Chapter 18 Solution

Exercise 71

1. (a) (i) The required probability

$$= \frac{3+2+4+3}{20}$$ (A1) for correct formula

$$= \frac{3}{5}$$ A1 N2

(ii) The required probability

$$= \frac{3+5}{3+3+5}$$ (A1) for correct formula

$$= \frac{8}{11}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{3+2+3+3}{20}\right)\left(\frac{3+2+3+3-1}{20-1}\right)$$ (A2) for correct formula

$$= \left(\frac{11}{20}\right)\left(\frac{10}{19}\right)$$

$$= \frac{11}{38}$$ A1 N3

[3]

2. (a) (i) The required probability

$$= \frac{2+10+3+5+10}{50}$$ (A1) for correct formula

$$= \frac{3}{5}$$ A1 N2

(ii) The required probability

$$= \frac{3+5+10}{10+3+5+10}$$ (A1) for correct formula

$$= \frac{9}{14}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{5+10}{50}\right)\left(\frac{5+10-1}{50-1}\right)$$ (A2) for correct formula

$$= \left(\frac{15}{50}\right)\left(\frac{14}{49}\right)$$

$$= \frac{3}{35}$$ A1 N3

[3]

3. (a) (i) The required probability

$$= \frac{2+1+5+3+4+2+1}{25}$$ (A1) for correct formula

$$= \frac{18}{25}$$ A1 N2

(ii) The required probability

$$= \frac{5}{1+5+2}$$ (A1) for correct formula

$$= \frac{5}{8}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{4+2+1}{25}\right)\left(\frac{4+2+1-1}{25-1}\right)$$ (A2) for correct formula

$$= \left(\frac{7}{25}\right)\left(\frac{6}{24}\right)$$

$$= \frac{7}{100}$$ A1 N3

[3]

4. (a) $\dfrac{5+15+a}{100}=\dfrac{6}{25}$ (M1) for setting equation

$20+a=24$

$a=4$ A1

$5+15+4+\cdots+15+b=100$

$b=6$ A1 N3

[3]

(b) The required probability

$=\dfrac{15+4+10+10+15+6}{15+4+5+5+10+10+15+6}$ (A1) for correct formula

$=\dfrac{6}{7}$ A1 N2

[2]

(c) The required probability

$=\left(\dfrac{6}{100}\right)\left(\dfrac{6-1}{100-1}\right)$ (A2) for correct formula

$=\left(\dfrac{6}{100}\right)\left(\dfrac{5}{99}\right)$

$=\dfrac{1}{330}$ A1 N3

[3]

Exercise 72

1. (a) (i) $a+9=13$ — (M1) for valid approach
$a=4$ — A1 N2

(ii) $21+4+b=30$ — (M1) for valid approach
$b=5$ — A1 N2

[4]

(b) The required probability
$=\frac{4}{30}$ — (M1) for valid approach
$=\frac{2}{15}$ — A1 N2

[2]

2. (a) (i) $17+15-h+10=40$ — (M1) for valid approach
$h=2$ — A1 N2

(ii) $2+k=15$ — (M1) for valid approach
$k=13$ — A1 N2

[4]

(b) The required probability
$=\frac{2}{40}$ — (M1) for valid approach
$=\frac{1}{20}$ — A1 N2

[2]

3. (a) (i) $p=0.4$ — A1 N1

(ii) $0.4+q=0.6$ — (M1) for valid approach
$q=0.2$ — A1 N2

[3]

(b) $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ — (M1) for valid approach
$0.9=\mathrm{P}(A)+0.6-0.4$ — (A1) for substitution
$\mathrm{P}(A)=0.7$ — A1 N3

[3]

4. (a) (i) $a = 0.3$ A1 N1

(ii) $0.3 + b = 1 - 0.6$ (M1) for valid approach
$b = 0.1$ A1 N2

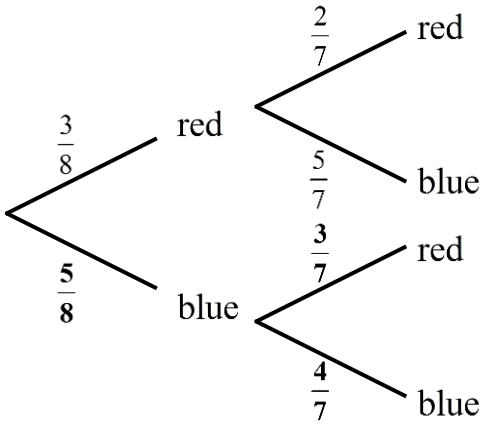
[3]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach
$1 - 0.3 = 0.6 + 0.2 - \mathrm{P}(A \cap B)$ (A1) for substitution
$\mathrm{P}(A \cap B) = 0.1$ A1 N3

[3]

Exercise 73

1. (a)



A3 N3

[3]

