Chapter 12 Solution

Exercise 51

1. $\int_0^k e^{4x+3}\mathrm{d}x = \frac{1}{4}e^3(e^{28}-1)$

Let $u = e^{4x+3}$. (M1) for substitution

$\frac{\mathrm{d}u}{\mathrm{d}x} = 4e^{4x+3} \Rightarrow \frac{1}{4}\mathrm{d}u = e^{4x+3}\mathrm{d}x$

$x = k \Rightarrow u = e^{4k+3}$

$x = 0 \Rightarrow u = e^{4(0)+3} = e^3$

$\therefore \int_{e^3}^{e^{4k+3}} \frac{1}{4}\mathrm{d}u = \frac{1}{4}e^3(e^{28}-1)$ (A2) for correct working

$\left[\frac{1}{4}u\right]_{e^3}^{e^{4k+3}} = \frac{1}{4}e^3(e^{28}-1)$ A1

$\frac{1}{4}e^{4k+3} - \frac{1}{4}e^3 = \frac{1}{4}e^{31} - \frac{1}{4}e^3$

$4k+3=31$

$4k=28$

$k=7$ A1

[5]

2. $\int_{-\frac{\pi}{3}}^{0} \frac{\sin^3 x}{\cos x} \mathrm{d}x = \frac{k}{8} - \ln 2$

$\int_{-\frac{\pi}{3}}^{0} \frac{\sin^2 x \cdot \sin x}{\cos x} \mathrm{d}x = \frac{k}{8} - \ln 2$

Let $u = \cos x$. (M1) for substitution

$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x \Rightarrow (-1)\mathrm{d}u = \sin x \mathrm{d}x$

$1 - u^2 = \sin^2 x$

$x = 0 \Rightarrow u = \cos 0 = 1$

$x = -\frac{\pi}{3} \Rightarrow u = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$\therefore \int_{\frac{1}{2}}^{1} \frac{1-u^2}{u}(-1)\mathrm{d}u = \frac{k}{8} - \ln 2$ (A2) for correct working

$\therefore \int_{\frac{1}{2}}^{1} \left(u - \frac{1}{u}\right)\mathrm{d}u = \frac{k}{8} - \ln 2$

$\left[\frac{1}{2}u^2 - \ln|u|\right]_{\frac{1}{2}}^{1} = \frac{k}{8} - \ln 2$ A1

$\left(\frac{1}{2}(1)^2 - \ln 1\right) - \left(\frac{1}{2}\left(\frac{1}{2}\right)^2 - \ln\frac{1}{2}\right) = \frac{k}{8} - \ln 2$

$\frac{1}{2} - \left(\frac{1}{8} + \ln 2\right) = \frac{k}{8} - \ln 2$ A1

$\frac{3}{8} - \ln 2 = \frac{k}{8} - \ln 2$

$k = 3$ A1

[6]

3. $\int_e^{e^c} \frac{1}{x(\ln x)^2}\,\mathrm{d}x = \frac{8}{9}$

Let $u = \ln x$. (M1) for substitution

$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \Rightarrow \mathrm{d}u = \frac{1}{x}\mathrm{d}x$

$x = e^c \Rightarrow u = \ln e^c = c$

$x = e \Rightarrow u = \ln e = 1$

$\therefore \int_1^c \frac{1}{u^2}\,\mathrm{d}u = \frac{8}{9}$ (A2) for correct working

$\left[-\frac{1}{u}\right]_1^c = \frac{8}{9}$ A1

$-\frac{1}{c} - \left(-\frac{1}{1}\right) = \frac{8}{9}$

$-\frac{1}{c} = -\frac{1}{9}$

$c = 9$ A1

[5]

4. $\int_k^4 \frac{(\sqrt{x}+x)(1+2\sqrt{x})}{2\sqrt{x}}\mathrm{d}x = 16$

$\int_k^4 (\sqrt{x}+x)\left(\frac{1}{2\sqrt{x}}+1\right)\mathrm{d}x = 16$

Let $u = \sqrt{x}+x$. (M1) for substitution

$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}+1 \Rightarrow \mathrm{d}u = \left(\frac{1}{2\sqrt{x}}+1\right)\mathrm{d}x$

