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Chapter

1

Functions

SUMMARY POINTs

✓ Odd and even functions:

$f(x)$ is odd if $f(-x) = -f(x)$

$f(x)$ is even if $f(-x) = f(x)$

✓ $f^{-1}(x)$ exists only when $f(x)$ is one-to-one in the restricted domain

✓ Absolute function:

$$|f(x)| = \begin{cases} f(x) \text{ if } x \geq 0 \\ -f(x) \text{ if } x < 0 \end{cases}$$

Solutions of Chapter 1

1 Paper 1 Section A – Absolute Signs

Example

(a) Solve the inequality $|3x+1| > x+1$.

[3]

(b) Solve the inequality $|3x+1| \geq |x+1|$.

[4]

Solution

(a) $|3x+1| > x+1$

$3x+1 > x+1$ or $3x+1 < -(x+1)$ M1

$3x+1 > x+1$ or $3x+1 < -x-1$

$2x > 0$ or $4x < -2$

$x > 0$ or $x < -\frac{1}{2}$ A2

[3]

(b) $|3x+1| \geq |x+1|$

$|3x+1|^2 \geq |x+1|^2$

$9x^2+6x+1 \geq x^2+2x+1$ M1

$8x^2+4x \geq 0$ (A1) for correct inequality

$4x(2x+1) \geq 0$

$\therefore x \leq -\frac{1}{2}$ or $x \geq 0$ A2

[4]

Exercise 1

1. (a) Solve the inequality $|2x+1| \leq 4x-1$.

[3]

(b) Solve the inequality $|2x+1| > |4x-1|$.

[4]

2. (a) Solve the inequality $\frac{|x|+1}{3} < 2|x| - 5$.

[3]

(b) Solve the inequality $\left|\frac{x+1}{3}\right| < |2x-5|$.

[4]

3. Let $f(x) = 2x + \frac{3}{x}$, where $x > 0$. Solve the inequality $f(|x|) > 5$.

[5]

4. Let $f(x) = \frac{x^3 - 14x + 8}{x+4}$, where $x < 0$. Solve the inequality $f(|x|) \geq |x| + 2$.

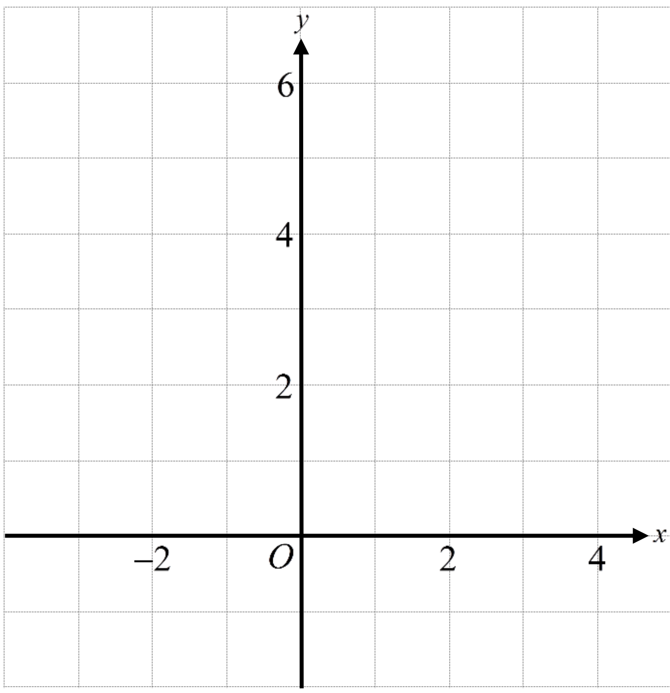
[5]

2 Paper 1 Section A – Absolute Rational Functions

Example

(a) Sketch the graph of $y = \left|\frac{2x+1}{x-2}\right|$, showing clearly any asymptotes and any points of intersection with the axes.

[4]



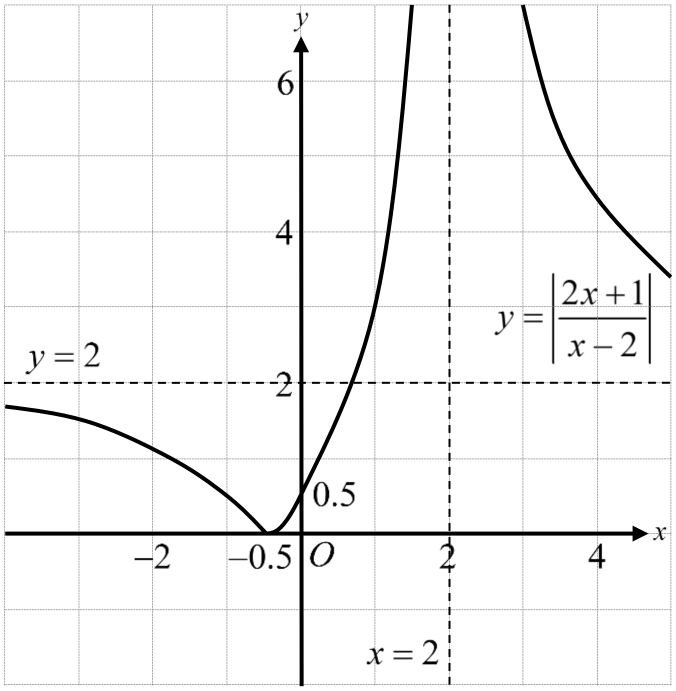
(b) Solve the equation $\left|\frac{2x+1}{x-2}\right| = 3$.