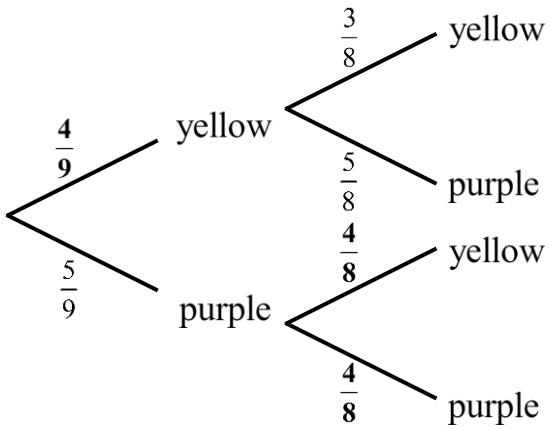
(b) The required probability

$$=\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)+\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$$ (M1)(A1) for correct formula

$$=\frac{15}{28}$$ A1 N3

[3]

2. (a)



A3 N3

[3]

(b) The required probability

$=\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)+\frac{5}{9}$ (M1)(A1) for correct formula

$=\frac{5}{6}$ A1 N3

[3]

3. (a) $x=\frac{5}{8}$ A1 N1

[1]

(b) $\mathrm{P}(B)=\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)+\left(\frac{5}{8}\right)\left(\frac{2}{5}\right)$ (M1)(A1) for correct formula

$\mathrm{P}(B)=\frac{13}{40}$ A1 N3

[3]

(c) $\mathrm{P}(A\,|\,B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A\,|\,B)=\frac{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}{\frac{13}{40}}$ (A1) for substitution

$\mathrm{P}(A\,|\,B)=\frac{3}{13}$ A1 N3

[3]

4. (a) $\mathrm{P}(B \mid A') = \frac{3}{5}$ A1 N1

[1]

(b) $\mathrm{P}(B) = \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{5}\right)$ (M1)(A1) for correct formula

$\mathrm{P}(B) = \frac{2}{3}$ A1 N3

[3]

(c) $\mathrm{P}(A' \mid B) = \frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A' \mid B) = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)}{\frac{2}{3}}$ (A1) for substitution

$\mathrm{P}(A' \mid B) = \frac{3}{5}$ A1 N3

[3]

Exercise 74

1. (a) $P(A) = P(A \cap B) + P(A \cap B')$ (M1) for valid approach

$P(A) = 0.08 + 0.12$

$P(A) = 0.2$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.08 = 0.2 \times P(B)$ (A1) for substitution

$P(B) = 0.4$ A1 N2

[2]

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.2 + 0.4 - 0.08$ (A1) for substitution

$P(A \cup B) = 0.52$ A1 N2

[2]

2. (a) $P(B) = P(A \cap B) + P(A' \cap B)$ (M1) for valid approach

$0.3 = P(A \cap B) + 0.15$

$P(A \cap B) = 0.15$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.15 = P(A) \times 0.3$ (A1) for substitution

$P(A) = 0.5$ A1 N2

[2]

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.5 + 0.3 - 0.15$ (A1) for substitution

$P(A \cup B) = 0.65$ A1 N2

[2]

3. (a) $P(A) = P(A \cap B) + P(A \cap B')$ (M1) for valid approach

$0.4 = P(A \cap B) + 0.28$

$P(A \cap B) = 0.12$ A1 N2

[2]

(b) $P(A \cap B) = P(A) \times P(B)$

$0.12 = 0.4 \times P(B)$ (A1) for substitution

$P(B) = 0.3$ (A1) for correct value

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.4 + 0.3 - 0.12$ (A1) for substitution

$P(A \cup B) = 0.58$ A1 N4

[4]

4. (a) $P(A \cap B) = P(A) \times P(B)$ (M1) for valid approach

$0.21 = 0.7P(B)$

$P(B) = 0.3$ A1 N2

[2]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$P(A \cup B) = 0.7 + 0.3 - 0.21$ A1

$P(A \cup B) = 0.79$ AG N0

[2]

(c) $P(A' \cap B') = 1 - P(A \cup B)$ (M1) for valid approach

$P(A' \cap B') = 1 - 0.79$ (A1) for substitution

$P(A' \cap B') = 0.21$ A1 N3

[3]

Exercise 75

1. (a) $\mathrm{P}(C \cap D) = \mathrm{P}(C) \times \mathrm{P}(D)$

$\mathrm{P}(C \cap D) = 2k^2 \times 3k^2$ (A1) for substitution

$\mathrm{P}(C \cap D) = 6k^4$ A1 N2

[2]

(b) $6k^4 = 0.0096$ (A1) for correct equation

$k^4 = 0.0016$

$k = 0.2$ A1 N2

[2]