$x = 4 \Rightarrow u = \sqrt{4}+4 = 6$

$x = k \Rightarrow u = \sqrt{k}+k$

$\therefore \int_{\sqrt{k}+k}^{6} u\mathrm{d}u = 16$ (A2) for correct working

$\left[\frac{1}{2}u^2\right]_{\sqrt{k}+k}^{6} = 16$ A1

$\frac{1}{2}(6)^2 - \frac{1}{2}(\sqrt{k}+k)^2 = 16$ A1

$36-(\sqrt{k}+k)^2 = 32$

$(\sqrt{k}+k)^2 = 4$

$\sqrt{k}+k = -2$ (*Rejected*) or $\sqrt{k}+k = 2$ A1

$(\sqrt{k})^2+\sqrt{k}-2 = 0$

$(\sqrt{k}+2)(\sqrt{k}-1) = 0$

$\sqrt{k} = -2$ (*Rejected*) or $\sqrt{k} = 1$

$k = 1$ A1

[7]

Exercise 52

1. $\int_1^4 \ln\frac{x}{4}\,dx = \left[(x)\left(\ln\frac{x}{4}\right)\right]_1^4 - \int_1^4 x\,d\left(\ln\frac{x}{4}\right)$ (A2) for correct working

$\int_1^4 \ln\frac{x}{4}\,dx = \left[x\ln\frac{x}{4}\right]_1^4 - \int_1^4 x\cdot\frac{1}{x}\,dx$

$\int_1^4 \ln\frac{x}{4}\,dx = \left[x\ln\frac{x}{4}\right]_1^4 - \int_1^4 dx$ (M1) for valid approach

$\int_1^4 \ln\frac{x}{4}\,dx = \left[x\ln\frac{x}{4}\right]_1^4 - [x]_1^4$ A1

$\int_1^4 \ln\frac{x}{4}\,dx = \left[x\ln\frac{x}{4} - x\right]_1^4$

$\int_1^4 \ln\frac{x}{4}\,dx = (4\ln 1 - 4) - \left(\ln\frac{1}{4} - 1\right)$

$\int_1^4 \ln\frac{x}{4}\,dx = -4 - \ln\frac{1}{4} + 1$

$\int_1^4 \ln\frac{x}{4}\,dx = -3 - \ln\frac{1}{4}$ A1

[5]

2. $\int_1^4 x\ln x\mathrm{d}x = k\ln 4 - \frac{15}{4}$

Let $\theta = x^2$. (M1) for valid approach

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = 2x \Rightarrow \frac{1}{2}\mathrm{d}\theta = 2x\mathrm{d}x$

$\therefore \int_1^4 \frac{1}{2}\ln x\mathrm{d}(x^2) = k\ln 4 - \frac{15}{4}$

$\left[\frac{1}{2}(\ln x)(x^2)\right]_1^4 - \frac{1}{2}\int_1^4 x^2\mathrm{d}(\ln x) = k\ln 4 - \frac{15}{4}$ (A2) for correct working

$\left[\frac{1}{2}x^2\ln x\right]_1^4 - \frac{1}{2}\int_1^4 x^2 \cdot \frac{1}{x}\mathrm{d}x = k\ln 4 - \frac{15}{4}$

$\left[\frac{1}{2}x^2\ln x\right]_1^4 - \frac{1}{2}\int_1^4 x\mathrm{d}x = k\ln 4 - \frac{15}{4}$

$\left[\frac{1}{2}x^2\ln x\right]_1^4 - \frac{1}{2}\left[\frac{1}{2}x^2\right]_1^4 = k\ln 4 - \frac{15}{4}$ A1

$\left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_1^4 = k\ln 4 - \frac{15}{4}$

$\left(\frac{1}{2}(4)^2\ln 4 - \frac{1}{4}(4)^2\right) - \left(\frac{1}{2}(1)^2\ln 1 - \frac{1}{4}(1)^2\right)$

$= k\ln 4 - \frac{15}{4}$

$(8\ln 4 - 4) - \left(-\frac{1}{4}\right) = k\ln 4 - \frac{15}{4}$ A1

$8\ln 4 - \frac{15}{4} = k\ln 4 - \frac{15}{4}$

$k = 8$ A1

[6]

