Applications and Interpretation Higher Level for IBDP Mathematics Practice Paper Set 1 – Paper 3 (60 Minutes)

Question – Answer Book

Instructions

1. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

2. A graphic display calculator is needed.

3. You are suggested to prepare a formula booklet of Applications and Interpretation for IBDP Mathematics when attempting the questions.

4. Supplementary answer sheets and graph papers will be supplied on request.

5. Unless otherwise specified, **ALL** working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

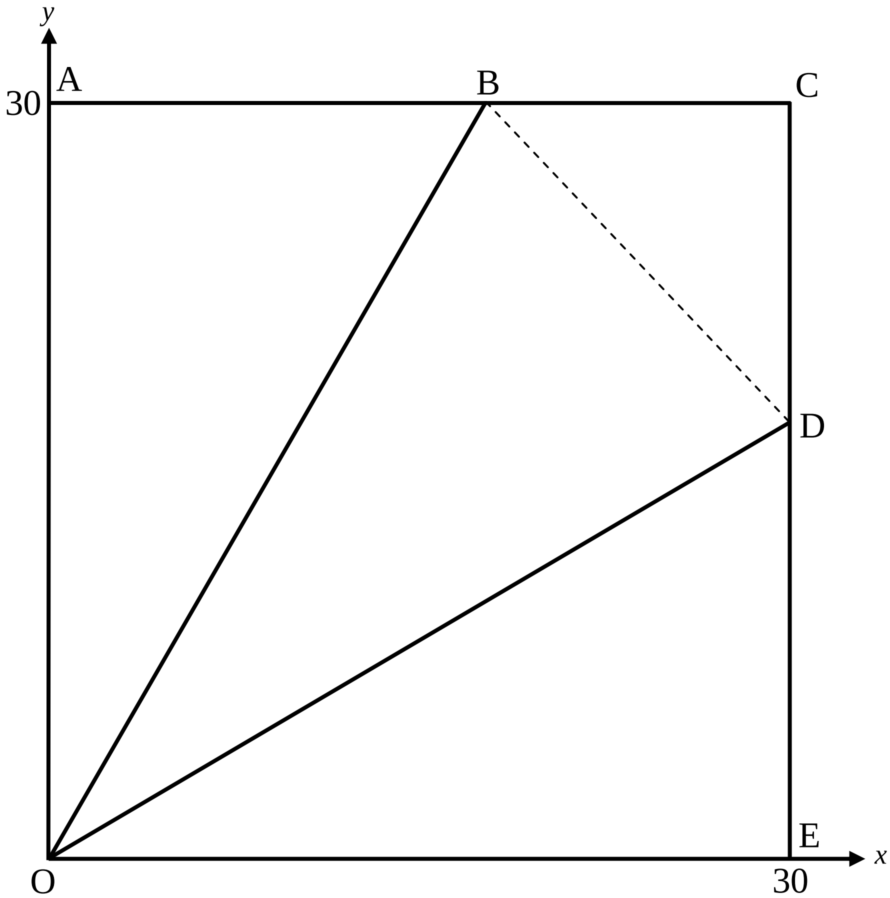
7. The diagrams in this paper are **NOT** necessarily drawn to scale.

8. Information to be read before you start the exam:

	Marker's Use Only	**Examiner's Use Only**	
Question Number	**Marks**	**Marks**	**Maximum Mark**
1			**28**
2			**27**
Overall			
Paper 3 Total			**55**

1. This question aims at investigating the design of a square farm by Voronoi diagrams and graph theory.

The diagram below shows the Voronoi diagram of a square farm OABCDE bounded by the coordinate axes, the lines $x=30$ and $y=30$, where 1 unit represents $1\,\mathrm{m}$.



The owner of the farm wishes to create two roads OB and OD to let his car to travel in the farm. He first drafts the directions of the roads such that $\mathrm{A\hat{O}B}=\mathrm{B\hat{O}D}=\mathrm{D\hat{O}E}=\frac{\pi}{6}\,\mathrm{rad}$. It is given that ABC and CDE are straight lines.

Let $\text{area of OAB} : \text{area of OBCD} : \text{area of ODE} = 1 : r : 1$.

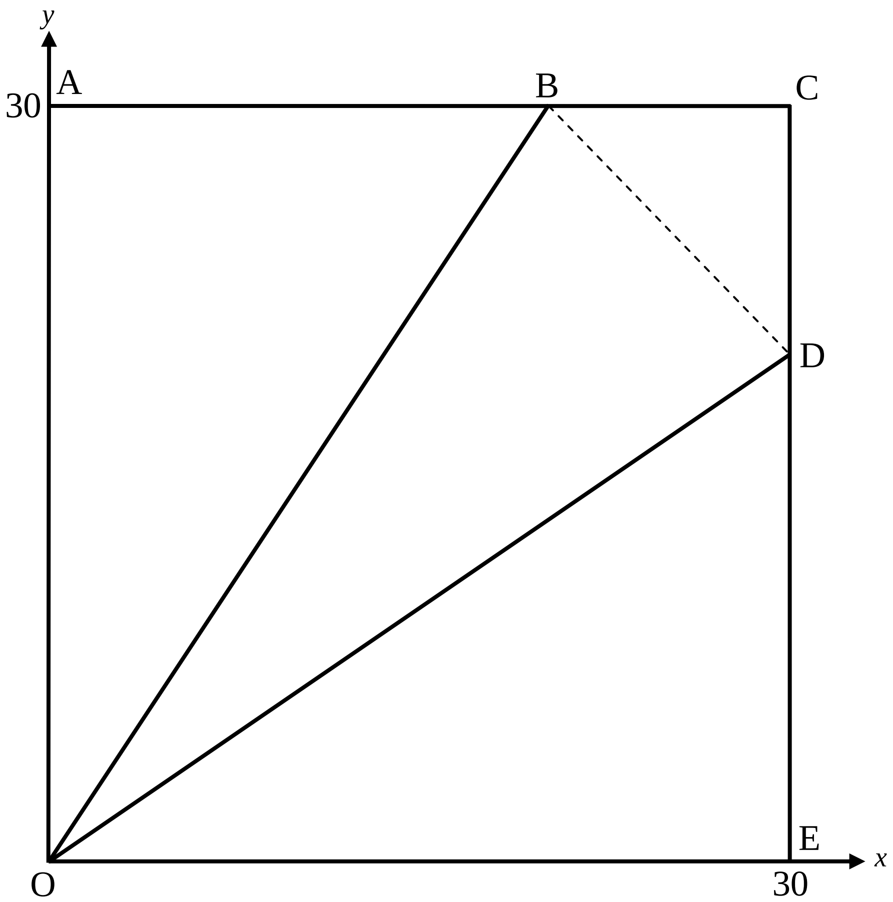
(a) (i) Find DE.