(c) $\mathrm{P}(C) = \mathrm{P}(C \cap D) + \mathrm{P}(C \cap D')$

$2(0.2)^2 = 6(0.2)^4 + \mathrm{P}(C \cap D')$ (A1) for substitution

$\mathrm{P}(C \cap D') = 0.0704$ A1 N2

[2]

(d) $\mathrm{P}(D' \mid C) = \dfrac{\mathrm{P}(D' \cap C)}{\mathrm{P}(C)}$

$\mathrm{P}(D' \mid C) = \dfrac{0.0704}{2(0.2)^2}$ (A1) for substitution

$\mathrm{P}(D' \mid C) = 0.88$ A1 N2

[2]

2. (a) $\mathrm{P}(E \cap F) = \mathrm{P}(E) \times \mathrm{P}(F)$

$\mathrm{P}(E \cap F) = 4k^3 \times k$ (A1) for substitution

$\mathrm{P}(E \cap F) = 4k^4$ A1 N2

[2]

(b) $4k^4 = \dfrac{1}{2500}$ (A1) for correct equation

$k^4 = \dfrac{1}{10000}$

$k = \dfrac{1}{10}$ A1 N2

[2]

(c) $\mathrm{P}(E \cup F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F)$

$\mathrm{P}(E \cup F) = 4\left(\dfrac{1}{10}\right)^3 + \dfrac{1}{10} - 4\left(\dfrac{1}{10}\right)^4$ (A1) for substitution

$\mathrm{P}(E \cup F) = \dfrac{259}{2500}$ A1 N2

[2]

3. (a) $P(A \cap B) = P(A) \times P(B)$ M1

$P(A \cap B) = 2k \times 1.5(2k)$ A1

$P(A \cap B) = 6k^2$ AG N0

[2]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$6k - 1 = 2k + 1.5(2k) - 6k^2$ A1

$6k - 1 = 5k - 6k^2$ A1

$6k^2 + k - 1 = 0$ AG N0

[2]

(c) $6k^2 + k - 1 = 0$

$(3k - 1)(2k + 1) = 0$ (A1) for factorization

$k = \frac{1}{3}$ or $k = -\frac{1}{2}$ (*Rejected*) A1 N2

[2]

(d) $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$P(B \mid A) = \frac{6\left(\frac{1}{3}\right)^2}{2\left(\frac{1}{3}\right)}$ (A1) for substitution

$P(B \mid A) = 1$ A1 N2

[2]

4. (a) $P(A \cap B) = P(A) \times P(B)$

$P(A \cap B) = (x)(3x)$ A1

$P(A \cap B) = 3x^2$ A1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$0.93 = x + 3x - 3x^2$ A1

$3x^2 - 4x + 0.93 = 0$

$300x^2 - 400x + 93 = 0$ AG N0

[4]

(b) $300x^2 - 400x + 93 = 0$

$(30x - 31)(10x - 3) = 0$ (A1) for factorization

$x = \frac{31}{30}$ (*Rejected*) or $x = \frac{3}{10}$ (A1) for correct value

$P(B) = 3\left(\frac{3}{10}\right) = \frac{9}{10}$ A1 N3

[3]

Exercise 76

1. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 = 0.4 + 0.65 - P(A \cap B)$ (A1) for substitution

$P(A \cap B) = 0.05$ A1 N2

[2]

(b) $P(A \cap B) + P(A' \cap B) = P(B)$ (M1) for valid approach

$0.05 + P(A' \cap B) = 0.65$

$P(A' \cap B) = 0.6$ A1 N2

[2]

(c) (i) $P(A \cap C) = P(A \mid C) \times P(C)$ (M1) for valid approach

$P(A \cap C) = 0.78 \times 0.7$ (A1) for substitution

$P(A \cap C) = 0.546$ A1 N3

(ii) $P(A \cap C) + P(A' \cap C) = P(C)$ (M1) for valid approach

$0.546 + P(A' \cap C) = 0.7$

$P(A' \cap C) = 0.154$ A1 N2

(iii) $P(A) + P(A') = 1$

$0.4 + P(A') = 1$ (A1) for substitution

$P(A') = 0.6$ A1 N2

(iv) $P(C \mid A') = \dfrac{P(C \cap A')}{P(A')}$

$P(C \mid A') = \dfrac{0.154}{0.6}$ (A1) for substitution

$P(C \mid A') = 0.256666666$

$P(C \mid A') = 0.257$ A1 N2

[9]

2. (a) $\mathrm{P}(A \cup T) = \mathrm{P}(A) + \mathrm{P}(T) - \mathrm{P}(A \cap T)$

$1 = 0.55 + 0.7 - \mathrm{P}(A \cap T)$ (A1) for substitution

$\mathrm{P}(A \cap T) = 0.25$

Thus, the required percentage is 25%. A1 N2

[2]