[3]

Solution

(a) For correct asymptotes A1
For correct intercepts A1
For correct shape A2

[4]



(b) $\left|\dfrac{2x+1}{x-2}\right| = 3$

$\dfrac{2x+1}{x-2} = 3$ or $\dfrac{2x+1}{x-2} = -3$ M1

$2x+1 = 3(x-2)$ or $2x+1 = -3(x-2)$

$2x+1 = 3x-6$ or $2x+1 = -3x+6$

$7 = x$ or $5x = 5$

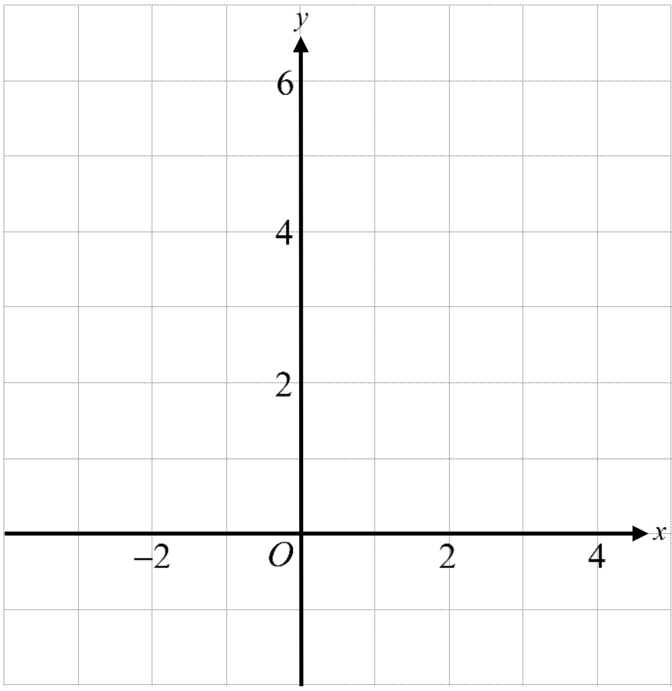
$x = 7$ or $x = 1$ A2

[3]

Exercise 2

1. (a) Sketch the graph of $y = \left|\frac{3x-1}{1-x}\right|$, showing clearly any asymptotes and any points of intersection with the axes.

[4]

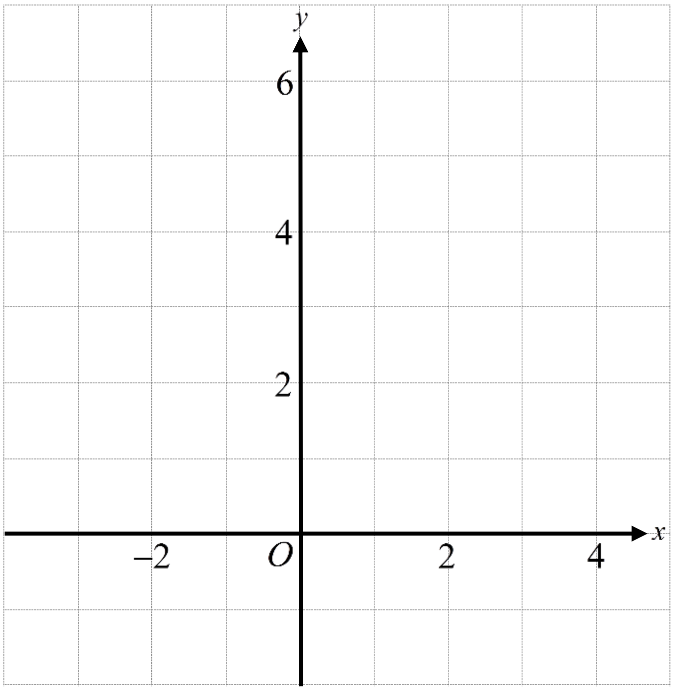


(b) Solve the equation $\left|\frac{3x-1}{1-x}\right| = 5$.

[3]

2. (a) Sketch the graph of $y = \left|\frac{1-2x}{x+1}\right|$, showing clearly any asymptotes and any points of intersection with the axes.

[4]



(b) It is given that the equation $\left|\frac{1-2x}{x+1}\right| = k$ has only one solution. Write down the possible values of k.

