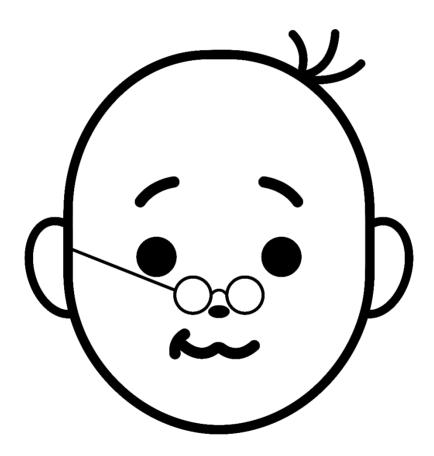
## **Your Intensive Notes Applications and Interpretation Standard Level** for IBDP Mathematics



## Algebra







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### **Topics Covered**

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### **Important Notes**

Standard form: A number in the format  $\pm a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$  (*k* is an integer)

Notes on GDC			
TEXAS TI-84 Plus CE	TEXAS TI-Nspire CX	CASIO fx-CG50	
$mode \rightarrow SCI$ on the 2nd row to	Doc→Setting & Status	SHIFT MENU	
express any number in its	→Document Settings…	→ <mark>Sci</mark> on the Display row	
standard form	→Scientific on the	$\rightarrow 3$ to express any number in	
	Exponential Format row to	its standard form	
	express any number in its		
	standard form		

### Example 1.1

A rectangle is 2376 cm long and 693 cm wide.

(a) Find the diagonal length of the rectangle, giving the answer in the form  $a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$ .

The required diagonal length  $= \sqrt{2376^{2} + 693^{2}}$ Pythagoras' theorem (A1) = 2475 cm a = 2.475 & k = 3 (A1)

(b) Find the area of the rectangle, giving the answer correct to two significant figures and in the form  $a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$ .

[2]

[2]

The required area = (2376)(693)=  $1646568 \text{ cm}^2$ =  $1600000 \text{ cm}^2$ =  $1.6 \times 10^6 \text{ cm}^2$ 

Base Length  $\times$  Height (A1)

Round off to 2 sig. fig. a = 1.6 & k = 6 (A1)









The base length and the height of a right-angled triangle are 1107 mm and 4920 mm respectively.

- (a) Find the length of the hypotenuse of the triangle, giving the answer in the form  $a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$ .
- (b) Using (a), find the perimeter of the triangle, giving the answer correct to two significant figures and in the form  $a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$ .
- (c) Find the area of the triangle, giving the answer correct to four significant figures and in the form  $a \times 10^k$ , where  $1 \le a < 10$  and  $k \in \mathbb{Z}$ .

[2]

[2]

[2]

### **Approximation and Error**

### **Important Notes**

Common rounding methods:

- $2.7 \stackrel{^{+1}}{1} \underline{8} 28 \rightarrow 2.72$  (Correct to 3 significant figures) 1.
- $2.71828 \rightarrow 2.718$  (Correct to 3 decimal places) 2.
- $2.71828 \rightarrow 3$  (Correct to the nearest integer) 3.

Exact and approximated values:

- $v_E$ : Exact value 1.
- $v_{A}$ : Approximated value corrected to the nearest unit  $\frac{d}{d}$ 2.
- <mark>d</mark>: Maximum absolute error 3.
- $\frac{1}{2}d$ : Lower bound (Least possible value) of  $v_E$ 4.

5. 
$$v_A + \frac{1}{2}d$$
: Upper bound of  $v_E$ 

6. 
$$v_A - \frac{1}{2}d \leq v_E < v_A + \frac{1}{2}d$$
: Range of  $v_E$ 

×100% : Percentage error 7.







### Example 1.2

In a charity booth, there is a transparent box completely filled with identical cubic blocks. Participants have to estimate the number of cubic blocks in the box. The box is 30 cm long, 30 cm wide and 20 cm tall.

[2]

[2]

[2]

The volume	
=(30)(30)(20)	V = lwh (M1)
$=18000 \text{ cm}^3$	(A1)

Sergio estimates the volume of one cubic block to be  $200 \text{ cm}^3$  and uses this value to estimate the number of cubic blocks in the box.

