AI HL Practice Set 3 Paper 3 Solution

1. (a) (i) $42.3\,\text{s}$ A1

(ii) $1.47\,\text{s}^2$ A1

[2]

(b) $\text{P}(P_1 + P_2 + P_3 < 40.5)$

$= 0.0688229545$ (A1) for correct value

$= 0.0688$ A1

[2]

(c) (i) The required variance

$= \frac{0.7^2}{5}$ (A1) for correct approach

$= 0.098\,\text{s}^2$ A1

(ii) The required variance

$= \frac{0.55^2}{5}$ (A1) for correct approach

$= 0.0605\,\text{s}^2$ A1

[4]

(d) (i) $\bar{P} - \bar{Q} \sim \text{N}\left(14.1 - 14.9, \frac{0.7^2}{5} + \frac{0.55^2}{5}\right)$ A2

$\text{P}(\bar{P} - \bar{Q} > 0)$ M1

$= 0.0222451001$ (A1) for correct value

$= 0.0222$ A1

(ii) $\text{P}(-0.2 < \bar{P} - \bar{Q} < 0.2)$ M1

$= 0.0598891201$ (A1) for correct value

$= 0.0599$ A1

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(e) (i) An unbiased estimate

$= \dfrac{13.9+14.7+13.5+14.0+14.2}{5}$ (A1) for correct approach

$= 14.06\,\text{s}$ A1

(ii) s_{n-1}

$= \sqrt{\dfrac{(13.9-14.06)^2+(14.7-14.06)^2}{+\cdots+(14.2-14.06)^2}{5-1}}$ (A1) for correct approach

$= 0.4393176527\,\text{s}$

$= 0.439\,\text{s}$ A1

[4]

(f) 95% confidence interval:

$(13.515, 14.605)$ A2

[2]

(g) (i) $H_0: \mu_d = 0$ A1

(ii) $H_1: \mu_d > 0$ A1

(iii) $p\text{-value} = 0.7875667907$ (A1) for correct value

$p\text{-value} = 0.788$ A1

(iv) -0.868 A1

(v) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. R1

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2. (a) (i) $(10, -10)$ A1

(ii) 50 A1

[2]

(b) (i) $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$

$= A_3 \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ (M1) for substitution

$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^3 \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ A1

$= \begin{pmatrix} 80 \\ -80 \end{pmatrix}$ A1

(ii) The required area

$= \dfrac{(80)(80)}{2}$ M1

$= 3200$ A1

(iii) $\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^n \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ M1A1

$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ A1

$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 10 \cdot 2^n \\ -10 \cdot 2^n \end{pmatrix}$ A1

$\therefore x_n - y_n$

$= 10 \cdot 2^n - (-10 \cdot 2^n)$

$= 20 \cdot 2^n$ A1

$= 5 \cdot 2^2 \cdot 2^n$ M1

$= 5 \cdot 2^{n+2}$ AG

[11]

(c) (i) $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$

$= A_4 \begin{pmatrix} 0 \\ -10 \end{pmatrix}$ (M1) for substitution

$$= \begin{pmatrix} 1.1^4 & 0 \\ 0 & 1.1^4 \end{pmatrix}\begin{pmatrix} 1.1^3 & 0 \\ 0 & 1.1^3 \end{pmatrix}\begin{pmatrix} 1.1^2 & 0 \\ 0 & 1.1^2 \end{pmatrix}$$

$$\begin{pmatrix} 1.1^1 & 0 \\ 0 & 1.1^1 \end{pmatrix}\begin{pmatrix} 0 \\ -10 \end{pmatrix}$$ A1

$$= \begin{pmatrix} 1.1^{10} & 0 \\ 0 & 1.1^{10} \end{pmatrix}\begin{pmatrix} 0 \\ -10 \end{pmatrix}$$ M1A1

$$= \begin{pmatrix} 0 \\ -10 \cdot 1.1^{10} \end{pmatrix}$$ A1

(ii) $y_n = -10 \cdot 1.1^{1+2+\cdots+n}$ A1

$y_n < -375$

$\therefore -10 \cdot 1.1^{1+2+\cdots+n} < -375$ (M1) for setting inequality

$1.1^{1+2+\cdots+n} > 37.5$

$1.1^{1+2+\cdots+n} - 37.5 > 0$

$1.1^{\frac{n}{2}(n+1)} - 37.5 > 0$ (A1) for correct inequality

By considering the graph of

$y = 1.1^{\frac{n}{2}(n+1)} - 37.5$, $n > 8.2351929$. (A1) for correct value

Thus, the least value of n is 9. A1

[10]

(d) (i) $\begin{pmatrix} \cos 180^\circ & \sin 180^\circ \\ \sin 180^\circ & -\cos 180^\circ \end{pmatrix}\begin{pmatrix} \cos 90^\circ & \sin 90^\circ \\ \sin 90^\circ & -\cos 90^\circ \end{pmatrix}$ A1

(ii) For correct quadrant A1

For correct points A1

[3]