[2]

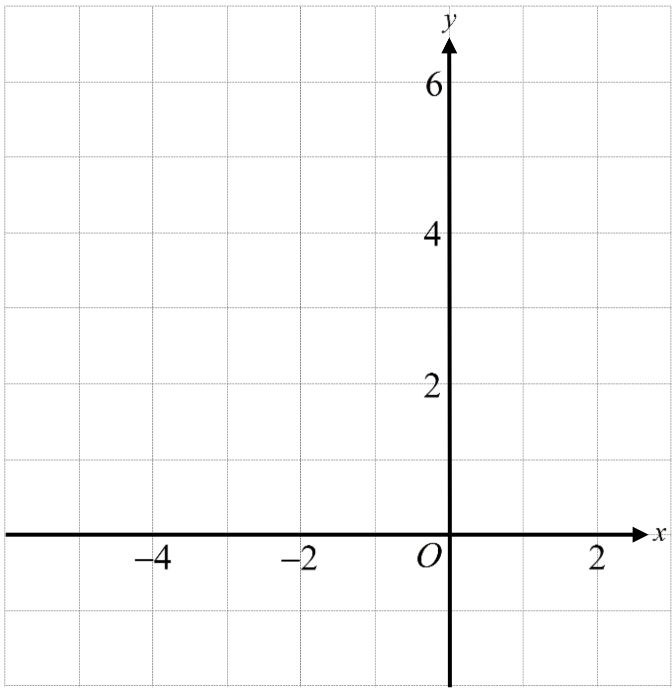
3. The function f is given by $f(x)=\frac{2x}{2-x}$, $x \neq 2$. It is given that $f^{-1}(x)=\frac{2x}{ax+2}$.

(a) Find the value of a.

[3]

(b) Sketch the graph of $y=\left|f^{-1}(x)\right|$, showing clearly any asymptotes and any points of intersection with the axes.

[4]

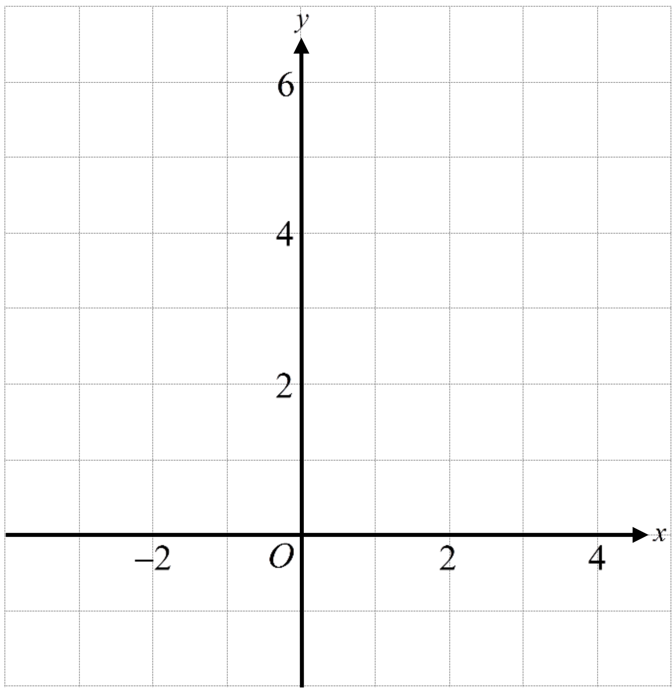


4. The function f is given by $f(x)=\dfrac{1}{3-x}$, where $x>-3$, $x\neq 3$. On the same coordinate plane, sketch the following graphs, showing clearly any asymptotes and any points of intersection with the axes.

(a) (i) $y=|f(x)|$,

(ii) $y=f(|x|)$.

[6]



(b) Write down the number of solutions of the equation $|f(x)|=f(|x|)$ for $-3\leq x\leq 0$.

[1]

3 Paper 1 Section A – Odd and Even Functions

Example

The function f and g are defined as $f(x)=x^6$ and $g(x)=\sqrt{\sqrt[3]{x}+4}$, $x\in\mathbb{R}$.

(a) Find the expression of $(g\circ f)(x)$.

[2]

(b) Show that $(g\circ f)(x)$ is an even function.

[2]

(c) Write down the range of $(g\circ f)(x)$.

[2]

Solution

(a) $(g\circ f)(x)=\sqrt{\sqrt[3]{f(x)}+4}$

$(g\circ f)(x)=\sqrt{\sqrt[3]{x^6}+4}$ M1

$(g\circ f)(x)=\sqrt{x^2+4}$ A1

[2]

(b) $(g\circ f)(-x)=\sqrt{(-x)^2+4}$ M1

$(g\circ f)(-x)=\sqrt{x^2+4}$ A1

$(g\circ f)(-x)=(g\circ f)(x)$

Thus, $(g\circ f)(x)$ is an even function. AG

[2]

(c) $\{y:y\geq 2\}$ A2

[2]

Exercise 3

1. The function f and g are defined as $f(x)=\dfrac{4x+1}{\sqrt{x}}$ and $g(x)=4x^2$, $x\in\mathbb{R}$, $x>0$.

(a) Find the expression of $(f\circ g)(x)$.

[2]

(b) Show that $(f\circ g)(x)$ is an odd function.

