Chapter 13 Solution

Exercise 41

1. (a) $\dfrac{AB}{\sin 48^\circ} = \dfrac{10}{\sin 114^\circ}$ (M1)(A1) for substitution

$AB = \dfrac{10\sin 48^\circ}{\sin 114^\circ}$

$AB = 8.134732862$

$AB = 8.13\ \text{cm}$ A1 N3

[3]

(b) $B\hat{A}C = 180^\circ - 114^\circ - 48^\circ$

$B\hat{A}C = 18^\circ$ (A1) for correct value

$\dfrac{BC}{\sin 18^\circ} = \dfrac{10}{\sin 114^\circ}$ (M1) for sine rule

$BC = \dfrac{10\sin 18^\circ}{\sin 114^\circ}$

$BC = 3.382612127$

$BC = 3.38\ \text{cm}$ A1 N3

[3]

2. (a) $B\hat{C}A = 180^\circ - 43^\circ - 92^\circ$ (M1) for valid approach

$B\hat{C}A = 45^\circ$

$\dfrac{AB}{\sin 45^\circ} = \dfrac{21}{\sin 43^\circ}$ (M1) for sine rule

$AB = \dfrac{21\sin 45^\circ}{\sin 43^\circ}$

$AB = 21.77313506$

$AB = 21.8\ \text{cm}$ A1 N3

[3]

(b) The area of ΔABC

$= \dfrac{1}{2}(AC)(AB)\sin B\hat{A}C$ (M1) for valid approach

$= \dfrac{1}{2}(21)(21.77313506)\sin 92^\circ$ (A1) for substitution

$= 228.4786503$

$= 228\ \text{cm}^2$ A1 N3

[3]

3. (a) $\frac{1}{2}(86)(\text{BC})\sin 40^\circ = 1900$ (M1)(A1) for substitution

$\text{BC} = 68.74128537$

$\text{BC} = 68.7\ \text{cm}$ A1 N3

[3]

(b) $\text{AB} = \sqrt{86^2 + 68.74128537^2 - 2(86)(68.74128537)\cos 40^\circ}$ (M1)(A1) for substitution

$\text{AB} = 55.35374433$

$\text{AB} = 55.4\ \text{cm}$ A1 N3

[3]

4. (a) $\frac{1}{2}(35)(54)\sin \text{B}\hat{\text{A}}\text{C} = 892$ (M1)(A1) for substitution

$\text{B}\hat{\text{A}}\text{C} = 70.7198401^\circ$

$\text{B}\hat{\text{A}}\text{C} = 70.7^\circ$ A1 N3

[3]

(b) $\text{BC} = \sqrt{35^2 + 54^2 - 2(35)(54)\cos 70.7198401^\circ}$ (M1)(A1) for substitution

$\text{BC} = 53.78560244$

$\text{BC} = 53.8\ \text{cm}$ A1 N3

[3]

Exercise 42

1. (a) $A\hat{P}B = 180^\circ - 45^\circ - 34^\circ$ (M1) for valid approach

$A\hat{P}B = 101^\circ$ A1 N2

[2]

(b) $\dfrac{PB}{\sin 45^\circ} = \dfrac{120}{\sin 101^\circ}$ (M1)(A1) for substitution

$PB = 86.44097797$

$PB = 86.4 \text{ m}$ A1 N3

[3]

(c) $\sin 34^\circ = \dfrac{PC}{86.44097797}$ (M1) for valid approach

$PC = 48.33718145$

$PC = 48.3 \text{ m}$ A1 N2

[2]

2. (a) $P\hat{R}Q = 69^\circ - 24^\circ$ (M1) for valid approach

$P\hat{R}Q = 45^\circ$ A1 N2

[2]

(b) $Q\hat{P}R = 180^\circ - 69^\circ - 90^\circ$ (M1) for valid approach

$Q\hat{P}R = 21^\circ$ A1 N2

[2]

