Formula List of Applications and Interpretation Standard Level for IBDP Mathematics

Analysis & Approaches Standard Level	**Analysis & Approaches Higher Level**
Applications & Interpretation Standard Level	**Applications & Interpretation Higher Level**

1 Standard Form

✓ Standard Form:

A number in the form $(\pm)a\times10^k$, where $1\le a<10$ and k is an integer

2 Approximation and Error

✓ Summary of rounding methods:

2.71828	Correct to 3 significant figures	Correct to 3 decimal places
Round off	2.7**2**	2.71**8**

✓ Consider a quantity measured as Q and correct to the nearest unit d:

$\frac{1}{2}d$: Maximum absolute error

$Q-\frac{1}{2}d\le A<Q+\frac{1}{2}d$: Range of the actual value A

$Q-\frac{1}{2}d$: Lower bound (Least possible value) of A

$Q+\frac{1}{2}d$: Upper bound of A

$\frac{\text{Maximum absolute error}}{Q}\times100\%$: Percentage error

3 Functions

✓ The function $y=f(x)$:

1. $f(a)$: Functional value when $x=a$
2. Domain: Set of values of x
3. Range: Set of values of y

✓ Properties of rational function $y=\frac{ax+b}{cx+d}$:

1. $y=\frac{1}{x}$: Reciprocal function
2. $y=\frac{a}{c}$: Horizontal asymptote
3. $x=-\frac{d}{c}$: Vertical asymptote

✓ Variations:

1. $y=kx, k\neq 0$: y is directly proportional to x
2. $y=\frac{k}{x}, k\neq 0$: y is inversely proportional to x

4 Quadratic Functions

✓ General form $y=ax^2+bx+c$, where $a\neq 0$:

$a>0$	The graph opens upward
$a<0$	The graph opens downward
c	y-intercept
$h=-\frac{b}{2a}$	x-coordinate of the vertex
$k=ah^2+bh+c$	y-coordinate of the vertex
	Extreme value of y
$x=h$	Equation of the axis of symmetry

✓ Other forms:

1. $y=a(x-h)^2+k$: Vertex form
2. $y=a(x-p)(x-q)$: Factored form with x-intercepts p and q

✓ $h=-\frac{b}{2a}=\frac{p+q}{2}$

✓ The x-intercepts of the quadratic function $y=ax^2+bx+c$ are the roots of the corresponding quadratic equation $ax^2+bx+c=0$

5 Exponential and Logarithmic Functions

✓ $y = a^x$: Exponential function, where $a \neq 1$

✓ $y = \log_a x$: Logarithmic function, where $a > 0$

✓ $y = \log x = \log_{10} x$: Common Logarithmic function

✓ $y = \ln x = \log_e x$: Natural Logarithmic function, where $e = 2.71828...$ is an exponential number

✓ Properties of the graphs of $y = a^x$:

$a > 1$	$0 < a < 1$
y-intercept $= 1$	
y increases as x increases	y decreases as x increases
y tends to zero as x tends to negative infinity	y tends to zero as x tends to positive infinity
Horizontal asymptote: $y = 0$	

6 Systems of Equations

✓ $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$: 2×2 system

✓ $\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ ix + jy + kz = l \end{cases}$: 3×3 system

✓ The above systems can be solved by PlySmlt2 in TI-84 Plus CE

7 Arithmetic Sequences

✓ Properties of an arithmetic sequence u_n:

1. u_1: First term
2. $d = u_2 - u_1 = u_n - u_{n-1}$: Common difference
3. $u_n = u_1 + (n-1)d$: General term (n th term)
4. $S_n = \frac{n}{2}[2u_1 + (n-1)d] = \frac{n}{2}[u_1 + u_n]$: The sum of the first n terms

✓ $\sum_{r=1}^{n} u_r = u_1 + u_2 + u_3 + \cdots + u_{n-1} + u_n$: Summation sign

8 Geometric Sequences

✓ Properties of a geometric sequence u_n:

1. u_1: First term
2. $r = u_2 \div u_1 = u_n \div u_{n-1}$: Common ratio
3. $u_n = u_1 \times r^{n-1}$: General term (n th term)
4. $S_n = \frac{u_1(1-r^n)}{1-r}$: The sum of the first n terms

9 Financial Mathematics

✓ Compound Interest:

PV: Present value

$r\%$: Interest rate per annum (per year)

n: Number of years

k: Number of compounded periods in one year

$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$: Future value

$I = FV - PV$: Interest

✓ Inflation:

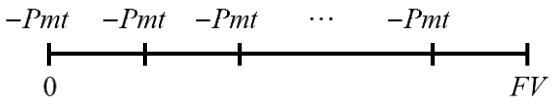
$i\%$: Inflation rate

$R\%$: Interest rate compounded yearly

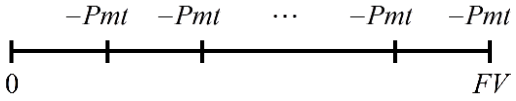
$(R-i)\%$: Real rate

✓ Annuity:

1. Payments at the beginning of each year

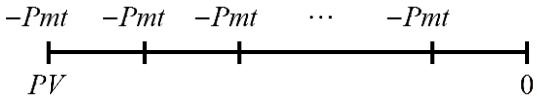


2. Payments at the end of each year

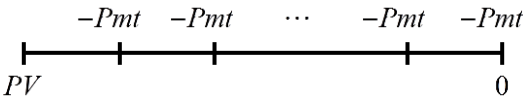


✓ Amortization:

1. Payments at the beginning of each year



2. Payments at the end of each year



10 Coordinate Geometry

✓ Consider the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a x-y plane:

1. $m = \frac{y_2 - y_1}{x_2 - x_1}$: Slope of PQ
2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$: Distance between P and Q
3. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$: Mid-point of PQ

✓ Consider the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on a x - y - z plane:

1. z -axis: The axis perpendicular to the x - y plane
2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$: Distance between P and Q
3. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$: Mid-point of PQ

✓ Forms of straight lines with slope m and y -intercept c:

1. $y = mx + c$: Slope-intercept form
2. $Ax + By + C = 0$: General form

✓ Ways to find the x -intercept and the y -intercept of a line:

1. Substitute $y = 0$ and make x the subject to find the x -intercept
2. Substitute $x = 0$ and make y the subject to find the y -intercept

11 Voronoi Diagrams

✓ Elements in Voronoi Diagrams:

Site: A given point

Cell of a site: A collection of points which is closer to the site than other sites

Boundary: A line dividing the cells

Vertex: An intersection of boundaries

✓ Related problems:

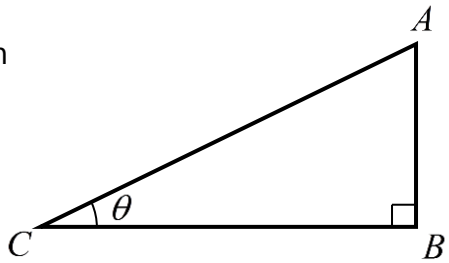
1. Nearest neighbor interpolation
2. Incremental algorithm
3. Toxic waste dump problem

12 Trigonometry

✓ Consider a right-angled triangle ABC:

$AB^2 + BC^2 = AC^2$: Pythagoras' Theorem

$$\begin{cases} \sin\theta = \dfrac{AB}{AC} \\ \cos\theta = \dfrac{BC}{AC} \\ \tan\theta = \dfrac{AB}{BC} \end{cases}$$: Trigonometric ratios



✓ Properties of a general trigonometric function $y = A\sin B(x - C) + D$:

1. $A = \dfrac{y_{\max} - y_{\min}}{2}$: Amplitude
2. $B = \dfrac{360^\circ}{\text{Period}}$
3. $D = \dfrac{y_{\max} + y_{\min}}{2}$
4. C can be found by substitution of a point on the graph