[2]

It is given that the coordinates of the minimum point of $y=(f\circ g)(x)$ are $(0.25, 4)$.

(c) Write down the range of $(f\circ g)(x)$.

[1]

2. The function f is defined as $f(x)=x^4-x^2$, $x\in\mathbb{R}$.

(a) Show that $f(x)$ is an even function.

[2]

The minimum point of f has x-coordinate $-\dfrac{1}{\sqrt{2}}$ for $x\le 0$.

(b) Find the range of $f(x)$.

[3]

3. The function f is defined as $f(x)=\dfrac{x}{x^2+0.19}$, $x\in\mathbb{R}$.

(a) Show that $f(x)$ is an odd function.

[2]

It is given that $f(a)=a$, $a\in\mathbb{R}$.

(b) Find the possible values of a.

[3]

(c) Write down the equation of the horizontal asymptote of $f(x)$.

[1]

4. The function f is defined as $f(x)=\dfrac{2-5x}{2-9x}$, $x\in\mathbb{R}$, $x\neq\dfrac{2}{9}$.

(a) Show that $f(|x|)$ is an even function.

[2]

(b) Write down the equation of the horizontal asymptote of $f(|x|)$.

[1]

(c) Write down the equations of the vertical asymptotes of $f(|x|)$.

[2]

4 Paper 2 Section A – Restricted Domains for f^{-1}

Example

The function f is defined as $f(x) = |x^2 - 4| - 5,\ x \geq -10$. The domain of f is restricted such that f^{-1} exists.

(a) State the largest possible domain of f.

[2]

(b) Find the expression of f^{-1}.

[3]

(c) State the domain of f^{-1}.

[1]

Solution

(a) $\{x : x \geq 2\}$ A2

[2]

(b) $f(x) = (x^2 - 4) - 5$ (A1) for correct function

$y = x^2 - 9$

$\Rightarrow x = y^2 - 9$ (M1) for swapping variables

$x + 9 = y^2$

$y = \sqrt{x+9}$

$\therefore f^{-1}(x) = \sqrt{x+9}$ A1

[3]

(c) $\{x : x \geq -5\}$ A1

[1]

Exercise 4

1. The function f is defined as $f(x)=10-2|x-7|$, $x\geq 0$. The domain of f is restricted such that f^{-1} exists.

(a) State the largest possible domain of f.

[2]

(b) Find the expression of f^{-1}.

[3]

2. The function f is defined as $f(x)=|x^3-8|$, $x\leq 20$. The domain of f is restricted such that f^{-1} exists.

(a) State the largest possible domain of f.

[2]

(b) Find the expression of f^{-1}.

[3]

(c) State the value of $f^{-1}\left(\frac{63}{8}\right)$.

[1]

3. The function f is defined as $f(x)=(2x+1)^2$, $-4\leq x\leq 4$. The domain of f is restricted such that f^{-1} exists.

(a) State the largest possible domain of f.

[2]

(b) Find the expression of f^{-1}.

[2]

(c) The function g is defined such that $(f\circ g)(x)=(4x+7)^2$. Find a possible expression of $g(x)$.

[2]

4. The function f is defined as $f(x) = -(x-3)^2 + 5$, $-5 \leq x \leq 5$. The domain of f is restricted such that f^{-1} exists.

(a) State the largest possible domain of f.

[2]

(b) Find the expression of f^{-1}.

[3]

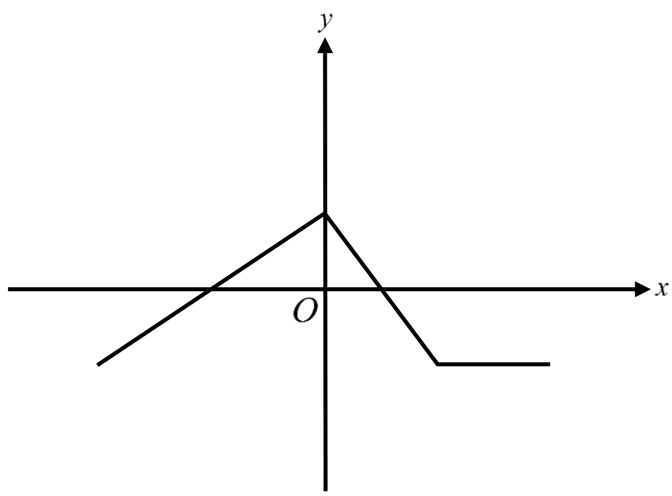
(c) The function g is defined such that $(g^{-1} \circ f^{-1})(x) = 2x$. Find the expression of $g(x)$.

[3]

5 Paper 1 –More about Function Sketching

Example

The following diagram shows the graph of $y = f(x)$:



It is given that the range of $f(x)$ is $\{y : -1 \leq y \leq 1\}$.