(b)	Find Sergio's estimated number of cubes in the box.	
-----	---	--

The estimated number	
$=\frac{18000}{200}$	Divided by 200 (M1)
= 90	(A1)

The actual number of cubic blocks in the box is 87.

(c) Find the percentage error in Sergio's estimated number of cubic blocks in the box.

The percentage error

$= \left \frac{90-87}{87}\right  \times 100\%$	$\left \frac{v_A - v_E}{v_E}\right  \times 100\% \text{ (M1)}$
= 3.448275862%	
= 3.45%	(A1)

A delivery container is a cuboid with dimensions 2 m, 0.8 m and 0.8 m.

Find the exact volume of the container. (a)

[2]

Zac estimates the dimensions of the container as 2.2 m, 1 m and 0.7 m, and uses these to estimate the volume of the container.

(b) Find the percentage error in Zac's estimated volume of the container.

[3]

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# **3** Arithmetic Sequences

### **Important Notes**

An arithmetic sequence is a sequence such that the next term is generated by adding or subtracting the same number from the previous term.

Properties of an arithmetic sequence  $u_1, u_2, u_3, \cdots$ :

- 1.  $u_1$ : First term
- 2.  $d = u_2 u_1 = u_n u_{n-1}$ : Common difference
- 3.  $u_n = u_1 + (n-1)d$ : General term (*n* th term)

4. 
$$S_n = \frac{n}{2} [2u_1 + (n-1)d] = \frac{n}{2} [u_1 + u_n]$$
: Sum of the first *n* terms

 $\sum_{r=1}^{n} u_{r} = u_{1} + u_{2} + u_{3} + \dots + u_{n-1} + u_{n}$ : Summation sign

Notes on GDC			
TEXAS TI-84 Plus CE	TEXAS TI-Nspire CX	CASIO fx-CG50	
y= to input the general term	Graph to input the general	Table to input the general	
$\rightarrow$ 2nd window to set the	term to generate a table	term to generate a table	
starting row	$\rightarrow$ ctrl T to generate a table	$\rightarrow$ F5 to set the starting row	
$\rightarrow$ 2nd graph to find the value	$\rightarrow$ menu 2 5 to set the	$\rightarrow$ F6 to find the value of the	
of the term needed	starting row to find the value	term needed	
	of the term needed		

### Example 1.3

Mady designs a decorative glass face for a museum. The glass face is made up of small triangular panes. The first level, at the bottom of the glass face, has seven triangular panes. The second level has nine triangular panes, and the third level has eleven triangular panes. Each additional level has two more triangular panes than the level below it.

(a) Find the number of triangular panes in the 10th level.

The number of triangular panes

10th term (M1)  $= u_{10}$ =7+(10-1)(2) $u_1 = 7 \& d = 2$  (A1) = 25(A1)

It is given that there are  $\frac{41}{1}$  triangular panes in the *r* th level.

(b) Find 
$$r$$
.  
 $u_r = 41$  Set up an equation  
 $\therefore 7 + (r-1)(2) = 41$  Correct equation (A1)  
 $2(r-1) = 34$   
 $r-1 = 17$   
 $r = 18$  (A1)

Show that the total number of triangular panes  $S_n$  in the first *n* (c) (i) levels is given by  $S_n = n^2 + 6n$ .

$$S_{n}$$

$$= \frac{n}{2} [2u_{1} + (n-1)d]$$

$$S_{n} = \frac{n}{2} [2u_{1} + (n-1)d] (M1)$$

$$= \frac{n}{2} [2(7) + (n-1)(2)]$$

$$u_{1} = 7 \& d = 2 (A1)$$

$$= \frac{n}{2} (14 + 2n - 2)$$

$$= \frac{n(2n+12)}{2}$$

$$= \frac{2n^{2} + 12n}{2}$$

$$2n^{2} + 12n (M1)$$

$$= n^{2} + 6n (AG)$$

Solution



 $S_n$ 







(AG)

[3]

[3]

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(ii) Hence, find the total number of panes in a glass face with 13 levels.

[2]

[3]

[3]

The total number of panes =  $S_{13}$ =  $13^2 + 6(13)$  n = 13 (M1) = 247 (A1)

Mady has 500 triangular panes to build the decorative glass face and does not want it to have any incomplete levels.

