

# Chapter 18 Solution

## Exercise 77

1. (a)  $P(X = 1) + P(X = 2) + P(X = 5) + P(X = 10) = 1$  (A1) for correct formula  
 $(r + s) + (r - s) + 0.15 + 0.05 = 1$   
 $2r = 0.8$   
 $r = 0.4$  A1  
 $E(X) = (1)(r + s) + (2)(r - s) + (5)(0.15) + (10)(0.05)$   
 $\therefore 2.35 = 0.4 + s + 2(0.4 - s) + 0.75 + 0.5$  (A1) for substitution  
 $2.35 = 2.45 - s$   
 $s = 0.1$  A1
- (b) (i)  $-2.35$  A1 [4]  
(ii)  $4.9275$  A1 [2]
2. (a)  $P(X = 10) + P(X = 20) + P(X = t) + P(X = 2t) = 1$  (A1) for correct formula  
 $0.2 + 0.5 + r + r^2 + 0.06 = 1$   
 $r^2 + r - 0.24 = 0$   
 $25r^2 + 25r - 6 = 0$   
 $(5r + 6)(5r - 1) = 0$   
 $r = -1.2$  (*Rejected*) or  $r = 0.2$  A1  
 $E(X) = (10)(0.2) + (20)(0.5) + tr + (2t)(r^2 + 0.06)$   
 $\therefore 24 = 2 + 10 + 0.2t + 0.2t$  (A1) for substitution  
 $12 = 0.4t$   
 $t = 30$  A1 [4]
- (b)  $4\sqrt{184} = q\sqrt{46}$  A1  
 $q = \frac{4\sqrt{184}}{\sqrt{46}}$   
 $q = 4(2)$   
 $q = 8$  A1 [2]

3. (a)  $P(X = -2) + P(X = 0) + P(X = 2) + P(X = 4) = 1$  (A1) for correct formula  
 $r^2 + 0.21 + r + 0.25 + 0.15 = 1$   
 $r^2 + r - 0.39 = 0$   
 $100r^2 + 100r - 39 = 0$   
 $(10r + 13)(10r - 3) = 0$   
 $r = -1.3$  (*Rejected*) or  $r = 0.3$  A1 [2]
- (b)  $E(X) = (-2)(r^2 + 0.21) + 0 + (2)(0.25) + (4)(0.15)$  (A1) for correct formula  
 $E(X) = -0.6 + 0 + 0.5 + 0.6$   
 $E(X) = 0.5$  A1 [2]
- (c)  $\text{Var}(Y) = a^2 \text{Var}(X)$  A1  
 $17.4 = 4.35a^2$   
 $a^2 = 4$   
 $a = -2$  (*Rejected*) or  $a = 2$  A1 [2]
- (d) 4.5 A1 [1]
4. (a)  $P(X = 5) + P(X = 10) + P(X = 15) + P(X = 20) = 1$  (A1) for correct formula  
 $0.4 + 0.4 - r + 0.19 + r^2 + r = 1$   
 $r^2 = 0.01$   
 $r = -0.1$  (*Rejected*) or  $r = 0.1$  A1 [2]
- (b)  $E(X) = (5)(0.4) + (10)(0.4 - r)$   
 $+ (15)(0.19 + r^2) + 20r$  (A1) for correct formula  
 $E(X) = 2 + 3 + 3 + 2$   
 $E(X) = 10$  A1 [2]
- (c)  $a = \frac{\sqrt{25}}{1}$  A1  
 $a = 5$  A1 [2]
- (d)  $E(X) = 5E(Y) + 20$   
 $10 = 5E(Y) + 20$  (A1) for substitution  
 $5E(Y) = -10$   
 $E(Y) = -2$  A1 [2]