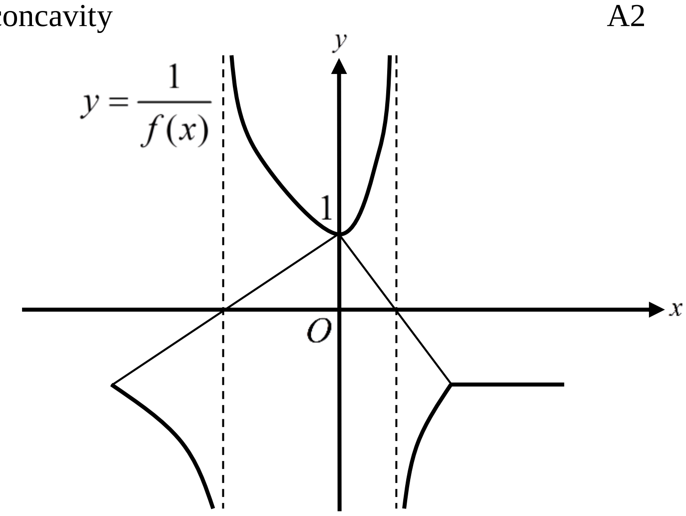
On the same diagram, sketch the graph of $y = \dfrac{1}{f(x)}$, clearly indicating any asymptotes and axes intercepts.

[5]

Solution

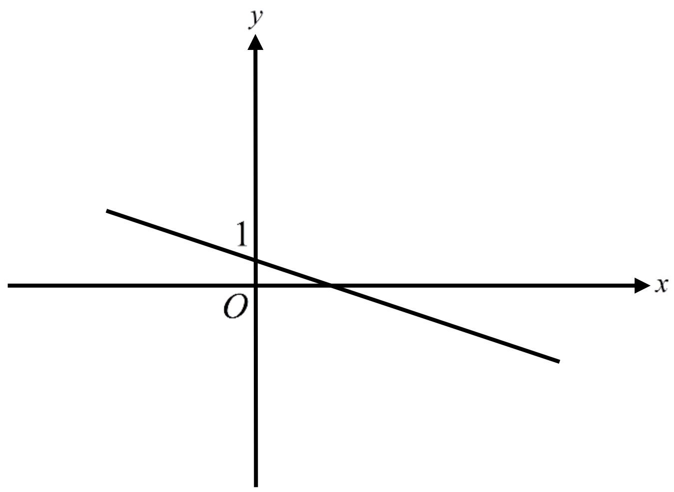
For correct y-intercept	A1
For correct asymptotes	A2
For correct concavity	A2

[5]



Exercise 5

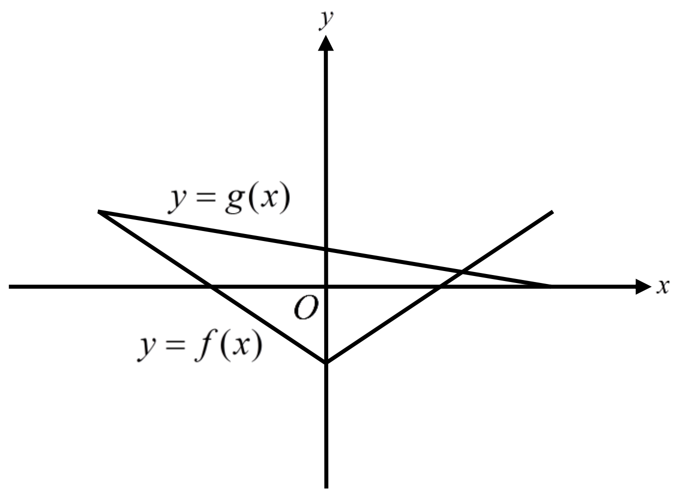
1. The following diagram shows the graphs of $y = f(x)$:



On the same diagram, sketch the graph of $y = (f(x))^2$, clearly indicating any intercepts and stationary point.

[4]

2. The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$:

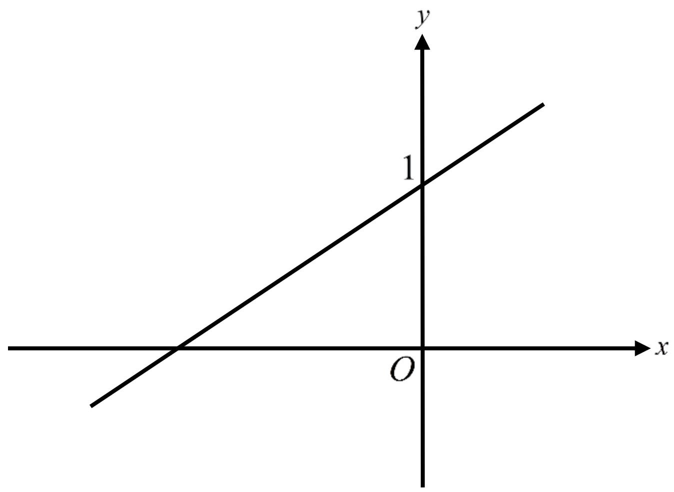


It is given that the range of $g(x)$ is $\{y : 0 \le y \le 1\}$.