(d) Find the greatest number of complete levels that Mady can build.

 $S_n = 500$ ∴  $n^2 + 6n = 500$   $n^2 + 6n - 500 = 0$ By considering the graph of  $y = n^2 + 6n - 500$ , the horizontal intercepts are -25.56103(*Rejected*) and 19.561028. Thus, the greatest number of complete levels that Mady can build is 19. (A1)

Each triangular pane has an area of  $2.27 \text{ m}^2$  and the maximum number of complete levels were built.

(e) Find the total area of the decorative glass face, giving the area to the nearest  $m^2$ .

The total area	
$= S_{19} \times 2.27$	Multiply by 2.27 (M1)
$= [19^2 + 6(19)](2.27)$	<i>n</i> =19 (M1)
$=1078.25 \text{ m}^2$	
$=1078 \text{ m}^2$	(A1)

In a party game, a number of apples are placed one metre apart in a straight line. Players are gathered at the starting position which is five metres before the first apple.

Each player collects a single apple by picking it up and bringing it back to the starting position. The nearest apple is collected first. The player then collects the first nearest apple and the game continues in this way until the signal is given for the end.

Fatima runs to get each apple and brings it back to the start. Let  $u_1$  metres be the distance she has to run in order to collect the first apple - that is, to pick up the first apple and bring it back to the starting position.

(a) Write down  $u_1$ .

(h)	(;)	Show that w 12 and w 14	
(u)	(1)	Show that $u_2 = 12$ and $u_3 = 14$ .	

(ii) Hence, write down the common difference d of the sequence.

The final apple Fatima collected was 20 metres from the starting position.

Find the total number of apples that Fatima collected. (c) (i)

[2]

[1]

[1]

[1]









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(ii) Find the total distance that Fatima ran to collect these apples.

Akash also plays the game. When the signal is given for the end of the game he has run 491 metres.

(d) (i) Find the total number of apples that Akash has collected.

[3]

[3]

(ii) Hence, find the distance between Akash and the starting position when the signal is given.

[3]

4

### **Geometric Sequences**

### **Important Notes**

A geometric sequence is a sequence such that the next term is generated by multiplying or being divided by the same number from the previous term.

Properties of a geometric sequence  $u_1, u_2, u_3, \cdots$ :

- 1.  $u_1$ : First term
- 2.  $r = u_2 \div u_1 = u_n \div u_{n-1}$ : Common ratio
- 3.  $u_n = u_1 \times r^{n-1}$ : General term (*n* th term)
- 4.  $S_n = \frac{u_1(1-r^n)}{1-r} = \frac{u_1(r^n-1)}{r-1}$ : Sum of the first *n* terms

Notes on GDC		
TEXAS TI-84 Plus CE	TEXAS TI-Nspire CX	CASIO fx-CG50
y= to input the general term	Graph to input the general	Table to input the general
$\rightarrow$ 2nd window to set the	term to generate a table	term to generate a table
starting row	→ctrl T to generate a table	$\rightarrow$ F5 to set the starting row
$\rightarrow$ 2nd graph to find the value	$\rightarrow$ menu 2 5 to set the	$\rightarrow$ F6 to find the value of the
of the term needed	starting row to find the value	term needed
	of the term needed	

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### Example 1.4

On Monday, Peter goes to a running track to train. He runs the first lap of the track in 100 seconds. Each lap Peter runs takes 1.07 times as long as his previous lap.

(a) Find the time, in seconds, Peter takes to run his fourth lap.

The required time  $= u_4$  4th term (M1)  $= u_1 \times r^{4-1}$   $= 100 \times 1.07^3$   $u_1 = 100 \& r = 1.07$  (A1) = 122.5043 s= 123 s (A1)

[3]

[3]

Peter runs his last lap in at least 183 seconds.

(b) (i) Find the least possible number of laps he has run on Monday.

 $u_n \ge 183$ Set up an inequality $\therefore 100 \times 1.07^{n-1} \ge 183$ Correct inequality (A1) $100 \times 1.07^{n-1} - 183 \ge 0$ By considering the graph of $y = 100 \times 1.07^{n-1} - 183$ , the graph is abovethe horizontal axis when n > 9.9318362. GDC approach (M1) $\therefore$  The least possible number of laps hehas run on Monday is 10.