(c) $\dfrac{PQ}{\sin 45^\circ} = \dfrac{5}{\sin 21^\circ}$ (M1)(A1) for substitution

$PQ = 9.865653194$

$PQ = 9.87 \text{ m}$ A1 N3

[3]

3. (a) $\cos R\hat{P}Q = \dfrac{8^2 + 6^2 - 9^2}{2(8)(6)}$ (M1)(A1) for substitution

$R\hat{P}Q = 78.58484226$

Thus, the angle of depression of R from P is 78.6°. A1 N3

[3]

(b) $\cos 78.58484226^\circ = \dfrac{x}{8}$ (M1) for valid approach

$x = 1.583333333$

$x = 1.58$ A1 N2

[2]

(c) $\sin 78.58484226^\circ = \dfrac{h}{8}$ (M1) for valid approach

$h = 7.841750797$

$h = 7.84$ A1 N2

[2]

4. (a) 29° A1 N1

[1]

(b) $\dfrac{\sin B\hat{F}A}{10} = \dfrac{\sin 29^\circ}{30}$ (M1)(A1) for substitution

$\sin B\hat{F}A = 0.161603206$

$B\hat{F}A = 9.299964331^\circ$ (A1) for correct value

The angle of elevation of F from B

$= O\hat{B}F$

$= 9.299964331^\circ + 29^\circ$

$= 38.299964331^\circ$

$= 38.3^\circ$ A1 N4

[4]

(c) $\sin 38.299964331^\circ = \dfrac{OF}{30}$ (M1) for valid approach

$OF = 18.5933563$

Thus, the height of the vertical flagpole is 18.6 m. A1 N2

[2]

Exercise 43

1. (a) 60° A1 N1

[1]

(b) $\frac{1}{2}(3)(BC)\sin 120^\circ = \frac{27\sqrt{3}}{4}$ (M1)(A1) for substitution

$BC = 9 \text{ cm}$ A1 N3

[3]

(c) The arc length of the sector BDC

$= 2\pi(9) \times \frac{60^\circ}{360^\circ}$ (A1) for substitution

$= 3\pi \text{ cm}$ A1 N2

[2]

2. (a) $\cos A\hat{B}C = \frac{10^2 + 10^2 - (10\sqrt{3})^2}{2(10)(10)}$ (M1)(A1) for substitution

$A\hat{B}C = 120^\circ$ A1 N3

[3]

(b) 60° A1 N1

[1]

(c) The perimeter of the figure ADC

$= 10\sqrt{3} + 2(10) + 2\pi(10) \times \frac{60^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 47.79248359$

$= 47.8 \text{ cm}$ A1 N3

[3]

3. (a) $AC^2 = 6^2 + 16^2 - 2 \times 6 \times 16 \times \cos 60^\circ$ M1A1

$AC^2 = 196$ A1

$AC = \sqrt{196}$

$AC = 14 \text{ cm}$ AG N0

[3]

(b) The area of this shape

$= \frac{1}{2}(6)(16)\sin 60^\circ + \pi\left(\frac{14}{2}\right)^2 \times \frac{180^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 118.5382394$

$= 119 \text{ cm}^2$ A1 N3

[3]

4. (a) $\frac{1}{2}(8)(\text{AC})\sin 60^{\circ} = 24\sqrt{3}$ M1A1

$\text{AC} = 12\text{ cm}$ AG N0

[2]

(b) $\text{AB} = \sqrt{12^2 + 8^2 - 2(12)(8)\cos\frac{\pi}{3}}$ (A1) for substitution

$\text{AB} = \sqrt{112}\text{ cm}$ A1 N2

[2]

(c) The perimeter of this shape

$= \sqrt{112} + 8 + 2\pi\left(\frac{12}{2}\right) \times \frac{180^{\circ}}{360^{\circ}}$ (M1)(A1) for substitution

$= 37.43256117$

$= 37.4\text{ cm}$ A1 N3

[3]

