-0)^2+(h-0)^2} = \sqrt{1025}$ — Correct equation (A1)

$49 + 576 + h^2 = 1025$

$h^2 = 400$

$h = 20$ — (A1)

Exercise 3.2

(a) Every position in the Voronoi cell of A has A to be the nearest farm. (A1)

(b) $x = 4$ (A1)

(c) (i) $(5, 5)$ (A1)

(ii) 4 (A1)

(iii) -0.25 (A1)

(iv) The equation of L:

$y - 5 = -0.25(x - 5)$ $\quad y - y_1 = m(x - x_1)$ (M1)

$y - 5 = -0.25x + 1.25$

$y = -0.25x + 6.25$ (A1)

(v) $\begin{cases} y = x - 1 \\ y = -0.25x + 6.25 \end{cases}$

$\therefore x - 1 = -0.25x + 6.25$ $\quad$ Equating y (M1)

$1.25x - 1 = 6.25$

$1.25x = 7.25$

$x = 5.8$

y

$= 5.8 - 1$

$= 4.8$

Thus, the coordinates of Q are $(5.8, 4.8)$. (A1)(A1)

(d) (i) The required radius

$= \sqrt{(6-4)^2 + (9-6)^2}$ $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (M1)

$= 3.605551275$ km

$= 3.61$ km (A1)

(ii) The required radius

$= \sqrt{(6-5.8)^2 + (9-4.8)^2}$ $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (M1)

$= 4.204759208$ km

$= 4.20$ km (A1)

(iii) Q (A1)

(e) D (A1)

Exercise 3.3

(a) (i) 8 m (A1)

(ii) 62 m (A1)

(b) (i) a

$= \dfrac{62-8}{2}$ $\quad \dfrac{y_{max}-y_{min}}{2}$ (M1)

$= 27$ (A1)

(ii) d

$= \dfrac{62+8}{2}$ $\quad \dfrac{y_{max}+y_{min}}{2}$ (M1)

$= 35$ (A1)

(iii) b

$= \dfrac{2\pi}{\text{Period}}$ $\quad \dfrac{2\pi}{\text{Period}}$ (M1)

$= \dfrac{2\pi}{\frac{60}{6}}$

$= \dfrac{\pi}{5}$ (A1)

(iv) $y = 27\sin\left(\dfrac{\pi}{5}(t-c)\right)+35$

$\therefore 8 = 27\sin\left(\dfrac{\pi}{5}(0-c)\right)+35$ $\quad t=0$ & $y=8$ (M1)

$27\sin\left(-\dfrac{\pi}{5}c\right)+27=0$

By considering the graph of

$y = 27\sin\left(-\dfrac{\pi}{5}c\right)+27$, the horizontal

intercept is 2.5. GDC approach (M1)

$\therefore c = 2.5$ (A1)

(c) $27\sin\left(\frac{\pi}{5}(t-2.5)\right)+35\geq 50$ | Correct inequality (A1)

$27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15\geq 0$

By considering the graph of

$y=27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15$, the graph is above

The horizontal axis when

$3.4374719<x<6.5625281$,

$13.437472<x<16.562528$

or $23.437472<x<26.562528$. | GDC approach (M1)

The amount of time

$=3(6.5625281-3.4374719)$ | $3(t_2-t_1)$ (A1)

$=9.3751686$ min

$=$ 9.38 min | (A1)

Exercise 3.4

(a) (i) $\overset{\frown}{ACB}$

$= 2\pi(8) \times \frac{135^\circ}{360^\circ}$ — $s = 2\pi r \times \frac{\theta}{360}$ (A1)

$= 18.84955592$ m

$= 18.8$ m — (A1)

(ii) The perimeter

$= 18.84955592 + 8 + 8$ — Sum of three sides (M1)

$= 34.84955592$ m

$= 34.8$ m — (A1)

(b) (i) The area of OACB

$= \pi(8)^2 \times \frac{135^\circ}{360^\circ}$ — $A = \pi r^2 \times \frac{\theta}{360}$ (A1)

$= 75.39822369$ m^2

$= 75.4$ m^2 — (A1)

(ii) The area of OAB

$= \frac{1}{2}(8)(8)\sin 135^\circ$ — $A = \frac{1}{2}ab\sin C$ (A1)

$= 22.627417$ m^2

$= 22.6$ m^2 — (A1)

(iii) The area of ACB

$= 75.39822369 - 22.627417$ — Difference of areas (M1)

