

# Solución de Práctica de Prueba 3 de AE NS Set 3

1. (a)  $r_1 + r_2 = -\frac{a_1}{1}$  (A1) por sustitución  
 $a_1 = -r_1 - r_2$  A1  
 $r_1 r_2 = \frac{a_0}{1}$  (A1) por sustitución  
 $a_0 = r_1 r_2$  A1

[4]

(b) (i)  $a_1$   
 $= -r_1 - r_2$   
 $= -(r_1 + r_2)$   
 $= -S_1$  A1

(ii)  $\frac{S_1^2 - S_2}{2}$   
 $= \frac{(r_1 + r_2)^2 - (r_1^2 + r_2^2)}{2}$   
 $= \frac{r_1^2 + 2r_1 r_2 + r_2^2 - r_1^2 - r_2^2}{2}$  M1A1  
 $= \frac{2r_1 r_2}{2}$   
 $= r_1 r_2$  A1  
 $= a_0$

$\therefore a_0 = \frac{S_1^2 - S_2}{2}$  AG

(c) (i)  $a_2 = -S_1$  A1

[4]

$$\begin{aligned}
\text{(ii)} \quad & \frac{S_1^2 - S_2}{2} \\
&= \frac{(r_1 + r_2 + r_3)^2 - (r_1^2 + r_2^2 + r_3^2)}{2} \\
&= \frac{r_1^2 + r_1r_2 + r_1r_3 + r_1r_2 + r_2^2 + r_2r_3}{2} \\
&= \frac{+r_1r_3 + r_2r_3 + r_3^2 - r_1^2 - r_2^2 - r_3^2}{2} \quad \text{M1A1} \\
&= \frac{2r_1r_2 + 2r_1r_3 + 2r_2r_3}{2} \\
&= r_1r_2 + r_1r_3 + r_2r_3 \quad \text{R1} \\
&= a_1 \\
&\therefore a_1 = \frac{S_1^2 - S_2}{2} \text{ es cierto.} \quad \text{A1}
\end{aligned}$$

[5]

$$\begin{aligned}
\text{(d)} \quad & ka_0 = S_1^3 - 3S_1S_2 + 2S_3 \\
& k(-r_1r_2r_3) = (r_1 + r_2 + r_3)^3 \\
& -3(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) + 2(r_1^3 + r_2^3 + r_3^3) \quad \text{(A1) por enfoque correcto} \\
& -kr_1r_2r_3 = (r_1 + r_2 + r_3)(r_1 + r_2 + r_3)^2 \\
& -3(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) + 2(r_1^3 + r_2^3 + r_3^3) \\
& -kr_1r_2r_3 \\
& = (r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2 + 2r_1r_2 + 2r_1r_3 + 2r_2r_3) \quad \text{M1A1} \\
& -3(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) + 2(r_1^3 + r_2^3 + r_3^3) \\
& \text{El coeficiente de } r_1r_2r_3 \text{ en el lado derecho} \\
& = 2 + 2 + 2 \quad \text{A1} \\
& = 6 \\
& \therefore k = -6 \quad \text{A1}
\end{aligned}$$

[5]

$$\begin{aligned}
\text{(e)} \quad & a_{n-1} = -S_1 \quad \text{A1} \\
& a_{n-2} = \frac{S_1^2 - S_2}{2} \quad \text{A1} \\
& a_{n-3} = -\frac{1}{6}S_1^3 + \frac{1}{2}S_1S_2 - \frac{1}{3}S_3 \quad \text{A1}
\end{aligned}$$

[3]

$$(f) \begin{cases} u + v + w = 14 \\ u^2 + v^2 + w^2 = 86 \\ u^3 + v^3 + w^3 = 560 \end{cases}$$

Sean  $S_1 = 14$ ,  $S_2 = 86$  y  $S_3 = 560$ .

(M1) por enfoque válido

$u$ ,  $v$  y  $w$  son las raíces de la ecuación

$x^3 + a_2x^2 + a_1x + a_0 = 0$ , donde  $a_2 = -S_1$ ,

$$a_1 = \frac{S_1^2 - S_2}{2} \text{ y } a_0 = -\frac{1}{6}S_1^3 + \frac{1}{2}S_1S_2 - \frac{1}{3}S_3. \quad \text{A1}$$

$$a_2 = -14 \quad \text{A1}$$

$$a_1 = \frac{14^2 - 86}{2}$$

$$a_1 = 55 \quad \text{A1}$$

$$a_0 = -\frac{1}{6}(14)^3 + \frac{1}{2}(14)(86) - \frac{1}{3}(560)$$

$$a_0 = -42 \quad \text{A1}$$

Por lo tanto,  $u$ ,  $v$  y  $w$  son las raíces de la

ecuación  $x^3 - 14x^2 + 55x - 42 = 0$ . R1

Considerando la gráfica de

$$y = x^3 - 14x^2 + 55x - 42, \quad x = 1, \quad x = 6 \text{ o } x = 7.$$

$$\therefore u = 1, \quad v = 6, \quad w = 7 \quad \text{A3}$$

[9]

2. (a)  $\cos(A+B)x + \cos(A-B)x$   
 $\cos Ax \cos Bx - \text{sen } Ax \text{ sen } Bx$  A2  
 $+ \cos Ax \cos Bx + \text{sen } Ax \text{ sen } Bx$  AG  
 $= 2 \cos Ax \cos Bx$

[2]