On the same diagram, sketch the graph of $y = \dfrac{g(x)}{f(x)}$, clearly indicating any asymptotes and axes intercepts.

[5]

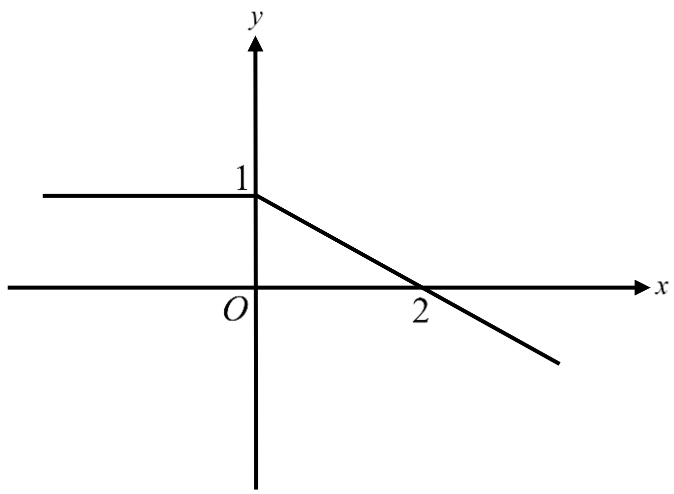
3. The following diagram shows the graphs of $y = f(x)$:



On the same diagram, sketch the graph of $y = \sqrt{f(x)}$, clearly indicating any intercepts.

[4]

4. The following diagram shows the graphs of $y = f(x)$:



On the same diagram, sketch the graph of $y = \frac{f(x)}{|x|}$, clearly indicating any vertical asymptote and axes intercepts.

[4]

Chapter

2

Polynomials

SUMMARY POINTs

✓ Number of roots of the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$:
The maximum number of roots of $f(x) = 0$ is n

✓ Sum and product of roots of $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$:

1. r_1, r_2, ..., r_n: Roots

2. $r_1 + r_2 + \cdots + r_n = -\dfrac{a_{n-1}}{a_n}$

3. $r_1 r_2 + r_1 r_3 + \cdots + r_1 r_n + r_2 r_3 + r_2 r_4 + \cdots + r_2 r_n + \cdots + r_{n-1} r_n = \dfrac{a_{n-2}}{a_n}$

...

4. $r_1 r_2 r_3 \cdots r_{n-1} r_n = (-1)^n \dfrac{a_0}{a_n}$

SUMMARY POINTs

✓ Factor theorem:

$(x-a)$ is a factor of $f(x)$ if $f(a)=0$

$(px-q)$ is a factor of $f(x)$ if $f\left(\frac{q}{p}\right)=0$

✓ Remainder theorem:

$f(a)$ is the remainder when $f(x)$ is divided by $(x-a)$

$f\left(\frac{q}{p}\right)$ is the remainder when $f(x)$ is divided by $(px-q)$

✓ Partial fractions:

1. $\dfrac{ax+b}{(cx+d)(ex+f)}$ can be expressed as $\dfrac{P}{cx+d}+\dfrac{Q}{ex+f}$
2. $\dfrac{ax+b}{(cx+d)^2}$ can be expressed as $\dfrac{P}{cx+d}+\dfrac{Q}{(cx+d)^2}$

Solutions of Chapter 2

6 Paper 1 Section A – Remainder Theorem

Example

When the polynomial $2x^3 + ax^2 + bx + 1$ is divided by $(x-1)$, the remainder is 9, and when divided by $(x+2)$, it is 3. Find the value of a and the value of b, where a, $b \in \mathbb{R}$.

[5]

Solution

$2(1)^3 + a(1)^2 + b + 1 = 9$	(M1) for remainder theorem
$2 + a + b + 1 = 9$	
$b = 6 - a$	A1
$2(-2)^3 + a(-2)^2 + b(-2) + 1 = 3$	
$-16 + 4a - 2b + 1 = 3$	
$\therefore -16 + 4a - 2(6-a) + 1 = 3$	(M1) for substitution
$-16 + 4a - 12 + 2a + 1 = 3$	
$6a = 30$	
$a = 5$	A1
$b = 6 - 5$	
$b = 1$	A1

[5]

Exercise 6

1. When the polynomial $ax^3 - x^2 + bx + 3$ is divided by $(x+2)$, the remainder is -13, and when divided by $(x-3)$, it is 27. Find the value of a and the value of b, where a, $b \in \mathbb{R}$.

[5]

2. The same remainder is found when $4x^3 - 2kx^2 - 5$ and $3x^3 - k^2x^2 + 15$ are divided by $x - 2$. Find the possible values of k.

[6]

3. When $4x^3 + px^2 + qx$ is divided by $x-2$ and $x+2$, the remainders are equal. Given that p, $q \in \mathbb{R}$.

(a) Find the value of q;

[3]

(b) Find the set of values of p.