$= 52.77080669$ m^2

$= 52.8$ m^2 — (A1)

Exercise 3.5

(a) $\dfrac{AC}{\sin A\hat{B}C} = \dfrac{AB}{\sin A\hat{C}B}$ — Sine rule (M1)

$\dfrac{AC}{\sin(360^\circ - 310^\circ)} = \dfrac{60}{\sin 83^\circ}$ — $360^\circ - 310^\circ$, 60 & 83° (A1)

$\dfrac{AC}{\sin 50^\circ} = \dfrac{60}{\sin 83^\circ}$

$AC = 46.30783819$ km

$AC = 46.3$ km — (A1)

(b) The area of ABC

$= \dfrac{1}{2}(AB)(AC)\sin B\hat{A}C$ — $A = \dfrac{1}{2}ab\sin C$ (M1)

$= \dfrac{1}{2}(60)(46.30783819)\sin(180^\circ - 83^\circ - 50^\circ)$ — 60, $46.3\cdots$ & $180^\circ - 83^\circ - 50^\circ$ (A1)

$= 1016.022266$ km²

$= 1020$ km² — (A1)

(c) The area of ACD

$= (1016.022266)(1.44)$

$= 1463.072063$ — 1463.072063 (A1)

$\dfrac{1}{2}(AC)(CD)\sin A\hat{C}D = 1463.072063$ — $A = \dfrac{1}{2}ab\sin C$ (M1)

$\dfrac{1}{2}(46.30783819)(75)\sin\theta = 1463.072063$ — 46.30783819 & 75 (A1)

$\sin\theta = 0.8425194643$

$\theta = 57.40713299^\circ$

$\theta = 57.4^\circ$ — (A1)

(d) $\dfrac{BC}{\sin B\hat{A}C} = \dfrac{AB}{\sin A\hat{C}B}$ — Sine rule (M1)

$\dfrac{BC}{\sin(180^\circ - 83^\circ - 50^\circ)} = \dfrac{60}{\sin 83^\circ}$ — $180^\circ - 83^\circ - 50^\circ$, 60 & 83° (A1)

$BC = 44.21076242$

$BD^2 = CD^2 + BC^2 - 2(CD)(BC)\cos B\hat{C}D$ — Cosine rule (M1)

$BD^2 = 75^2 + 44.21076242^2$
$-2(75)(44.21076242)\cos(83^\circ + 57.40713299^\circ)$ — $75, 44.21076242$ & $83^\circ + 57.40713299^\circ$ (A1)

$BD = 112.6492983$ km

$BD = 113$ km — (A1)

Exercise 3.6

(a) The volume

$=\frac{2}{3}\pi r^3$ — $V=\frac{2}{3}\pi r^3$ (M1)

$=\frac{2}{3}\pi(20)^3$ — $r=20$ (A1)

$=16755.16082$ cm³

$=16800$ cm³ — (A1)

(b) The total surface area

$=3\pi r^2$ — $A=3\pi r^2$ (M1)

$=3\pi(20)^2$ — $r=20$ (A1)

$=3769.911184$ cm²

$=3770$ cm² — (A1)

(c) $V=\frac{1}{3}Ah$ — $V=\frac{1}{3}Ah$ (M1)

$(16755.16082)(4)=\frac{1}{3}(\text{AD})^2(47)$ — $V=67020.64328$ & $h=47$ (A1)

$\text{AD}^2=4277.913401$

$\text{AD}=65.40575969$ cm

$\text{AD}=65.4$ cm — (A1)

(d) $\tan \text{O}\hat{\text{M}}\text{V}=\frac{\text{OV}}{\text{OM}}$ — Tangent ratio (M1)

$\tan \text{O}\hat{\text{M}}\text{V}=\frac{47}{65.40575969 \div 2}$

$\text{O}\hat{\text{M}}\text{V}=55.1695737^\circ$

Thus, the required angle is 55.2°. — (A1)

(e) OD

$= \sqrt{OM^2 + DM^2}$

$= \sqrt{\left(\frac{65.40575969}{2}\right)^2 + \left(\frac{65.40575969}{2}\right)^2}$

$= 46.24885621$ — 46.24885621 (A1)

$\tan O\hat{D}V = \frac{OV}{OD}$ — Tangent ratio (M1)

$\tan O\hat{D}V = \frac{47}{46.24885621}$

$O\hat{D}V = 45.46152242^\circ$

Thus, the required angle is 45.5°. (A1)