(ii) Hence, find the total time, in minutes, run by Peter on Monday.

[4]  
The total time  

$$= S_{10}$$

$$= \frac{u_1(1-r^{10})}{1-r} \qquad S_n = \frac{u_1(1-r^n)}{1-r} \text{ (M1)}$$

$$= \frac{(100)(1-1.07^{10})}{1-1.07} \qquad u_1 = 100 \text{ \& } r = 1.07 \text{ (A1)}$$

$$= 1381.644796 \text{ s}$$

$$= \frac{1381.644796}{60} \text{ min} \qquad 1 \text{ s} = \frac{1}{60} \text{ min} \text{ (A1)}$$

$$= 23.02741327 \text{ min}$$

$$= 23.0 \text{ min} \qquad \text{(A1)}$$

On Tuesday, Peter takes Zoey to train. They both run the first lap of the track in 100 seconds. Each lap Zoey runs takes her 15 seconds longer than her previous lap.

#### (c) Find the time, in seconds, Zoey takes to run her fifth lap.

The required time	
$= v_5$	5th term (M1)
=100+(5-1)(15)	$v_1 = 100 \& d = 15$ (A1)
$=160  \mathrm{s}$	(A1)

After a certain number of laps, Peter takes more time per lap than Zoey.

Find the number of the lap when this happens.

[3]  

$$u_n > v_n$$
 Set up an inequality  
 $\therefore 100 \times 1.07^{n-1} > 100 + (n-1)(15)$  Correct inequality (A1)  
 $100 \times 1.07^{n-1} - 100 - 15(n-1) > 0$   
By considering the graph of  
 $y = 100 \times 1.07^{n-1} - 100 - 15(n-1)$ , the graph is  
above the horizontal axis when  $n < 1$   
(*Rejected*) or  $n > 22.072137$ . GDC approach (M1)  
 $\therefore$  Peter takes more time per lap than Zoey  
during the 23 rd lap. (A1)

(d)









[3]

A new café opened and during the first week their profit was \$1200. The café's profit increases by 8% every week.

(a)	Find the café's profit during the tenth week.		101
(b)	(i)	Show that the café's <b>total</b> profit for the first <i>n</i> weeks is given by $S_n = -15000(1-1.08^n)$ .	[3]
	(ii)	"Hence, find the café's <b>total</b> profit for the first eleven weeks.	[2]

[2]

A new dessert shop opened at the same time as the café. During the first week their profit was also \$1200. The dessert shop's profit increases by \$180 every week.

(c) Find the dessert shop's profit during the seventh week.

[3] In the *m* th week, the café's **total** profit exceeds the dessert shop's **total** profit for the first time since they both opened.

Find *m*. (d)

[3]

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# **5** Systems of Equations

### **Important Notes**

Common systems of equations:

1.  $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$  2×2 system,  $a, b, c, d, e, f \in \mathbb{R}$ 2.  $\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \end{cases}$  3×3 system,  $a, b, c, d, e, f, g, h, i, j, k, l \in \mathbb{R}$  $ix + jy + kz = l \end{cases}$ 

### Example 1.5

The total revenue P for selling *x* boxes of chocolate milk, *y* boxes of coffee milk and *z* boxes of oat milk can be modelled by P = 6x + 5y + 9z, where 6, 5 and 9 are the prices of a box of chocolate milk, coffee milk and oat milk respectively. It is given that

- The total number of boxes of milk sold is 28.
- The total revenue is \$238.
- There are twice as many boxes of coffee milk as boxes of chocolate milk.
- (a) (i) Write down the system of three equations in x, y and z.

 $\begin{cases} x + y + z = 28 \\ 6x + 5y + 9z = 238 \\ 2x - y = 0 \end{cases}$  (A1)(A1)(A1)

[3]

[3]

[1]

(ii) Hence, find the values of  $\frac{x}{y}$ ,  $\frac{y}{z}$  and  $\frac{z}{z}$ .

$$x = 7$$
,  $y = 14$  and  $z = 7$  (A1)(A1)(A1)

(b) Write down the total price of three boxes of oat milk and ten boxes of chocolate milk.