(ii) Show that the area of the triangle ODE is $260\,\mathrm{m}^2$.

(iii) Hence, write down r.

[4]

Consider the case when $r=1$.



(b) (i) Find DE.

(ii) Hence, find $\mathrm{D\hat{O}E}$ in radians.

(iii) Write down $\mathrm{B\hat{O}D}$ in radians.

[5]

The straight line BD is the boundary of the Voronoi cells of $\mathrm{C}(30, 30)$ and F, where F is the site of the region OBD.

(c) (i) State the geometric relationship between BD and CF.

(ii) Find the coordinates of the mid-point of BD.

(iii) Hence, write down the coordinates of F.

[5]

The owner can walk along OA, ABC, CDE, OE, OB, OD and BD. One scarecrow is placed at each of the six positions O, A, B, C, D and E.

(d) Write down the adjacency matrix $\mathbf{M}$ of the graph.

[3]

(e) Hence, find the total number of walks of length at most 3 from C to E.

[2]

The following table shows the least weight of a path connecting any two vertices.

	O	A	B	C	D	E
O	-	30	36.1	p	36.1	30
A	30	-	20	30	34.1	q
B	36.1	20	-	10	14.1	34.1
C	p	30	10	-	10	30
D	36.1	34.1	14.1	10	-	20
E	30	q	34.1	30	20	-

(f) Write down the value of

(i) p;

(ii) q.

[2]

(g) Using the nearest neighbour algorithm, starting and finishing at O, find an upper bound of the total distance of a cycle that passes through all six positions of scarecrows.

[3]

(h) Using the deleted vertex algorithm by deleting the vertex C, find a lower bound of the total distance of a cycle that passes through all six positions of scarecrows, giving the answer correct to one decimal place.

[4]

2. This question aims at investigating the results of a Mathematics mock examination.

Three hundred students attended a Mathematics mock examination. The following table shows the distribution of their final grades, where 1 represents the lowest grade and 7 represents the highest grade.

Grade	1	2	3	4	5	6	7
Frequency	12	27	58	103	45	35	20

The grades of two students are randomly selected.

(a) (i) Find the probability that both grades are either 5, 6 or 7.

(ii) Given that the probability that both grades are either 5, 6 or 7, find the probability that both grades are the same.

[5]

The organizer of the mock examination claims that 18% of the students attending the mock examination takes the Mathematics Higher level course. A sample of 25 students are interviewed, and 7 of them takes the Mathematics Higher level course.

A hypothesis test is conducted at a 5% significance level to test whether there are actually more than 18% of the students attending the mock examination takes the Mathematics Higher level course.

(b) (i) Write down the null hypothesis of the test.

(ii) Write down the alternative hypothesis of the test.

(iii) Find the p-value.

(iv) Hence, state the conclusion of the test with a reason.

[6]

The following table shows the distribution of the actual scores:

Score (x)	$0 \le x \le 20$	$20 < x \le 40$	$40 < x \le 60$	$60 < x \le 80$	$80 < x \le 100$
Observed Frequency	20	72	140	45	23
Expected Frequency	21.7	77.4	116.7	$84.2 - f$	f

(c) Write down the unbiased estimates for the population

(i) mean;

(ii) standard deviation;

(iii) variance.

[3]

A χ^2 goodness of fit test is conducted at a 5% significance level to determine whether the examination score can be modelled by a normal distribution with parameters values evaluated in (c).

(d) (i) Write down the null hypothesis of the test.

(ii) Write down f.

(iii) Hence, write down the degree of freedom of the test.

(iv) Find the p-value.

(v) Hence, state the conclusion of the test with a reason.

[7]

The organizer starts an online system such that students who are interested in participating in the mock examination in the following month can reserve the quota. The number of students, X, reserving the quota follows a Poisson distribution with parameter λ per hour.

A hypothesis test is conducted at a particular significance level to test whether λ is less than 11.

(e) (i) Write down the null hypothesis of the test.

(ii) Write down the alternative hypothesis of the test.

[2]

The null hypothesis is rejected if it is observed that at most five students reserving the quotas in a particular hour.

(f) Find the probability that a Type I error is made.

[2]

The actually value of λ is 7.

(g) Find the probability that a Type II error is made.

[2]

END OF PAPER