AI SL Practice Set 3 Paper 2 Solution

1. (a) $a = 5.6$ A1 N1

$b = 34.8$ A1 N1

[2]

(b) The estimated hardness

$= 5.6(6.3) + 34.8$ (A1) for substitution

$= 70.08$ A1 N2

[2]

(c) The required probability

$= \frac{120 - 56}{120}$ (M1) for valid approach

$= \frac{8}{15}$ A1 N2

[2]

(d) (i) Let X be the number of selected ingots of the hardness at least 65, where

$X \sim \mathrm{B}\left(10, \frac{8}{15}\right)$.

The required probability

$= \mathrm{P}(X = 5)$ (M1) for valid approach

$= 0.2406733955$

$= 0.241$ A1 N2

(ii) The required probability

$= \mathrm{P}(X < 4)$ (M1) for valid approach

$= 0.1226252054$

$= 0.123$ A1 N2

(iii) $\frac{16}{3}$ A1 N1

[5]

(e) (i) $H_1: \mu_1 \neq \mu_2$ A1 N1

(ii) $p\text{-value} = 0.0741679182$ (A1) for correct value

$p\text{-value} = 0.0742$ A1 N2

(iii) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. R1 N2

[5]

2. (a) The volume

$= \pi r^2 h$

$= \pi(4)^2(15)$ (A1) for substitution

$= 240\pi \text{ cm}^3$ A1 N2

[2]

(b) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(15)$ (A1) for substitution

$= 152\pi \text{ cm}^2$ A1 N2

[2]

(c) 26 A1 N1

[1]

(d) $l^2 h = \pi r^2 h$ (M1) for setting equation

$l^2 = \pi r^2$

$\therefore l^2 = \pi(4)^2$ (A1) for substitution

$l = \sqrt{16\pi}$

$l = 7.089815404 \text{ cm}$

$l = 7.09 \text{ cm}$ A1 N3

[3]

(e) The total surface area of the new container

$= 2l^2 + 4lh$ M1

$= 2(7.089815404)^2 + 4(7.089815404)(15)$ A1

$= 525.9198891 \text{ cm}^2$

$> 152\pi \text{ cm}^2$ R1

Thus, the claim is agreed. A1 N0

[4]

3. (a) (i) H_0: The punctuality of buses and the locations of bus stops are independent. A1 N1

(ii) H_1: The punctuality of buses and the locations of bus stops are not independent. A1 N1

[2]

(b) 8 A1 N1

[1]

(c) $\chi^2_{calc} = 19.37210492$ (A1) for correct value

$\chi^2_{calc} = 19.4$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 15.507$. R1 N2

[2]

(e) (i) The required probability

$= \frac{48}{500}$ (A1) for correct formula

$= \frac{12}{125}$ A1 N2

(ii) The required probability

$= \frac{15+13+8+11+8}{500}$ (A1) for correct formula

$= \frac{11}{100}$ A1 N2

(iii) The required probability

$= \frac{11}{15+13+8+11+8}$ (A1) for correct formula

$= \frac{1}{5}$ A1 N2

[6]

(f) The required probability

$= \left(\frac{74}{500}\right)\left(\frac{74-1}{500-1}\right)\left(\frac{74-2}{500-2}\right)$ (A2) for correct formula

$= 0.0031303088$

$= 0.00313$ A1 N3

[3]

4. (a) $P(0)=116$

$\therefore a+b\times c^{0}=116$ — (M1) for setting equation

$a+b=116$ — A1 N2

[2]

(b) $P(1)=172$

$\therefore a+b\times c^{-1}=172$ — (M1) for setting equation

$a+\frac{b}{c}=172$ — A1 N2

[2]

(c) (i) $\log_{c}81=4$

$\therefore c^{4}=81$ — M1

$c^{4}=3^{4}$ — A1

$c=3$ — AG N0

(ii) The system is $\begin{cases} a+b=116 \\ a+\frac{1}{3}b=172 \end{cases}$. — (M1) for valid approach

Solving, we have $a=200$ and $b=-84$. — A2 N3

[5]

(d) The number of elephants

$=200-84\times 3^{-3}$ — (M1) for substitution

$=196.8888889$

$=197$ — A1 N2

[2]

(e) 200 — A1 N1

[1]

(f) $200-84\times 3^{-t}>195$ — (M1) for setting inequality

$5-84\times 3^{-t}>0$

By considering the graph of $y=5-84\times 3^{-t}$,

$t=2.5681297$.

Thus, the number of years needed is 2.57 years. — A1 N2

[2]

(g) By considering the graphs of $y=200-84\times3^{-t}$, $y=170$, $y=180$ and $y=190$, y reaches 170, 180 and 190 at $t_1=0.9372$, $t_2=1.3062702$ and $t_3=1.9372$ respectively. M1A1

$\because 2(t_2-t_1)$

$=2(1.3062702-0.9372)$

$=0.7381404$

$\neq t_3-t_2$ R1

Thus, the claim is disagreed. A1 N0

[4]

5. (a) (i) $(4, 8)$ A2 N2

(ii) $\{y : 4 \le y \le 8,\ y \in \mathbb{R}\}$ A2 N2

[4]

(b) $f'(x)$

$= -0.25(2x) + 2(1) + 0$ (A1) for correct derivatives

$= -0.5x + 2$ A1 N2

[2]

(c) $f'(x) = -1$ M1

$\therefore -0.5x + 2 = -1$ A1

$-0.5x = -3$

$x = 6$ A1

$f(6)$

$= -0.25(6)^2 + 2(6) + 4$ M1

$= 7$

Thus, the coordinates of P are $(6, 7)$. AG N0

[4]

(d) The equation of the tangent:

$y - 7 = -1(x - 6)$ (A1) for substitution

$y - 7 = -x + 6$

$x + y - 13 = 0$ A1 N2

[2]

(e) (i) 4 A1 N1

(ii) 5.75 A1 N1

[2]

(f) The estimate of $\int_0^8 f(x)\mathrm{d}x$

$= \frac{1}{2}(1)\left[4 + 4 + 2\begin{pmatrix} 5.75 + 7 + 7.75 \\ +8 + 7.75 + 7 + 5.75 \end{pmatrix}\right]$ (A2) for substitution

$= 53$ A1 N3

[3]

(g) Underestimate A1 N1

[1]