Exercise 44

1. (a) The length of arc ABC

$= 2\pi(55) - 2\pi(55) \times \frac{155^\circ}{360^\circ}$ (M1)(A1) for substitution

$= 196.7858732$

$= 197$ cm A1 N3

[3]

(b) The perimeter of sector OABC

$= 196.7858732 + 55 + 55$ (M1) for valid approach

$= 306.7858732$

$= 307$ cm A1 N2

[2]

(c) The area of sector OABC

$= \pi(55)^2 - \pi(55)^2 \times \frac{155^\circ}{360^\circ}$ (M1) for valid approach

$= 5411.611512$

$= 5410 \text{ cm}^2$ A1 N2

[2]

2. (a) The length of arc ABC

$= 2\pi(20) \times \frac{54.5^\circ}{360^\circ}$ (A1) for substitution

$= 19.02408885$ cm (A1) for correct value

The perimeter of sector OABC

$= 19.02408885 + 20 + 20$ (M1) for valid approach

$= 59.02408885$

$= 59.0$ cm A1 N4

[4]

(b) The area of sector OABC

$= \pi(20)^2 \times \frac{54.5^\circ}{360^\circ}$ (A1) for substitution

$= 190.2408885$

$= 190 \text{ cm}^2$ A1 N2

[2]

3. (a) $2\pi(8.6)\times\frac{\theta^\circ}{360^\circ}=9.46$ (A1) for correct equation

$\theta=63.02535746$

$\theta=63.0$ A1 N2

[2]

(b) The reflex $\hat{AOC}$

$=360^\circ-63.02535746^\circ$ (M1) for valid approach

$=296.9746425^\circ$ (A1) for correct value

The area of sector OADC

$=\pi(8.6)^2\times\frac{296.9746425^\circ}{360^\circ}$ (A1) for substitution

$=191.6741927$

$=192\text{ cm}^2$ A1 N4

[4]

4. (a) $\pi(\text{OC})^2\times\frac{114.5^\circ}{360^\circ}=14$ (A1) for correct equation

$\text{OC}^2=14.01119499$

$\text{OC}=3.743153081$

$\text{OC}=3.74\text{ cm}$ A1 N2

[2]

(b) The reflex $\hat{AOC}$

$=360^\circ-114.5^\circ$ (M1) for valid approach

$=245.5^\circ$ (A1) for correct value

The area of sector OADC

$=\pi(3.743153081)^2\times\frac{245.5^\circ}{360^\circ}$ (A1) for substitution

$=30.01746725$

$=30.0\text{ cm}^2$ A1 N4

[4]

Exercise 45

1. (a) The required area

$= \pi(125)^2 \times \frac{142^\circ}{360^\circ}$ (A1) for substitution

$= 19362.24639$

$= 19400 \text{ cm}^2$ A1 N2

[2]

(b) The required area

$= \frac{1}{2}(125)(125)\sin 142^\circ$ (A1) for substitution

$= 4809.855276$

$= 4810 \text{ cm}^2$ A1 N2

[2]

(c) The required area

$= 19362.24639 - 4809.855276$ (M1) for valid approach

$= 14552.39111$

$= 14600 \text{ cm}^2$ A1 N2

[2]

2. (a) The required length

$= 2\pi(1740) \times \frac{80^\circ}{360^\circ}$ (A1) for substitution

$= 2429.498319$

$= 2430 \text{ cm}$ A1 N2

[2]

(b) $AB = \sqrt{1740^2 + 1740^2 - 2(1740)(1740)\cos 80^\circ}$ (M1)(A1) for substitution

$AB = 2236.900882$

$AB = 2240 \text{ cm}$ A1 N3

[3]

(c) The required perimeter

$= 2429.498319 + 2236.900882$ (M1) for correct approach

$= 4666.399201$

$= 4670 \text{ cm}$ A1 N2

[2]

3. (a) $\cos A\hat{O}B = \dfrac{20^2 + 20^2 - 32^2}{2(20)(20)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.28$