3. Let $\theta = \cos 4x$. (M1) for valid approach

$$\frac{d\theta}{dx} = -4\sin 4x \Rightarrow -\frac{1}{4}d\theta = \sin 4x dx$$

$$\therefore \int x^2 \sin 4x dx = \int -\frac{1}{4}x^2 d(\cos 4x)$$

$$\int x^2 \sin 4x dx = \left(-\frac{1}{4}x^2\right)(\cos 4x) - \int \cos 4x d\left(-\frac{1}{4}x^2\right)$$ (A2) for correct working

$$\int x^2 \sin 4x dx = -\frac{1}{4}x^2 \cos 4x + \int \frac{1}{2}x \cos 4x dx$$

Let $\alpha = \sin 4x$. (M1) for valid approach

$$\frac{d\alpha}{dx} = 4\cos 4x \Rightarrow \frac{1}{4}d\alpha = \cos 4x dx$$

$$\therefore \int x^2 \sin 4x dx = -\frac{1}{4}x^2 \cos 4x + \int \frac{1}{8}x d(\sin 4x)$$

$$\int x^2 \sin 4x dx = -\frac{1}{4}x^2 \cos 4x$$

$$+\left(\frac{1}{8}x\right)(\sin 4x) - \int \sin 4x d\left(\frac{1}{8}x\right)$$ (A1) for correct working

$$\int x^2 \sin 4x dx = -\frac{1}{4}x^2 \cos 4x + \frac{1}{8}x \sin 4x - \int \frac{1}{8}\sin 4x dx$$

$$\int x^2 \sin 4x dx = -\frac{1}{4}x^2 \cos 4x + \frac{1}{8}x \sin 4x + \frac{1}{32}\cos 4x + C$$ A1

[6]

4. Let $\theta = \sin x$. (M1) for valid approach

$$\frac{d\theta}{dx} = \cos x \Rightarrow d\theta = \cos x dx$$

$$\therefore \int e^{3x} \cos x dx = \int e^{3x} d(\sin x)$$

$$\int e^{3x} \cos x dx = e^{3x} \sin x - \int \sin x d(e^{3x})$$ (A2) for correct working

$$\int e^{3x} \cos x dx = e^{3x} \sin x - \int 3 \sin x e^{3x} dx$$

Let $\alpha = \cos x$. (M1) for valid approach

$$\frac{d\alpha}{dx} = -\sin x \Rightarrow d\alpha = -\sin x dx$$

$$\therefore \int e^{3x} \cos x dx = e^{3x} \sin x + \int 3e^{3x} d(\cos x)$$

$$\int e^{3x} \cos x dx = e^{3x} \sin x + 3e^{3x} \cos x - \int \cos x d(3e^{3x})$$ (A1) for correct working

$$\int e^{3x} \cos x dx = e^{3x} \sin x + 3e^{3x} \cos x - \int 9e^{3x} \cos x dx$$

$$\therefore 10 \int e^{3x} \cos x dx = e^{3x} \sin x + 3e^{3x} \cos x + C$$ A1

$$\int_0^{\pi} e^{3x} \cos x dx = \frac{1}{10} \left[e^{3x} \sin x + 3e^{3x} \cos x \right]_0^{\pi}$$

$$\int_0^{\pi} e^{3x} \cos x dx = \frac{1}{10} \begin{bmatrix} (e^{3\pi} \sin \pi + 3e^{3\pi} \cos \pi) \\ -(e^{3(0)} \sin 0 + 3e^{3(0)} \cos 0) \end{bmatrix}$$ A1

$$\int_0^{\pi} e^{3x} \cos x dx = \frac{1}{10} (-3e^{3\pi} - 3)$$

$$\int_0^{\pi} e^{3x} \cos x dx = -\frac{3}{10} (e^{3\pi} + 1)$$ AG

[7]

Exercise 53

1. (a) $f(x)=\dfrac{1}{(x+2)(x-4)}$

Let $\dfrac{1}{(x+2)(x-4)}\equiv\dfrac{A}{x+2}+\dfrac{B}{x-4}$, where A and B are constants.