✓ Properties of graphs of trigonometric functions:

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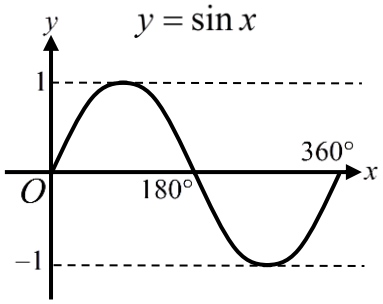
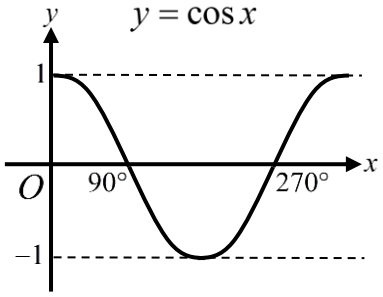
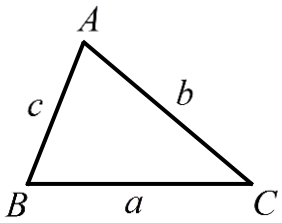
<tr>
<td> 
</td>
<td>1. Amplitude = 1
2. Period = 360°
3. $-1 \le \sin x \le 1$</td>
</tr>
<tr>
<td> 
</td>
<td>1. Amplitude = 1
2. Period = 360°
3. $-1 \le \cos x \le 1$</td>
</tr>
</table>

13 2-D Trigonometry

✓ Consider a triangle ABC:

1. $\frac{\sin A}{a}=\frac{\sin B}{b}$ or $\frac{a}{\sin A}=\frac{b}{\sin B}$: Sine rule
2. $a^2=b^2+c^2-2bc\cos A$

   or $\cos A=\frac{b^2+c^2-a^2}{2bc}$: Cosine rule
3. $\frac{1}{2}ab\sin C$: Area of the triangle ABC

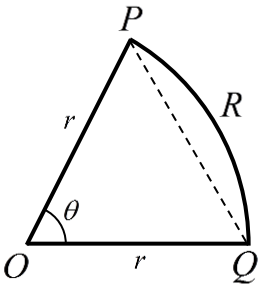


✓ Consider a sector $OPRQ$ with centre O, radius r and $\angle POQ=\theta^\circ$:

$2\pi r\times\frac{\theta^\circ}{360^\circ}$: Arc length PRQ

$\pi r^2\times\frac{\theta^\circ}{360^\circ}$: Area of the sector $OPRQ$

$\pi r^2\times\frac{\theta^\circ}{360^\circ}-\frac{1}{2}r^2\sin\theta^\circ$: Area of the segment PRQ



14 Areas and Volumes

✓ For a cube of side length l:

1. $6l^2$: Total surface area
2. l^3: Volume

✓ For a cuboid of side lengths a, b and c:

1. $2(ab+bc+ac)$: Total surface area
2. abc: Volume

✓ For a prism of height h and cross-sectional area A:

1. Ah: Volume

✓ For a cylinder of height h and radius r:

1. $2\pi r^2 + 2\pi rh$: Total surface area
2. $2\pi rh$: Lateral surface area
3. $\pi r^2 h$: Volume

✓ For a pyramid of height h and base area A:

1. $\frac{1}{3}Ah$: Volume

✓ For a circular cone of height h and radius r:

1. $l = \sqrt{r^2 + h^2}$: Slant height
2. $\pi r^2 + \pi rl$: Total surface area
3. πrl: Curved surface area
4. $\frac{1}{3}\pi r^2 h$: Volume

✓ For a sphere of radius r:

1. $4\pi r^2$: Total surface area
2. $\frac{4}{3}\pi r^3$: Volume

✓ For a hemisphere of radius r:

1. $3\pi r^2$: Total surface area
2. $2\pi r^2$: Curved surface area
3. $\frac{2}{3}\pi r^3$: Volume