$A\hat{O}B = 106.2602047^\circ$

$A\hat{O}B = 106^\circ$ A1 N3

[3]

(b) The area of the sector AOB

$= \pi(20)^2 \times \dfrac{106.2602047^\circ}{360^\circ}$ (A1) for substitution

$= 370.9180872$

$= 371 \text{ cm}^2$ A1 N2

[2]

(c) The area of the shaded region

$= 370.9180872 - 192$ (M1) for valid approach

$= 178.9180872$

$= 179 \text{ cm}^2$ A1 N2

[2]

4. (a) $\cos A\hat{O}B = \dfrac{40^2 + 40^2 - 60^2}{2(40)(40)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.125$

$A\hat{O}B = 97.18075578^\circ$

$A\hat{O}B = 97.2^\circ$ A1 N3

[3]

(b) The length of the minor arc AB

$= 2\pi(40) \times \dfrac{97.18075578^\circ}{360^\circ}$ (A1) for substitution

$= 67.84496632$

$= 67.8 \text{ cm}^2$ A1 N2

[2]

(c) The perimeter of the shaded segment

$= (2\pi(40) - 67.84496632) + 60$ (M1) for valid approach

$= 243.482446$

$= 243 \text{ cm}$ A1 N2

[2]

Exercise 46

1. (a) The bearing of C from E

$= 360^\circ - (180^\circ - 77^\circ)$ (M1) for valid approach

$= 257^\circ$ A1 N2

[2]

(b) (i) $A\hat{C}E = 180^\circ - 77^\circ$

$A\hat{C}E = 103^\circ$ (A1) for correct value

$A\hat{E}C = 180^\circ - 103^\circ - 51^\circ$

$A\hat{E}C = 26^\circ$ A1 N2

(ii) $\frac{AE}{\sin 103^\circ} = \frac{800}{\sin 26^\circ}$ (M1)(A1) for substitution

$AE = 1778.164593$

$AE = 1780$ km A1 N3

[5]

(c) $DE = \sqrt{1778.164593^2 + 1350^2 - 2(1778.164593)(1350)\cos 51^\circ}$ (M1)(A1) for substitution

$DE = 1401.061804$

$DE = 1400$ km A1 N3

[3]

(d) (i) B lies on AC such that $BE \perp AC$. (M1) for valid approach

$\sin B\hat{A}E = \frac{BE}{AE}$

$BE = 1381.893432$

$BE = 1380$ km A1 N2

(ii) The time required

$= \frac{DE}{62}$ (M1) for valid approach

$= \frac{1401.061804}{62}$

$= 22.59777103$ h

The speed of the boat

$= \frac{BE}{22.59777103}$ (M1) for valid approach

$= \frac{1381.893432}{22.59777103}$

$= 61.15175829$

$= 61.2$ km/h A1 N3

[5]

2. (a) $A\hat{D}C = 160^\circ - 90^\circ$

$A\hat{D}C = 70^\circ$ (A1) for correct value

$\frac{AC}{\sin 70^\circ} = \frac{15}{\sin 58^\circ}$ (M1) for sine rule

$AC = 16.62097866$

$AC = 16.6\text{ km}$ A1 N3

[3]

(b) $D\hat{A}C = 180^\circ - 70^\circ - 58^\circ$

$D\hat{A}C = 52^\circ$ (A1) for correct value

The area of the triangle DAC

$= \frac{1}{2}(15)(16.62097866)\sin 52^\circ$ (A1) for substitution

$= 98.2313244$

$= 98.2\text{ km}^2$ A1 N3

[3]

(c) (i) The area of the triangle ABC

$= 2(98.2313244)$

$= 196.4626488$

$= 196\text{ km}^2$ A1 N1

(ii) $\frac{1}{2}(16.620979)(BC)\sin 56^\circ = 196.4626488$ (M1)(A1) for correct equation

$BC = 28.51538144$

$BC = 28.5\text{ km}$ A1 N3

[4]