$$\frac{1}{(x+2)(x-4)}\equiv\frac{A(x-4)}{(x+2)(x-4)}+\frac{B(x+2)}{(x+2)(x-4)} \qquad \text{M1}$$

$$\frac{1}{(x+2)(x-4)}\equiv\frac{Ax-4A+Bx+2B}{(x+2)(x-4)}$$

$1\equiv(A+B)x+(-4A+2B)$ A1

$0=A+B$

$B=-A$

$1=-4A+2B$

$\therefore 1=-4A+2(-A)$ A1

$1=-6A$

$A=-\dfrac{1}{6}$

$\therefore B=-\left(-\dfrac{1}{6}\right)$

$B=\dfrac{1}{6}$

$$\therefore \frac{1}{(x+2)(x-4)}\equiv-\frac{1}{6(x+2)}+\frac{1}{6(x-4)} \qquad \text{A1}$$

[4]

(b) $\displaystyle\int f(x)\mathrm{d}x=\int\left(-\frac{1}{6(x+2)}+\frac{1}{6(x-4)}\right)\mathrm{d}x$

$$\int f(x)\mathrm{d}x=-\frac{1}{6}\ln(x+2)+\frac{1}{6}\ln(x-4)+C \qquad \text{A1}$$

[1]

2. (a) $f(x)=\dfrac{3}{x(x-5)}$

Let $\dfrac{3}{x(x-5)}\equiv\dfrac{A}{x}+\dfrac{B}{x-5}$, where A and B are constants.

$\dfrac{3}{x(x-5)}\equiv\dfrac{A(x-5)}{x(x-5)}+\dfrac{Bx}{x(x-5)}$ M1

$\dfrac{3}{x(x-5)}\equiv\dfrac{Ax-5A+Bx}{x(x-5)}$

$3\equiv(A+B)x-5A$ A1

$0=A+B$

$B=-A$ A1

$3=-5A$

$A=-\dfrac{3}{5}$

$\therefore B=-\left(-\dfrac{3}{5}\right)$

$B=\dfrac{3}{5}$

$\therefore\dfrac{3}{x(x-5)}\equiv-\dfrac{3}{5x}+\dfrac{3}{5(x-5)}$ A1

[4]

(b) $\int_6^{11}f(t)\mathrm{d}t=\int_6^{11}\left(-\dfrac{3}{5t}+\dfrac{3}{5(t-5)}\right)\mathrm{d}t$

$\int_6^{11}f(t)\mathrm{d}t=\left[-\dfrac{3}{5}\ln t+\dfrac{3}{5}\ln(t-5)\right]_6^{11}$ A1

$\int_6^{11}f(t)\mathrm{d}t=\left(-\dfrac{3}{5}\ln 11+\dfrac{3}{5}\ln(11-5)\right)$

$-\left(-\dfrac{3}{5}\ln 6+\dfrac{3}{5}\ln(6-5)\right)$

$\int_6^{11}f(t)\mathrm{d}t=-\dfrac{3}{5}\ln 11+\dfrac{3}{5}\ln 6+\dfrac{3}{5}\ln 6-\dfrac{3}{5}\ln 1$

$\int_6^{11}f(t)\mathrm{d}t=-\dfrac{3}{5}\ln 11+\dfrac{6}{5}\ln 6$ A1

[2]

3. (a) $x=-2,\ x=-1$ A2

[2]

(b) $f(x)=\dfrac{x+4}{(x+2)(x+1)}$

Let $\dfrac{x+4}{(x+2)(x+1)}\equiv\dfrac{A}{x+2}+\dfrac{B}{x+1}$, where A and B are constants.

$\dfrac{x+4}{(x+2)(x+1)}\equiv\dfrac{A(x+1)}{(x+2)(x+1)}+\dfrac{B(x+2)}{(x+2)(x+1)}$ M1

$\dfrac{x+4}{(x+2)(x+1)}\equiv\dfrac{Ax+A+Bx+2B}{(x+2)(x+1)}$

$x+4\equiv(A+B)x+(A+2B)$ A1

$1=A+B$

$B=1-A$

$4=A+2B$

$\therefore 4=A+2(1-A)$ A1

$2=-A$

$A=-2$

$\therefore B=1-(-2)$

$B=3$

$\therefore \dfrac{x+4}{(x+2)(x+1)}\equiv-\dfrac{2}{x+2}+\dfrac{3}{x+1}$ A1

[4]