\$87 (A1)

In a softball league, each team gains x points for a win, y points for a draw and z points for a loss. There are twenty matches in a year. The following table shows the performances of three teams in 2023:

Team	Win	Draw	Loss	Points
A	8	7	5	41
В	6	4	10	22
С	13	7	0	66

(a) (i) Write down the system of three equations in x, y and z.

- (ii) Hence, find the values of x, y and z.
- Interpret the meaning of z. (b)

[1]

[3]

[3]









**6** Financial Mathematics

### **Important Notes**

Properties of compound interest:

- 1. *PV*: Present value
- 2. r%: Nominal annual interest rate
- 3. **k**: Number of compounded periods in one year
- 4. *n*: Number of years

5. 
$$FV = PV\left(1 + \frac{r\%}{k}\right)^{kn}$$
: Future value

6. I = FV - PV: Interest

Properties of inflation rates:

- 1. *i*%: Inflation rate per year
- 2. r%: Nominal annual interest rate
- 3. (r-i)%: Real rate

Types of annuities:

1. Payments at the beginning of each time period

2. Payments at the end of each time period

$$-PMT - PMT \cdots -PMT - PMT$$

$$PV = 0$$

$$FV$$

Types of amortizations:

1. Payments at the beginning of each time period

2. Payments at the end of each time period

Notes on GDC				
TEXAS TI-84 Plus CE	TEXAS TI-Nspire CX	CASIO fx-CG50		
N: Number of time periods	N: Number of time periods	n: Number of time periods		
I%: Annual interest rate	I%: Annual interest rate	I%: Annual interest rate		
PV: Present value	PV: Present value	PV: Present value		
PMT: Payment amount	Pmt: Payment amount	PMT: Payment amount		
FV: Future value	FV: Future value	FV: Future value		
P/Y: Number of annual	PpY: Number of annual	P/Y: Number of annual		
payments	payments	payments		
C/Y: Number of compounded	CpY: Number of compounded	C/Y: Number of compounded		
periods (equals to P/Y)	periods (equals to P/Y)	periods (equals to P/Y)		
PMT END/BEGIN: Time of	PmtAt END/BEGIN: Time of	$\rightarrow$ SHIFT MENU to choose		
payment in each time period	payment in each time period	payment at the beginning/end		
$\rightarrow$ alpha enter to solve the	$\rightarrow$ enter to solve the unknown	of each time period		
unknown quantity	quantity	$\rightarrow$ F1 to F5 to solve the		
		unknown quantity		











### Example 1.6

On 1st January 2024, Judy invests \$22000 in an account that pays a nominal annual interest rate of 6%, compounded **half-yearly**. It is given that there is no further deposit to or any withdrawal from the account.

(a) (i) Find the amount that Judy will have in her account after 3 years.

[3]

[2]

GDC approach (M1)(A1)

(A1)

C/Y(CpY) = 2 PMT(PmtAt): END FV = 26269.15052Thus, the amount after 3 years is \$26300.

By financial solver:

N(n) = 6I% = 6

FV = ?

PV = -22000PMT(Pmt) = 0

P/Y(PpY) = 2

(ii) Hence, find the interest that Judy can earn after 3 years.

The interest = 26269.15052 - 22000 I = FV - PV (M1) = \$4269.150524= \$4270 (A1) (b) Find the year in which the amount of money in Judy's account will become double the amount she invested.

By financial solver: N(n) = ?I% = 6PV = -22000PMT(Pmt) = 0GDC approach (M1)(A1) FV = 44000P/Y(PpY) = 2C/Y(CpY) = 2PMT(PmtAt): END N = 23.44977225 The number of years 23.44977225 2 =11.72488613 11.72488613 (A1) Thus, the required year is 2035. (A1)

It is given that the rate of inflation during these 3 years is  $\frac{2\%}{2\%}$  per year.

Write down the value of the real interest rate. (c) (i)

4%

N(n) = 6I% = 4

By financial solver:

(ii) Hence, find the real value of the amount that Judy will have in her account after 3 years.