## Exercise 78

1. (a)  $E(X) = (18)(0.343) + (24)(0.189)$  (A1) for correct formula  
 $+ (33)(0.0265) + (39)(0.4415)$   
 $E(X) = 28.803$  A1 [2]
- (b)  $E(X^2) = (18^2)(0.343) + (24^2)(0.189)$  (A1) for correct formula  
 $+ (33^2)(0.0265) + (39^2)(0.4415)$   
 $E(X^2) = 920.376$  A1 [2]
- (c) The standard deviation of  $X$   
 $= \sqrt{E(X^2) - E(X)^2}$  (A1) for correct formula  
 $= \sqrt{920.376 - 28.803^2}$   
 $= 9.526971764$   
 $= 9.53$  A1 [2]
2. (a)  $E(X) = (4)(0.45) + (9)(0.2) + (16)(0.2) + (25)(0.15)$  (A1) for correct formula  
 $E(X) = 10.55$  A1 [2]
- (b)  $E(Y) = (-\sqrt{4})(0.45) + (-\sqrt{9})(0.2)$  (A1) for correct formula  
 $+ (-\sqrt{16})(0.2) + (-\sqrt{25})(0.15)$   
 $E(Y) = -3.05$  A1 [2]
- (c) The standard deviation of  $Y$   
 $= \sqrt{E(Y^2) - E(Y)^2}$  (A1) for correct formula  
 $= \sqrt{E(X) - E(Y)^2}$   
 $= \sqrt{10.55 - (-3.05)^2}$   
 $= 1.116915395$   
 $= 1.12$  A1 [2]

3. (a)  $P(X = 1) + P(X = 2) + P(X = 3)$   
 $+ P(X = 4) + P(X = 5) + P(X = 6) = 1$  (A1) for correct formula
- $$\frac{1}{42k} + \frac{2}{42k} + \frac{3}{42k} + \frac{4}{42k} + \frac{5}{42k} + \frac{6}{42k} = 1$$
- $$\frac{21}{42k} = 1$$
- $$k = \frac{1}{2}$$
- A1 [2]
- (b)  $E(X) = (1)\left(\frac{1}{21}\right) + (2)\left(\frac{2}{21}\right) + (3)\left(\frac{3}{21}\right)$   
 $+ (4)\left(\frac{4}{21}\right) + (5)\left(\frac{5}{21}\right) + (6)\left(\frac{6}{21}\right)$  (A1) for correct formula
- $$E(X) = \frac{13}{3}$$
- A1 [2]
- (c)  $\text{Var}(X) = E(X^2) - E(X)^2$  (A1) for correct formula
- $$\text{Var}(X) = (1^2)\left(\frac{1}{21}\right) + (2^2)\left(\frac{2}{21}\right) + (3^2)\left(\frac{3}{21}\right)$$
- $$+ (4^2)\left(\frac{4}{21}\right) + (5^2)\left(\frac{5}{21}\right) + (6^2)\left(\frac{6}{21}\right) - \left(\frac{13}{3}\right)^2$$
- (A1) for substitution
- $$\text{Var}(X) = \frac{20}{9}$$
- A1 [3]

4. (a)  $P(X = 3) + P(X = 4) + P(X = 5)$   
 $+ P(X = 6) + P(X = 7) = 1$  (A1) for correct formula

$$\left(\frac{3}{16} - \frac{9}{5k^2}\right) + \left(\frac{4}{16} - \frac{9}{5k^2}\right) + \left(\frac{5}{16} - \frac{9}{5k^2}\right)$$

$$+ \left(\frac{6}{16} - \frac{9}{5k^2}\right) + \left(\frac{7}{16} - \frac{9}{5k^2}\right) = 1$$

$$\frac{25}{16} - \frac{9}{k^2} = 1$$

(M1) for valid approach

$$-\frac{9}{k^2} = -\frac{9}{16}$$

$$k^2 = 16$$

$$k = -4 \text{ (Rejected) or } k = 4$$

A1

[3]

(b)  $\text{Var}(X) = E(X^2) - E(X)^2$  (A1) for correct formula

$$\text{Var}(X) = (3^2)\left(\frac{3}{40}\right) + (4^2)\left(\frac{11}{80}\right)$$

$$+ (5^2)\left(\frac{1}{5}\right) + (6^2)\left(\frac{21}{80}\right) + (7^2)\left(\frac{13}{40}\right)$$

(A2) for substitution

$$- \left[ \left( 3\left(\frac{3}{40}\right) + 4\left(\frac{11}{80}\right) \right. \right. \\ \left. \left. + 5\left(\frac{1}{5}\right) + 6\left(\frac{21}{80}\right) + 7\left(\frac{13}{40}\right) \right)^2 \right]$$

$$\text{Var}(X) = \frac{133}{4} - \left(\frac{45}{8}\right)^2$$

$$\text{Var}(X) = \frac{103}{64}$$

A1

[4]