(d) (i) $\frac{DC}{\sin 52^\circ} = \frac{15}{\sin 58^\circ}$ (A1) for substitution

$DC = 13.93807893$

$DC = 13.9\text{ km}$ A1 N2

(ii) $BD = \sqrt{\begin{array}{l}13.93807893^2 + 28.51538144^2 \\ -2(13.93807893)(28.51538144) \\ \cos(58^\circ + 56^\circ)\end{array}}$ (A1) for substitution

$BD = 36.47892111$

$BD = 36.5\text{ km}$ A1 N2

(iii) $\frac{28.51538144}{1} = \frac{36.47892111}{T}$ (M1) for valid approach

$T = 1.279271722$

Thus, the time taken is 1.28 hours. A1 N2

[6]

3. (a) $\hat{ABC} = 360^\circ - 312^\circ$

$\hat{ABC} = 48^\circ$ (A1) for correct value

$\frac{AC}{\sin 48^\circ} = \frac{60}{\sin 83^\circ}$ (A1) for substitution

$AC = 44.9235428$

$AC = 44.9\text{ km}$ A1 N3

[3]

(b) The area of the triangle ABC

$= \frac{1}{2}(44.9235428)(60)\sin 49^\circ$ (M1)(A1) for substitution

$= 1017.126844$

$= 1020\text{ km}^2$ A1 N3

[3]

(c) The area of the triangle ACD

$= 1.5(1017.126844)$

$= 1525.690266$ (A1) for correct value

$\frac{1}{2}(DC)(AC)\sin\theta^\circ = 1525.690266$ (M1) for setting equation

$\frac{1}{2}(83)(44.9235428)\sin\theta^\circ = 1525.690266$ (A1) for correct equation

$\sin\theta^\circ = 0.8183597859$

$\theta^\circ = 54.92093749^\circ$

$\theta^\circ = 54.9^\circ$ A1 N4

[4]

(d) $\frac{BC}{\sin(180^\circ - 48^\circ - 83^\circ)} = \frac{60}{\sin 83^\circ}$

$BC = 45.62263905$ (A1) for correct value

$BD = \sqrt{\begin{array}{l}83^2 + 45.62263905^2 \\ -2(83)(45.62263905) \\ \times\cos(54.92093749^\circ + 83^\circ)\end{array}}$ (M1)(A1) for substitution

$BD = 120.7954013$

$\frac{BD}{\text{Speed of Q}} = \frac{BC + DC}{50}$ (M1) for valid approach

$\frac{120.7954013}{\text{Speed of Q}} = \frac{45.62263905 + 83}{50}$

$\text{Speed of Q} = 46.95728613$

$\text{Speed of Q} = 47.0\text{ km/h}$ A1 N5

[5]

4. (a) $B\hat{A}D = 220^\circ - 180^\circ$

$B\hat{A}D = 40^\circ$ (A1) for correct value

$B\hat{D}A = 180^\circ - 40^\circ - 61^\circ$

$B\hat{D}A = 79^\circ$ (A1) for correct value

$\frac{BD}{\sin 40^\circ} = \frac{80}{\sin 79^\circ}$ (A1) for substitution

$BD = 52.38547754$

$BD = 52.4 \text{ km}$ A1 N4

[4]

(b) The area of the triangle ABD

$= \frac{1}{2}(52.38547754)(80)\sin 61^\circ$ (M1)(A1) for substitution

$= 1832.694841$

$= 1830 \text{ km}^2$ A1 N3

[3]

(c) The area of the triangle BCD is 1832.694841 km^2.