15 Differentiation

✓ $\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$: Derivative of the function $y = f(x)$ (First derivative)

✓ Rules of differentiation:

1. $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
2. $f(x) = p(x) + q(x) \Rightarrow f'(x) = p'(x) + q'(x)$
3. $f(x) = cp(x) \Rightarrow f'(x) = cp'(x)$

✓ Relationship between graph properties and the derivatives:

1. $f'(x) > 0$ for $a \le x \le b$: $f(x)$ is increasing in the interval
2. $f'(x) < 0$ for $a \le x \le b$: $f(x)$ is decreasing in the interval
3. $f'(a) = 0$: $(a, f(a))$ is a stationary point of $f(x)$
4. $f'(a) = 0$ and $f'(x)$ changes from positive to negative at $x = a$: $(a, f(a))$ is a maximum point of $f(x)$
5. $f'(a) = 0$ and $f'(x)$ changes from negative to positive at $x = a$: $(a, f(a))$ is a minimum point of $f(x)$

✓ Tangents and normals:

1. $f'(a)$: Slope of tangent at $x = a$
2. $\frac{-1}{f'(a)}$: Slope of normal at $x = a$
3. $y - f(a) = f'(a)(x - a)$: Equation of tangent at $x = a$
4. $y - f(a) = \left(\frac{-1}{f'(a)}\right)(x - a)$: Equation of normal at $x = a$

16 Integration and Trapezoidal Rule

✓ Integrals of a function $y = f(x)$:

1. $\int f(x)\mathrm{d}x$: Indefinite integral of $f(x)$
2. $\int_a^b f(x)\mathrm{d}x$: Definite integral of $f(x)$ from a to b

✓ Rules of integration:

1. $\int x^n \mathrm{d}x = \frac{1}{n+1}x^{n+1} + C$
2. $\int (p'(x) + q'(x))\mathrm{d}x = p(x) + q(x) + C$
3. $\int cp'(x)\mathrm{d}x = cp(x) + C$

✓ $\int_a^b f(x)\mathrm{d}x$: Area under the graph of $f(x)$ and above the x-axis, between $x = a$ and $x = b$, where $f(x) \ge 0$

✓ Trapezoidal Rule:

a, b $(a<b)$: End points

n: Number of intervals

$h=\frac{b-a}{n}$: Interval width

$\int_a^b f(x)\mathrm{d}x$ can be estimated by $\frac{1}{2}h\left[f(x_0)+f(x_n)+2(f(x_1)+f(x_2)+\ldots+f(x_{n-1}))\right]$

✓ Estimation by Trapezoidal Rule:

1. The estimation overestimates if the estimated value is greater than the actual value of $\int_a^b f(x)\mathrm{d}x$
2. The estimation underestimates if the estimated value is less than the actual value of $\int_a^b f(x)\mathrm{d}x$

17 Statistics

✓ Relationship between frequencies and cumulative frequencies:

Data	Frequency	Data less than or equal to	Cumulative frequency
10	f_1	10	f_1
20	f_2	20	f_1+f_2
30	f_3	30	$f_1+f_2+f_3$

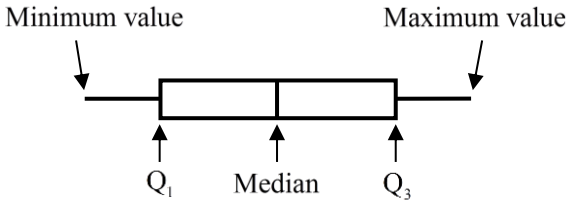
✓ Measures of central tendency for a data set $\{x_1, x_2, x_3, \cdots, x_n\}$ arranged in ascending order:

1. $\overline{x}=\frac{x_1+x_2+x_3+\cdots+x_n}{n}$: Mean
2. The datum or the average value of two data at the middle: Median
3. The datum appears the most: Mode

✓ Measures of dispersion for a data set $\{x_1, x_2, x_3, \cdots, x_n\}$ arranged in ascending order:

1. $x_n - x_1$: Range
2. Two subgroups A and B can be formed from the data set such that all data of the subgroup A are less than or equal to the median, while all data of the subgroup B are greater than or equal to the median
3. $Q_1 =$ The median of the subgroup A: Lower quartile
4. $Q_3 =$ The median of the subgroup B: Upper quartile
5. $Q_3 - Q_1$: Inter-quartile range (IQR)
6. $\sigma = \sqrt{\dfrac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\cdots+(x_n-\bar{x})^2}{n}}$: Standard deviation

✓ Box-and-whisker diagram:



✓ A datum x is defined to be an outlier if $x < Q_1 - 1.5\text{IQR}$ or $x > Q_3 + 1.5\text{IQR}$

✓ Coding of data:

1. Only the mean, the median, the mode and the quartiles will change when each datum of the data set is added or subtracted by a value
2. All measures of central tendency and measures of dispersion will change when each datum of the data set is multiplied or divided by a value

18 Probability

✓ Terminologies:

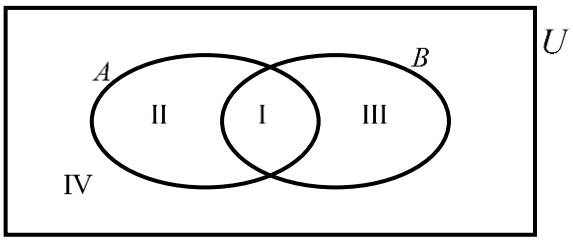
1. U : Universal set
2. A: Event
3. x: Outcome of an event
4. $n(U)$: Total number of elements
5. $n(A)$: Number of elements in A

✓ Formulae for probability:

1. $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
2. $\mathrm{P}(A')=1-\mathrm{P}(A)$
3. $\mathrm{P}(A\mid B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$
4. $\mathrm{P}(A)=\mathrm{P}(A\cap B)+\mathrm{P}(A\cap B')$
5. $\mathrm{P}(A'\cap B')+\mathrm{P}(A\cup B)=1$
6. $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)$ and $\mathrm{P}(A\cap B)=0$ if A and B are mutually exclusive
7. $\mathrm{P}(A\cap B)=\mathrm{P}(A)\cdot\mathrm{P}(B)$ and $\mathrm{P}(A\mid B)=\mathrm{P}(A)$ if A and B are independent

✓ Venn diagram:

1. Region I: $A\cap B$
2. Region II: $A\cap B'$
3. Region III: $A'\cap B$
4. Region IV: $(A\cup B)'$



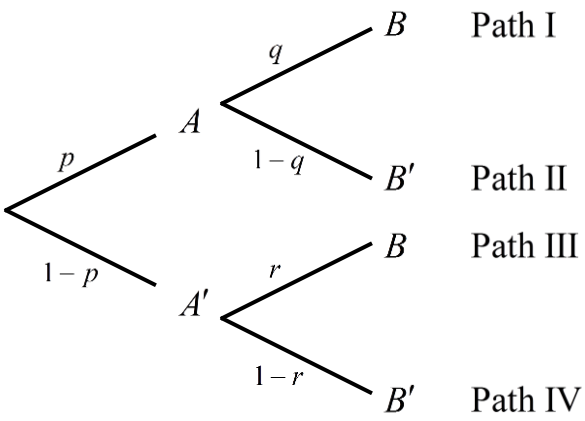
✓ Tree diagram:

1. Path I: $\mathrm{P}(A\cap B)=pq$
2. Path I + Path III:

$=\mathrm{P}(B)$

$=\mathrm{P}(A\cap B)+\mathrm{P}(A'\cap B)$

$=pq+(1-p)r$



19 Discrete Probability Distributions

✓ Properties of a discrete random variable X:

X	x_1	x_2	...	x_n
$\mathrm{P}(X=x)$	$\mathrm{P}(X=x_1)$	$\mathrm{P}(X=x_2)$	...	$\mathrm{P}(X=x_n)$