(c) $\int_4^5 f(x)\mathrm{d}x=\int_4^5\left(-\dfrac{2}{x+2}+\dfrac{3}{x+1}\right)\mathrm{d}x$

$\int_4^5 f(x)\mathrm{d}x=\left[-2\ln(x+2)+3\ln(x+1)\right]_4^5$ A1

$\int_4^5 f(x)\mathrm{d}x=(-2\ln(5+2)+3\ln(5+1))$

$-(-2\ln(4+2)+3\ln(4+1))$

$\int_4^5 f(x)\mathrm{d}x=-2\ln 7+3\ln 6+2\ln 6-3\ln 5$

$\int_4^5 f(x)\mathrm{d}x=-2\ln 7+5\ln 6-3\ln 5$ A1

[2]

4. (a) 0 A1

[1]

(b) $f(x)=\dfrac{1-2x}{(x+5)(x-3)}$

Let $\dfrac{1-2x}{(x+5)(x-3)}\equiv\dfrac{A}{x+5}+\dfrac{B}{x-3}$, where A and B are constants.

$$\frac{1-2x}{(x+5)(x-3)}\equiv\frac{A(x-3)}{(x+5)(x-3)}+\frac{B(x+5)}{(x+5)(x-3)}$$ M1

$$\frac{1-2x}{(x+5)(x-3)}\equiv\frac{Ax-3A+Bx+5B}{(x+5)(x-3)}$$

$$1-2x\equiv(A+B)x+(-3A+5B)$$ A1

$$-2=A+B$$

$$B=-2-A$$

$$1=-3A+5B$$

$$\therefore 1=-3A+5(-2-A)$$ A1

$$1=-3A-10-5A$$

$$11=-8A$$

$$A=-\frac{11}{8}$$

$$\therefore B=-2-\left(-\frac{11}{8}\right)$$

$$B=-\frac{5}{8}$$

$$\therefore \frac{1-2x}{(x+5)(x-3)}\equiv-\frac{11}{8(x+5)}-\frac{5}{8(x-3)}$$ A1

[4]

(c) $$\int_4^{12} f(x)\mathrm{d}x=\int_4^{12}\left(-\frac{11}{8(x+5)}-\frac{5}{8(x-3)}\right)\mathrm{d}x$$

$$\int_4^{12} f(x)\mathrm{d}x=\left[-\frac{11}{8}\ln(x+5)-\frac{5}{8}\ln(x-3)\right]_4^{12}$$ A1

$$\int_4^{12} f(x)\mathrm{d}x=\left(-\frac{11}{8}\ln(12+5)-\frac{5}{8}\ln(12-3)\right)$$
$$-\left(-\frac{11}{8}\ln(4+5)-\frac{5}{8}\ln(4-3)\right)$$

$$\int_4^{12} f(x)\mathrm{d}x=-\frac{11}{8}\ln 17-\frac{5}{8}\ln 9+\frac{11}{8}\ln 9+\frac{5}{8}\ln 1$$

$$\int_4^{12} f(x)\mathrm{d}x=-\frac{11}{8}\ln 17+\frac{3}{4}\ln 9$$ A1

[2]

Exercise 54

1. $x = 10\sec\theta$

$\dfrac{dx}{d\theta} = 10\sec\theta\tan\theta \Rightarrow dx = 10\sec\theta\tan\theta d\theta$ M1

$\sec\theta = \dfrac{x}{10}$

$\theta = \text{arcsec}\dfrac{x}{10}$ A1

$\therefore \int \dfrac{1}{x\sqrt{x^2-100}} dx = \int \dfrac{10\sec\theta\tan\theta d\theta}{10\sec\theta\sqrt{100\sec^2\theta - 100}}$ A1

$\int \dfrac{1}{x\sqrt{x^2-100}} dx = \int \dfrac{\tan\theta}{10\tan\theta} d\theta$ A1

$\int \dfrac{1}{x\sqrt{x^2-100}} dx = \int \dfrac{1}{10} d\theta$

$\int \dfrac{1}{x\sqrt{x^2-100}} dx = \dfrac{1}{10}\theta + C$ A1

$\int \dfrac{1}{x\sqrt{x^2-100}} dx = \dfrac{1}{10}\text{arcsec}\dfrac{x}{10} + C$ AG

[5]

2. $\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \frac{\pi}{k}$

Let $x = \sin\theta$.