(b) $\mathrm{P}(A \cup T) - \mathrm{P}(A \cap T)$ (M1) for valid approach

$= 1 - 0.25$

$= 0.75$

Thus, the required percentage is 75%. A1 N2

[2]

(c) (i) The required probability

$= \mathrm{P}(M \cap A)$

$= \mathrm{P}(A \mid M) \times \mathrm{P}(M)$ (M1) for valid approach

$= 0.72 \times 0.63$ (A1) for substitution

$= 0.4536$ A1 N3

(ii) $\mathrm{P}(M \cap A) + \mathrm{P}(M' \cap A) = \mathrm{P}(A)$ (M1) for valid approach

$0.4536 + \mathrm{P}(M' \cap A) = 0.55$

$\mathrm{P}(M' \cap A) = 0.0964$

Thus, the required probability is 0.0964. A1 N2

(iii) $\mathrm{P}(M) + \mathrm{P}(M') = 1$

$0.63 + \mathrm{P}(M') = 1$ (A1) for substitution

$\mathrm{P}(M') = 0.37$

Thus, the required probability is 0.37. A1 N2

(iv) The required probability

$= \mathrm{P}(A \mid M')$

$= \dfrac{\mathrm{P}(A \cap M')}{\mathrm{P}(M')}$

$= \dfrac{0.0964}{0.37}$ (A1) for substitution

$= 0.26054054$

$= 0.261$ A1 N2

[9]

3. (a) $\mathrm{P}(F \cup R) = \mathrm{P}(F) + \mathrm{P}(R) - \mathrm{P}(F \cap R)$

$1 = 0.85 + 0.45 - \mathrm{P}(F \cap R)$ (A1) for substitution

$\mathrm{P}(F \cap R) = 0.3$

Thus, the required percentage is 30%. A1 N2

[2]

(b) $\mathrm{P}(F \cap R) + \mathrm{P}(F' \cap R) = \mathrm{P}(R)$ (M1) for valid approach

$0.3 + \mathrm{P}(F' \cap R) = 0.45$

$\mathrm{P}(F' \cap R) = 0.15$

Thus, the required percentage is 15%. A1 N2

[2]

(c) $\mathrm{P}(F \cup R) - \mathrm{P}(F \cap R) = 1 - 0.3$ (M1) for valid approach

$\mathrm{P}(F \cup R) - \mathrm{P}(F \cap R) = 0.7$

Thus, the required percentage is 70%. A1 N2

[2]

(d) (i) The required probability

$= \mathrm{P}(R \mid F)$

$= \dfrac{\mathrm{P}(R \cap F)}{\mathrm{P}(F)}$

$= \dfrac{0.3}{0.85}$ (M1) for substitution

$= 0.352941176$

$= 0.353$ A1 N2

(ii) The required probability

$= \dfrac{0.15}{0.7}$ (M1) for substitution

$= 0.214285714$

$= 0.214$ A1 N2

[4]

(e) (i) 54% A1 N1

(ii) $\mathrm{P}(F \cap T) + \mathrm{P}(F \cap T') = \mathrm{P}(F)$ (M1) for valid approach

$0.54 + \mathrm{P}(F \cap T') = 0.85$

$\mathrm{P}(F \cap T') = 0.31$

Thus, the required percentage is 31%. A1 N2

[3]

4. (a) $\mathrm{P}(F \cup T) = \mathrm{P}(F) + \mathrm{P}(T) - \mathrm{P}(F \cap T)$

$1 - 0.25 = 0.35 + 0.5 - \mathrm{P}(F \cap T)$ (A1) for substitution

$\mathrm{P}(F \cap T) = 0.1$

Thus, the required percentage is 10%. A1 N2

[2]

(b) $\mathrm{P}(F \cap T) + \mathrm{P}(F' \cap T) = \mathrm{P}(T)$ (M1) for valid approach

$0.1 + \mathrm{P}(F' \cap T) = 0.5$

$\mathrm{P}(F' \cap T) = 0.4$

Thus, the required percentage is 40%. A1 N2

[2]

(c) 75% A1 N1

[1]

(d) (i) The required probability

$= \dfrac{0.4}{0.75}$ (A1) for substitution

$= 0.533333333$

$= 0.533$ A1 N2

(ii) The required probability

$= \mathrm{P}(Q \mid T)$

$= \dfrac{\mathrm{P}(Q \cap T)}{\mathrm{P}(T)}$

$= \dfrac{0.1}{0.5}$ (A1) for substitution

$= 0.2$ A1 N2

[4]

(e) (i) 38% A1 N1

(ii) $\mathrm{P}(T \cap G) + \mathrm{P}(T \cap G') = \mathrm{P}(T)$ (M1) for valid approach

$0.38 + \mathrm{P}(T \cap G') = 0.5$

$\mathrm{P}(T \cap G') = 0.12$

Thus, the required percentage is 12%. A1 N2

[3]