[1]

4. Let $f(x) = x^4 + 4x^2 - 3x + 2$. The remainder when $f(x)$ is divided by $(x-p)$ is q, and the remainder when $f(x)$ is divided by $(x+p)$ is $q+12$, where p and q are constants. Find the value of p and the value of q.

[5]

7 Paper 1 Section A – Factor Theorem

Example

The polynomial $P(x) = ax^3 + 6x^2 + bx - 12$ is divisible by $(x-1)$ and by $(x+4)$. Find the value of a and the value of b, where a, $b \in \mathbb{R}$.

[5]

Solution

$P(1) = 0$	
$a(1)^3 + 6(1)^2 + b(1) - 12 = 0$	(M1) for factor theorem
$b = 6 - a$	A1
$P(-4) = 0$	
$a(-4)^3 + 6(-4)^2 + b(-4) - 12 = 0$	
$-64a + 96 - 4b - 12 = 0$	
$\therefore -64a + 96 - 4(6 - a) - 12 = 0$	(M1) for substitution
$-64a + 96 - 24 + 4a - 12 = 0$	
$-60a = -60$	
$a = 1$	A1
$b = 6 - 1$	
$b = 5$	A1

[5]

Exercise 7

1. The polynomial $f(x) = ax^3 + bx^2 - 13x + 6$ is divisible by $(x-2)$ and by $(x+3)$. Find the value of a and the value of b, where a, $b \in \mathbb{R}$.

[5]

2. The cubic polynomial $x^3 + px^2 + qx + 48$ has a factor $(x-4)$ and leaves a remainder 105 when divided by $(x+3)$. Find the value of p and the value of q, where p, $q \in \mathbb{R}$.

[5]

3. Consider the polynomial $p(x) = x^3 + mx - 30$.

(a) Given that $p(x)$ has a factor $(x-5)$, find the value of m.

[2]

(b) Hence or otherwise, factorize $p(x)$ as a product of linear factors.

[3]

4. Consider the polynomial $q(x) = 2x^3 + (k+9)x^2 + kx + (k+1)$. It is given that $q(x)$ has a factor $(x+3)$.

(a) Find the value of k.

[2]

(b) Hence or otherwise, factorize $q(x)$ as a product of linear factors.

[3]

8 Paper 1 Section A – Sum and Product of Roots

Example

The equation $3x^3+24x^2+x+5=k$ has roots r_1, r_2 and r_3. Given that $r_1+r_2+r_3=r_1r_2r_3$, find the value of k.

[5]

Solution

$3x^3+24x^2+x+5=k$	
$3x^3+24x^2+x+(5-k)=0$	(M1) for valid approach
$r_1+r_2+r_3=r_1r_2r_3$	
$\therefore -\frac{24}{3}=-\frac{5-k}{3}$	M1A2
$24=5-k$	
$k=-19$	A1

[5]

Exercise 8

1. The equation $2x^3+30x^2+kx+20=x-5$ has roots r_1, r_2 and r_3. Given that $r_1r_2+r_2r_3+r_1r_3=2r_1r_2r_3$, find the value of k.

[5]

2. The equation $x^4+kx^3+3x^2+2x+1=-x^4+4x^3+13$ has roots r_1, r_2, r_3 and r_4. Given that $r_1r_2r_3r_4+r_1+r_2+r_3+r_4=0$, find the value of k.

[5]

3. Let $f(x) = 6x^3 - 18x^2 + 36x + 72$, $x \in \mathbb{R}$. r_1, r_2 and r_3 are the roots of $f(x) = 0$.

(a) Find the value of $\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_1 r_3}$.

[4]

A new polynomial is defined by $g(x) = f(x-3)$.

(b) Find the sum of the roots of the equation $g(x) = 0$.

[2]

4. Let $g(x) = -x^3 + 71x^2 - 10x - 15$, $x \in \mathbb{R}$. r_1, r_2 and r_3 are the roots of $g(x) = 0$.

(a) Find the value of $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.

[4]

A new polynomial is defined by $h(x) = 2g(x)$.

(b) Find the product of the roots of the equation $h(x) = 0$.

[2]

9 Paper 1 Section A – Transformations of Roots

Example

The real root of the equation $x^3 + 2x - 5 = 0$ is 1.3283, correct to four decimal places. Determine the real root for each of the following.

(a) $(x-5)^3 + 2(x-5) - 5 = 0$

[2]

(b) $\frac{1}{8}x^3 + x - 5 = 0$

[2]

Solution

(a) The required real root

$= 1.3283 + 5$ (M1) for valid approach

$= 6.3283$ A1

[2]

(b) The required real root

$= 2(1.3283)$ (M1) for valid approach

$= 2.6566$ A1

[2]

Exercise 9

1. The real root of the equation $4x^3 + 19x^2 + 10x - 50 = 0$ is 1.25. Determine the real root for each of the following.

