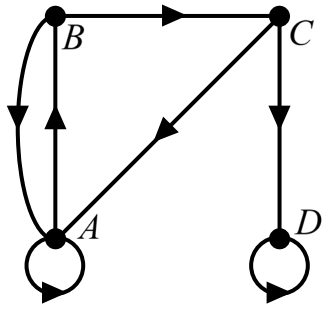
AI HL Practice Set 2 Paper 3 Solution

1. (a) For correct number of directed edges A1
For correct number of loops A1
For correct directions A2

[4]



(b) The column sum represents the in-degree of the corresponding vertex. A1

[1]

(c) (i) $\mathbf{M} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ A2

(ii) $\mathbf{T} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}$ A2

[4]

(d) (i) The player is definitely at the state A before his tosses the coin for the first time. R1

(ii) $\mathbf{v}_1 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0.5 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix}$ A2

(iii) There are four scenarios that the player will be at the state A after the coin is tossed for three times:
For any two scenarios correct R1
For all scenarios correct R1
1. Getting three consecutive tails
2. Getting one head followed by two consecutive tails
3. Getting heads and tails alternatively, starting with a tail
4. Getting two consecutive heads followed by one tail
Also, as the probability for each scenario is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, $\alpha_1 = 4\left(\frac{1}{8}\right) = \frac{1}{2}$. R1

(iv) $\alpha_2 : \alpha_3 : \alpha_4 = 2:1:1$ A1

[7]

(e) Let $\mathbf{v}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$ be the steady state probability vector, where $e+f+g+h=1$.

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$$ M1

$$\begin{pmatrix} \frac{1}{2}e+\frac{1}{2}f+\frac{1}{2}g \\ \frac{1}{2}e \\ \frac{1}{2}f \\ \frac{1}{2}g+h \end{pmatrix}=\begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix}$$ A1

$\frac{1}{2}g+h=h$

$g=0$

$\frac{1}{2}f=0$

$f=0$

$\frac{1}{2}e=0$

$e=0$ A1

$0+0+0+h=1$ M1

$h=1$

Thus, the steady state probability vector is

$\mathbf{v}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$. AG

[4]

(f) (i) The required probability

$$= (1)\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^3$$ M1

$$= \frac{2}{81}$$ A1

(ii) The required probability

$$= \left(1-\left(\frac{1}{3}\right)^3\right)\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^3$$ M1A2

$$= \frac{52}{2187}$$ A1

(iii) The required probability

$$= 1-\left(\frac{1}{3}\right)^3-\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^3-\frac{2}{81}$$

$$-(1)^2\left(1-\frac{1}{3}\right)\left(\frac{1}{3}\right)^3-\frac{52}{2187}$$ M1A2

$$= \frac{1892}{2187}$$ A1

[10]

2. (a) (i) 0.212 A1

(ii) $\bar{W} \sim \mathrm{N}\left(300, \frac{10^2}{12}\right)$ A1

The required probability

$= \mathrm{P}(\bar{W} < 292)$

$= 0.002791866$ (A1) for correct value

$= 0.00279$ A1

[4]

(b) (i) $6000\ \mathrm{g}$ A1

(ii) The required variance

$= 20(10^2)$ (A1) for substitution

$= 2000\ \mathrm{g}^2$ A1

(iii) The required probability

$= 0.0126736174$ (A1) for correct value

$= 0.0127$ A1

[5]

(c) (i) $H_0 : \rho = 0$ A1

(ii) $H_1 : \rho < 0$ A1

(iii) $p\text{-value} = 0.009830306$ (A1) for correct value

$p\text{-value} = 0.00983$ A1

(iv) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1

[6]

(d) (i) $a = -1.533333333$

$a = -1.53$ A1

$b = 510.7333333$

$b = 511$ A1

(ii) a represents the average increase of the maximum walking speed of a crab when its weight is increased by 1 gram. A1

[3]

(e)	(i)	$H_0: \mu = 300$	A1
	(ii)	$H_1: \mu \neq 300$	A1
	(iii)	$z = -1.16$	A1
	(iv)	p-value $= 0.2452782275$	(A1) for correct value
		p-value $= 0.245$	A1
	(v)	The null hypothesis is not rejected.	A1
		As p-value > 0.1.	R1

[7]