(A1)

[3]

[1]

[4]

PV = -22000PMT(Pmt) = 0GDC approach (M1)(A1) FV = ?P/Y(PpY) = 2C/Y(CpY) = 2PMT(PmtAt): END FV = 24775.57322 Thus, the real amount after 3 years is **\$24800**. (A1)

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On 1st January 2025, April invests \$10000 in an account that pays a nominal annual interest rate of 8%, compounded **quarterly**. It is given that there is no further deposit to or any withdrawal from the account.

- (a) (i) Find the amount that April will have in her account after 4 years.
  - (ii) Hence, find the interest that April can earn after 4 years.
- (b) Find the year in which the amount of money in April's account will become 2.5 times the amount she invested.

[4]

[3]

[2]

It is given that the rate of inflation during these 4 years is 3% per year.

(c)	(i)	Write down the value of the real interest rate.	
			[1]
	(ii)	Hence, find the real value of the amount that April will have in her account after 4 years.	
		5	[3]

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### Example 1.7

Takumi is going to purchase a boat. He is suggested to choose one of the two options to repay the loan of \$2100000:

Option 1: A total of 144 equal monthly payments have to be paid at the end of each month, with a nominal annual interest rate of 6%, compounded monthly Option 2: A deposit of \$400000 has to be paid at the beginning of the loan, followed by monthly payments of \$20000 at the end of each month until the loan is fully repaid, with a nominal annual interest rate of 5.4%, compounded monthly

(a) If Takumi selects the option 1, find

(i)	the amount of monthly payment,
-----	--------------------------------

By financial solver: N(n) = 144I% = 6PV = 2100000PMT(Pmt) = ?GDC approach (M1)(A1) FV = 0P/Y(PpY) = 12C/Y(CpY) = 12PMT(PmtAt): END PMT = -20492.85449Thus, the amount of monthly payment is **\$20500**. (A1) (ii) the total amount to be paid, [2] The total amount =(20492.85449)(144) $PMT \times N$  (A1) = \$2950971.047 = \$2950000 (A1) (iii) the amount of interest paid. [2] The amount of interest = 2950971.047 - 2100000I = FV - PV (M1) = \$850971.0466 = \$851000 (A1)

[3]

(b) If Takumi selects the option 2, find

(i)	the number of months to repay the loan,		[0]
	By financial solver: N(n) = ? I% = 5.4 PV = 1700000 PMT(Pmt) = -20000 FV = 0 P / Y(PpY) = 12 C / Y(CpY) = 12 PMT(PmtAt) : END N = 107.3689045	GDC approach (M1)(A1	[3] )
	Thus, the required number of months is $108$ .	(A1)	
(ii)	the exact total amount to be paid,		[2]
	The exact total amount		[~]
	= 400000 + (20000)(108) $= $2560000$	$400000 + PMT \times N$ (A1) (A1)	
(iii)	the amount of interest paid.		
	The amount of interest = 2560000 - 2100000 = \$460000	I = FV - PV (M1) (A1)	[2]
_			

(c) By considering the amounts of interest paid in both options in both options, state the better option for Takumi and explain the answer. [2]

The amount of interest paid in option 2 is less				
than that in option 1.	Comparing interest (R1)			
Thus, the option 2 is better.	(A1)			









Jun Ryeol is going to buy a flat. He is suggested to choose one of the two options to repay the loan of 300000 USD with a nominal annual interest rate of 3.3%, compounded monthly:

Option 1: A total of 270 equal monthly payments have to be paid at the end of each month

Option 2: A monthly payment of 2250 USD has to be paid at the end of each month until the loan is fully repaid

(a) If Jun Ryeol selects the option 1, find

(i)	the amount of monthly payment,	
(ii)	the total amount to be paid,	[3]
(")	the total amount to be paid,	[2]
(iii)	the amount of interest paid.	

[2]

(b) If Jun Ryeol selects the option 2, find

	(i)	the number of months to repay the loan,	
	(ii)	the exact total amount to be paid,	[3]
	()	,	[2]
	(iii)	the amount of interest paid.	[2]
(c)	By considering the amounts of monthly payment in both options, state the better option for Jun Ryeol and explain the answer.		

[2]

**CLICK HERE** 