(b)  $\int_0^\pi \cos Ax \cos Bx dx$   
 $= \frac{1}{2} \int_0^\pi (\cos(A+B)x + \cos(A-B)x) dx$  (A1) por sustitución  
 $= \frac{1}{2} \left[ \frac{1}{A+B} \text{sen}(A+B)x + \frac{1}{A-B} \text{sen}(A-B)x \right]_0^\pi$  A1  
 $= \frac{1}{2} \left[ \left( \frac{1}{A+B} \text{sen}(A+B)\pi + \frac{1}{A-B} \text{sen}(A-B)\pi \right) \right]$  M1  
 $\quad \left[ - \left( \frac{1}{A+B} \text{sen } 0 + \frac{1}{A-B} \text{sen } 0 \right) \right]$   
 $= 0$  A1

[4]

(c) (i)  $\frac{1}{z}$   
 $= \frac{1}{\cos \theta + i \text{sen } \theta}$   
 $= \frac{\cos \theta - i \text{sen } \theta}{(\cos \theta + i \text{sen } \theta)(\cos \theta - i \text{sen } \theta)}$  M1  
 $= \frac{\cos \theta - i \text{sen } \theta}{\cos^2 \theta - i^2 \text{sen}^2 \theta}$  A1  
 $= \frac{\cos \theta - i \text{sen } \theta}{\cos^2 \theta + \text{sen}^2 \theta}$   
 $= \cos \theta - i \text{sen } \theta$  A1

(ii)  $z + \frac{1}{z} = (\cos \theta + i \text{sen } \theta) + (\cos \theta - i \text{sen } \theta)$  (M1) por enfoque válido  
 $z + \frac{1}{z} = 2 \cos \theta$   
 $\cos \theta = \frac{1}{2} \left( z + \frac{1}{z} \right)$  A1

[5]

(d)  $\cos^3 \theta$

$$= \left(\frac{1}{2}\right)^3 \left(z + \frac{1}{z}\right)^3$$

$$= \frac{1}{8} \left( z^3 + \binom{3}{1} z^2 \cdot \frac{1}{z} + \binom{3}{2} z \cdot \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 \right) \quad \text{M1A1}$$

$$= \frac{1}{8} \left( \frac{\cos 3\theta + i \operatorname{sen} 3\theta + 3 \cos \theta + 3i \operatorname{sen} \theta}{\cos \theta + i \operatorname{sen} \theta} + \frac{1}{\cos 3\theta + i \operatorname{sen} 3\theta} \right) \quad \text{A1}$$

$$= \frac{1}{8} \left( \frac{\cos 3\theta + i \operatorname{sen} 3\theta + 3 \cos \theta + 3i \operatorname{sen} \theta}{+3(\cos \theta - i \operatorname{sen} \theta) + \cos 3\theta - i \operatorname{sen} 3\theta} \right) \quad \text{A1}$$

$$= \frac{1}{8} (2 \cos 3\theta + 6 \cos \theta)$$

$$= \frac{1}{4} (\cos 3\theta + 3 \cos \theta) \quad \text{AG}$$

[4]

(e)  $\cos^n \theta$

$$= \left(\frac{1}{2}\right)^n \left(z + \frac{1}{z}\right)^n$$

$$= \frac{1}{2^n} \left( z^n + \binom{n}{1} z^{n-1} \cdot \frac{1}{z} + \binom{n}{2} z^{n-2} \cdot \left(\frac{1}{z}\right)^2 \right. \quad \text{M1A1}$$

$$\left. + \dots + \binom{n}{n-1} z \cdot \left(\frac{1}{z}\right)^{n-1} + \left(\frac{1}{z}\right)^n \right)$$

$$= \frac{1}{2^n} \left( z^n + \binom{n}{1} z^{n-2} + \binom{n}{2} z^{n-4} \right. \quad \text{M1}$$

$$\left. + \dots + \binom{n}{n-1} \frac{1}{z^{n-2}} + \frac{1}{z^n} \right)$$

$$= \frac{1}{2^n} \sum_{r=0}^n \binom{n}{r} \cos(n-2r)\theta \quad \text{A2}$$

[5]

$$\begin{aligned}
\text{(f)} \quad & \int_0^\pi \cos 6x \cos^5 x dx \\
& = \int_0^\pi \cos 6x \left( \frac{1}{2^5} \sum_{r=0}^5 \binom{5}{r} \cos(5-2r)x \right) dx && \text{(A1) por sustitución} \\
& = \frac{1}{32} \int_0^\pi \cos 6x \left( \cos 5x + 5 \cos 3x + 10 \cos x \right. \\
& \quad \left. + 10 \cos(-x) + 5 \cos(-3x) + \cos(-5x) \right) dx && \text{M1} \\
& = \frac{1}{32} \int_0^\pi \cos 6x \left( \cos 5x + 5 \cos 3x + 10 \cos x \right. \\
& \quad \left. + 10 \cos x + 5 \cos 3x + \cos 5x \right) dx && \text{A1} \\
& = \frac{1}{32} \int_0^\pi \cos 6x (2 \cos 5x + 10 \cos 3x + 20 \cos x) dx \\
& = \frac{1}{32} \int_0^\pi 2 \cos 6x \cos 5x dx + \frac{1}{32} \int_0^\pi 10 \cos 6x \cos 3x dx \\
& \quad + \frac{1}{32} \int_0^\pi 20 \cos 6x \cos x dx \\
& = \frac{1}{16} \int_0^\pi \cos 6x \cos 5x dx + \frac{5}{16} \int_0^\pi \cos 6x \cos 3x dx && \text{A1} \\
& \quad + \frac{5}{8} \int_0^\pi \cos 6x \cos x dx \\
& = \frac{1}{16} (0) + \frac{5}{16} (0) + \frac{5}{8} (0) \\
& = 0 && \text{A1}
\end{aligned}$$

[5]