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \cos\theta \Rightarrow \mathrm{d}x = \cos\theta\,\mathrm{d}\theta$ (M1) for substitution

$x = \frac{\sqrt{2}}{2} \Rightarrow \theta = \arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

$x = \frac{1}{2} \Rightarrow \theta = \arcsin\frac{1}{2} = \frac{\pi}{6}$

$\therefore \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta\,\mathrm{d}\theta = \frac{\pi}{k}$ (A2) for correct working

$\therefore \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos\theta}{\cos\theta}\,\mathrm{d}\theta = \frac{\pi}{k}$ A1

$[\theta]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{\pi}{k}$ A1

$\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{k}$

$\frac{\pi}{12} = \frac{\pi}{k}$

$k = 12$ A1

[6]

3. (a) $-x^2-2x+8=-(x^2+2x-8)$

$-x^2-2x+8=-(x^2+2x+1-9)$ (M1) for valid approach

$-x^2-2x+8=9-(x^2+2x+1)$

$-x^2-2x+8=9-(x+1)^2$ A1

[2]

(b) $\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{9-(x+1)^2}}\,\mathrm{d}x$

Let $x+1=3\sin\theta$. A1

$x=3\sin\theta-1$

$\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\cos\theta\Rightarrow \mathrm{d}x=3\cos\theta\,\mathrm{d}\theta$ M1

$x=\frac{3\sqrt{2}}{2}-1\Rightarrow\theta=\sin^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$

$x=-\frac{3\sqrt{2}}{2}-1\Rightarrow\theta=\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$

$\therefore\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{3\cos\theta}{\sqrt{9-9\sin^2\theta}}\,\mathrm{d}\theta$ A1

$\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{3\cos\theta}{3\cos\theta}\,\mathrm{d}\theta$ A1

$\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\mathrm{d}\theta$

$\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=[\theta]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ A1

$\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)$

$\int_{-\frac{3\sqrt{2}}{2}-1}^{\frac{3\sqrt{2}}{2}-1}\frac{1}{\sqrt{-x^2-2x+8}}\,\mathrm{d}x=\frac{\pi}{2}$ AG

[5]

4. (a) $x^2+8x+20=x^2+8x+16+4$ (M1) for valid approach

$x^2+8x+20=(x+4)^2+4$ A1

[2]

(b) $\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\int_{-4}^{-2}\frac{1}{((x+4)^2+4)^2}\mathrm{d}x$

Let $x+4=2\tan\theta$. A1

$x=2\tan\theta-4$

$\frac{\mathrm{d}x}{\mathrm{d}\theta}=2\sec^2\theta \Rightarrow \mathrm{d}x=2\sec^2\theta\mathrm{d}\theta$ M1

$x=-2\Rightarrow\theta=\arctan 1=\frac{\pi}{4}$

$x=-4\Rightarrow\theta=\arctan 0=0$

$\therefore\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\int_{0}^{\frac{\pi}{4}}\frac{2\sec^2\theta}{(4\tan^2\theta+4)^2}\mathrm{d}\theta$ A1

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\int_{0}^{\frac{\pi}{4}}\frac{2\sec^2\theta}{16\sec^4\theta}\mathrm{d}\theta$ A1

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\int_{0}^{\frac{\pi}{4}}\frac{1}{8}\cos^2\theta\mathrm{d}\theta$

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\int_{0}^{\frac{\pi}{4}}\frac{1}{8}\left(\frac{1+\cos 2\theta}{2}\right)\mathrm{d}\theta$ A1

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\frac{1}{16}\int_{0}^{\frac{\pi}{4}}(1+\cos 2\theta)\mathrm{d}\theta$

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\frac{1}{16}\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}}$ A1

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x$

$=\frac{1}{16}\left(\frac{\pi}{4}+\frac{1}{2}\sin 2\left(\frac{\pi}{4}\right)\right)-\frac{1}{16}\left(0+\frac{1}{2}\sin 2(0)\right)$

$\int_{-4}^{-2}\frac{1}{(x^2+8x+20)^2}\mathrm{d}x=\frac{\pi}{64}+\frac{1}{32}$ AG

[6]