Exercise 2.1

(a) (i) $P(0)=50$ (A1)

(ii) $P^{-1}(70)=5$ (A1)

(b) $P(50)$

$=0.8(50)^2+50$ $P(50)$ (M1)

$=2050$

Thus, the range of P is $50 \le P \le 2050$, $N \in \mathbb{R}$. (A1)

(c) The price of the journey is 370 dollars when the passenger lives 20 kilometres from the airport. (A1)

(d) $P=0.8a^2+50$

$\rightarrow a=0.8P^2+50$ Interchange a and P (M1)

$a-50=0.8P^2$

$1.25a-62.5=P^2$

$P=\sqrt{1.25a-62.5}$

$\therefore P^{-1}(a)=\sqrt{1.25a-62.5}$ (A1)

(e) Vertical stretch of scale factor 1.2 (A1)
followed by an upward translation by 5 units (A1)

(f) $Q(a)$

$=1.2P(a)+5$

$=1.2(0.8a^2+50)+5$ $1.2(0.8a^2+50)+5$ (M1)

$=0.96a^2+65$ (A1)

Exercise 2.2

(a) Let $P=\frac{k}{A}$, where $k\neq 0$. $P=\frac{k}{A}$ (M1)

$15=\frac{k}{16}$

$k=240$

$\therefore P=\frac{240}{A}$ (A1)

(b) $\$3$ (A1)

(c) The price of a tetrahedron model of a large surface area will approach $\$0$. (A1)

(d) α

$=\frac{14400}{P^2}$

$=\frac{14400}{\left(\frac{240}{A}\right)^2}$ $\frac{14400}{\left(\frac{240}{A}\right)^2}$ (M1)

$=0.25A^2$ (A1)

Exercise 2.3

(a) $19\ \text{cm}^2$ (A1)

(b) (i) $x > 18,\ x \in \mathbb{R}$ (A1)

(ii) $A > 0,\ y \in \mathbb{R}$ (A1)

(c) (i) $A = 55$

$\therefore 0.5x^2 - 9.5x + 9 = 55$

$0.5x^2 - 9.5x - 46 = 0$ Correct equation (A1)

By considering the graph of $y = 0.5x^2 - 9.5x - 46$, the horizontal intercept is 23. GDC approach (M1)

$\therefore x = 23$ (A1)

(ii) The length of the longest side

$= \sqrt{(23-18)^2 + (23-1)^2}$

$= \sqrt{509}$ $\sqrt{509}$ (A1)

The required perimeter

$= (23-18) + (23-1) + \sqrt{509}$ The sum of 3 sides (M1)

$= 49.56102835\ \text{cm}$

$= 49.6\ \text{cm}$ (A1)

Exercise 2.4

(a) $P = P_0 e^{kt}$

$\therefore P_0(1-10\%) = P_0 e^{k(1)}$ — $P_1 = P_0(1-10\%)$ & $t = 1$ (A1)

$0.9 = e^k$

$k = \ln 0.9$ — $b = e^x \Leftrightarrow x = \ln b$ (M1)

$k = -0.1053605157$

$\therefore k = -0.1054$ (A1)

(b) The growth rate is negative. (R1)

(c) $0.5P_0 = P_0 e^{-0.1053605157t}$ — Correct equation (A1)

$0.5 = e^{-0.1053605157t}$

$e^{-0.1053605157t} - 0.5 = 0$

By considering the graph of

$y = e^{-0.1053605157t} - 0.5$, the horizontal intercept is

6.5788135. — GDC approach (M1)

$\therefore$ The least number of complete years is 66. (A1)

(d) t

$= 71 - 18.8\log_{10} 3000$ — $Q = 3000$ (M1)

$= 5.630120411$

$\therefore$ The number of complete years is 6. (A1)

Exercise 2.5

(a) (i) 0.864 (A1)

(ii) The vertical intercept

$= \ln 128$

$= 4.852030264$

$= 4.85203$ (A1)

(b) $\ln A = \ln 128 + 0.864 \ln t$

$\ln A = \ln 128 + \ln t^{0.864}$ $n \ln a = \ln a^n$ (M1)

$\ln A = \ln(128t^{0.864})$ $\ln a + \ln b = \ln(ab)$ (M1)

$A = 128t^{0.864}$ (A1)

(c) The required area

$= 128(7)^{0.864}$ $t = 7$ (M1)

$= 687.6614461 \text{ mm}^2$

$= 688 \text{ mm}^2$ (A1)

Exercise 2.6

(a) The gradient

$=\frac{1-(-9)}{-8-0}$ $m=\frac{y_2-y_1}{x_2-x_1}$ (M1)

$=-\frac{5}{4}$ (A1)

(b) (i) $(-4,-4)$ (A1)

(ii) The exact distance

$=\sqrt{(-4-(-8))^2+(-4-1)^2}$ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (M1)

$=\sqrt{41}$ (A1)

(c) The required slope

$=-1\div-\frac{5}{4}$ $m_1\times m_2=-1$ (M1)

$=\frac{4}{5}$

The equation:

$y-1=\frac{4}{5}(x-(-8))$ $y-y_1=m(x-x_1)$ (M1)

$5(y-1)=4(x+8)$

$5y-5=4x+32$

$4x-5y+37=0$ (A1)