1. $\mathrm{P}(X=x_1)+\mathrm{P}(X=x_2)+\cdots+\mathrm{P}(X=x_n)=1$
2. $\mathrm{E}(X)=x_1\mathrm{P}(X=x_1)+x_2\mathrm{P}(X=x_2)+\cdots+x_n\mathrm{P}(X=x_n)$: Expected value of X
3. $\mathrm{E}(X)=0$ if a fair game is considered

20 Binomial Distribution

✓ Properties of a random variable $X \sim \mathrm{B}(n, p)$ following binomial distribution:

1. Only two outcomes from every independent trial (Success and failure)
2. n : Number of trials
3. p : Probability of success
4. X : Number of successes in n trials

✓ Formulae for binomial distribution:

1. $\mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$ for $0 \le r \le n,\ r \in \mathbb{Z}$
2. $\mathrm{E}(X) = np$: Expected value of X
3. $\mathrm{Var}(X) = np(1-p)$: Variance of X
4. $\sqrt{np(1-p)}$: Standard deviation of X
5. $\mathrm{P}(X \le r) = \mathrm{P}(X < r+1) = 1 - \mathrm{P}(X \ge r+1)$

21 Normal Distribution

✓ Properties of a random variable $X \sim \mathrm{N}(\mu, \sigma^2)$ following normal distribution:

1. μ : Mean
2. σ : Standard deviation
3. The mean, the median and the mode are the same
4. The normal curve representing the distribution is a bell-shaped curve which is symmetric about the middle vertical line
5. $\mathrm{P}(X < \mu) = \mathrm{P}(X > \mu) = 0.5$
6. The total area under the curve is 1

22 Bivariate Analysis

✓ Correlations:

Positive	Strong	$0.75 < r < 1$
	Moderate	$0.5 < r < 0.75$
	Weak	$0 < r < 0.5$
No		$r = 0$
Negative	Weak	$-0.5 < r < 0$
	Moderate	$-0.75 < r < -0.5$
	Strong	$-1 < r < -0.75$

where r is the correlation coefficient

✓ Linear regression:

$y = ax + b$: Regression line of y on x

✓ Correlation Coefficient for ranked data:

r_s: Spearman's Rank Correlation Coefficient

23 Statistical Tests

✓ Hypothesis test:

H_0: Null hypothesis

H_1: Alternative hypothesis

C: Critical value in the hypothesis test

α: Significance level

- ✓ χ^2 test for independence for a contingency table with r rows and c columns:
  $n = rc$: Total number of data
  O_i $(i = 1, 2, \ldots, n)$: Observed frequencies
  E_i $(i = 1, 2, \ldots, n)$: Expected frequencies
  $v = (r-1)(c-1)$: Degree of freedom
  $\chi^2_{calc} = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$: χ^2 test statistic
  H_0: Two variables are independent
  H_1: Two variables are not independent
  H_0 is rejected if $\chi^2_{calc} > C$ or the p-value is less than the significance level
  H_0 is not rejected if $\chi^2_{calc} < C$ or the p-value is greater than the significance level

- ✓ χ^2 goodness of fit test for a contingency table with 1 row and c columns:
  $v = c - 1$: Degree of freedom
  H_0: The data follows an assigned distribution
  H_1: The data does not follow an assigned distribution
  H_0 is rejected if $\chi^2_{calc} > C$ or the p-value is less than the significance level
  H_0 is not rejected if $\chi^2_{calc} < C$ or the p-value is greater than the significance level

- ✓ Two sample t test:
  μ_1, μ_2: The population means of two groups of data
  H_0: $\mu_1 = \mu_2$
  H_1: $\mu_1 > \mu_2$, $\mu_1 < \mu_2$ (for 1-tailed test), $\mu_1 \neq \mu_2$ (for 2-tailed test)
  H_0 is rejected if the p-value is less than the significance level
  H_0 is not rejected if the p-value is greater than the significance level

Notes