(a) $4(x+6)^3 + 19(x+6)^2 + 10(x+6) - 50 = 0$

[2]

(b) $-4x^3 + 19x^2 - 10x - 50 = 0$

[2]

2. The real root of the equation $4x^3+17x^2+30x+27=0$ is -2.25. Determine the real root for each of the following.

(a) $4x^3-17x^2+30x-27=0$

[2]

(b) $108x^3+153x^2+90x+27=0$

[2]

3. Consider the function $f(x)=-x^4+4x^3+5x+3$, $x\in\mathbb{R}$. The graph of f is translated four units to the right to form the function $g(x)$. Express $g(x)$ in the form $ax^4+bx^3+cx^2+dx+e$, where a, b, c, d, $e\in\mathbb{Z}$.

[5]

4. Consider the function $f(x)=2x^4+5x^2-2$, $x\in\mathbb{R}$. The graph of f is translated one unit to the left and then stretched vertically with scale factor 2 to form the function $g(x)$. Express $g(x)$ in the form $ax^4+bx^3+cx^2+dx+e$, where a, b, c, d, $e\in\mathbb{Z}$.

[5]

10 Paper 2 Section A – Number of Solutions

Example

Find the values of r such that the equation $x^3 - 6x^2 + 9x = r - 3$ has two distinct real solutions.

[4]

Solution

$x^3 - 6x^2 + 9x = r - 3$

$x^3 - 6x^2 + 9x + 3 = r$ (M1) for valid approach

By considering the graph of $y = x^3 - 6x^2 + 9x + 3$, the local maximum is $(1, 7)$ and the local minimum is $(3, 3)$. (M1) for valid approach

Thus, $r = 3$ or $r = 7$. A2

[4]

Exercise 10

1. Find the range of values of r such that the equation $-2x^3 - 3x^2 + 12x + r - 7 = 0$ has three distinct real solutions.

[4]

2. Find the range of values of k such that the equation $x^4 - 8x^3 + 16x^2 = k - 4x$ has no real solution.

[4]

3. Find the range of values of r such that the equation $x^2 + 180 = \frac{r + 24x^2}{x}$ has at most two distinct real solutions.

[4]

4. Find the range of values of k such that the equation $8x^2(2x + 1) = k + 192x + x^4$ has four distinct real solutions.

[4]

11 Paper 2 Section A – Partial Fractions

Example

Let $f(x)=\dfrac{4}{(x-1)(x-6)}$, $x\neq 1$, $x\neq 6$.

(a) Express $f(x)$ in partial fractions.

[4]

(b) Write down the y-intercept of $f(x)$.

[1]

(c) Write down the equations of the vertical asymptotes of $f(x)$.

[2]

Solution

(a) Let $\dfrac{4}{(x-1)(x-6)}\equiv\dfrac{A}{x-1}+\dfrac{B}{x-6}$, where A and B are constants.

$$\frac{4}{(x-1)(x-6)}\equiv\frac{A(x-6)}{(x-1)(x-6)}+\frac{B(x-1)}{(x-1)(x-6)}$$ M1

$$\frac{4}{(x-1)(x-6)}\equiv\frac{Ax-6A+Bx-B}{(x-1)(x-6)}$$

$4\equiv(A+B)x+(-6A-B)$ A1

$0=A+B$

$B=-A$

$4=-6A-B$

$\therefore 4=-6A-(-A)$ A1

$4=-5A$

$A=-\dfrac{4}{5}$

$\therefore B=-\left(-\dfrac{4}{5}\right)$

$B=\dfrac{4}{5}$

$$\therefore\frac{4}{(x-1)(x-6)}\equiv-\frac{4}{5(x-1)}+\frac{4}{5(x-6)}$$ A1

[4]

(b) $\frac{2}{3}$ A1

[1]

(c) $x=1,\ x=6$ A2

[2]

Exercise 11

1. Let $f(x)=\frac{x+1}{(x-4)(3x-1)}$, $x\neq 4$, $x\neq \frac{1}{3}$.

(a) Express $f(x)$ in partial fractions.

[4]

(b) Write down the x-intercept of $f(x)$.

[1]

(c) Write down the equations of the vertical asymptotes of $f(x)$.

[2]

2. Let $f(x)=\frac{3-x}{2x^2+5x+2}$, $x\neq a$, $x\neq b$, $a<b$.

(a) Write down the value of a and the value of b.

[2]

(b) Express $f(x)$ in partial fractions.

[4]

3. Let $f(x)=\frac{x^2-x-1}{(x+3)(x+7)}$, $x\neq -3$, $x\neq -7$. $f(x)$ can be expressed in the form $A+\frac{B}{x+3}+\frac{C}{x+7}$.

(a) Find the values of A, B and C.

[6]

(b) Write down the equation of the horizontal asymptote of $f(x)$.

[1]

4. Let $f(x)=\dfrac{x^2+3}{(2-x)(5-3x)}$, $x\neq 2$, $x\neq\dfrac{5}{3}$. $f(x)$ can be expressed in the form $A+\dfrac{B}{2-x}+\dfrac{C}{5-3x}$.

(a) Find the values of A, B and C.

[6]

Let $g(x)=\dfrac{1}{f(x)}$.

(b) Show that $g(x)$ has no vertical asymptote.

[1]