$\frac{1}{2}(CD)(BD)\sin\theta^\circ = 1832.694841$ (M1) for setting equation

$\frac{1}{2}(72)(52.38547754)\sin\theta^\circ = 1832.694841$ (A1) for substitution

$\sin\theta^\circ = 0.9717996746$ (A1) for correct value

$\theta^\circ = 180^\circ - 76.36074617^\circ$

$\theta^\circ = 103.6392538^\circ$

$\theta^\circ = 104^\circ$ A1 N4

[4]

(d) $BC = \sqrt{72^2 + 52.38547754^2 - 2(72)(52.38547754)\cos 103.6392538^\circ}$ (A1) for substitution

$BC = 98.52440125$ (A1) for correct value

The total distance

$= 98.52440125 + 80$

$= 178.52440125$

The minimum time required

$= \frac{178.52440125}{70}$ (M1) for valid approach

$= 2.550348589 \text{ h}$ (A1) for correct value

$= 2$ hours 33 minutes A1 N5

[5]

Exercise 47

1. (a) $\frac{\sin A\hat{C}B}{20.8} = \frac{\sin 71.5^\circ}{26.6}$ (M1)(A1) for substitution

$A\hat{C}B = 47.86330515^\circ$

$A\hat{C}B = 47.9^\circ$ A1 N3

[3]

(b) (i) $B\hat{A}C = 180^\circ - 71.5^\circ - 47.86330515^\circ$ (M1) for valid approach

$B\hat{A}C = 60.63669485^\circ$

$B\hat{A}C = 60.6^\circ$ A1 N2

(ii) $\frac{BC}{\sin 60.63669485^\circ} = \frac{26.6}{\sin 71.5^\circ}$ (M1)(A1) for substitution

$BC = 24.44592146$

$BC = 24.4\ \text{km}$ A1 N3

[5]

(c) (i) $\cos B\hat{O}C = \frac{14^2 + 14^2 - 24.44592146^2}{2(14)(14)}$ (M1)(A1) for substitution

$B\hat{O}C = 121.634431^\circ$ (A1) for correct value

Reflex $B\hat{O}C$

$= 360^\circ - 121.634431^\circ$

$= 238.365569^\circ$

$= 238^\circ$ A1 N4

(ii) The required area

$= \pi(14)^2 \times \frac{238.365569^\circ}{360^\circ}$ (A1) for substitution

$= 407.7058723$

$= 408\ \text{cm}^2$ A1 N2

[6]

2. (a) $\dfrac{AC}{\sin 50^\circ} = \dfrac{43.2}{\sin 70^\circ}$ (M1)(A1) for substitution

$AC = 35.21696266$

$AC = 35.2$ cm A1 N3

[3]

(b) $B\hat{A}C = 180^\circ - 50^\circ - 70^\circ$ (M1) for valid approach

$B\hat{A}C = 60^\circ$ (A1) for correct value

$\dfrac{BC}{\sin 60^\circ} = \dfrac{43.2}{\sin 70^\circ}$ (M1)(A1) for substitution

$BC = 39.81333536$

$BC = 39.8$ cm A1 N5

[5]

(c) (i) $\cos B\hat{O}C = \dfrac{23^2 + 23^2 - 39.81333536^2}{2(23)(23)}$ (M1)(A1) for substitution

$B\hat{O}C = 119.8813635^\circ$

$B\hat{O}C = 120^\circ$ A1 N3

(ii) The area of the sector $OBDC$

$= \pi(23)^2 \times \dfrac{119.8813635^\circ}{360^\circ}$ (A1) for substitution

$= 553.4198315$

$= 553 \text{ cm}^2$ A1 N2

(iii) The required area

$= 553.4198315 - \dfrac{1}{2}(23)(23)\sin 119.8813635^\circ$ (M1)(A1) for substitution

$= 324.0827669$

$= 324 \text{ cm}^2$ A1 N3

[8]

3. (a) $\dfrac{AC}{\sin 40^\circ} = \dfrac{11}{\sin 21^\circ}$ (M1)(A1) for substitution

$AC = 19.73017876$

$AC = 19.7\ \text{cm}$ A1 N3

[3]

(b) $O\hat{A}C = 180^\circ - 40^\circ - 21^\circ$ (M1) for valid approach

$O\hat{A}C = 119^\circ$

$\dfrac{OC}{\sin 119^\circ} = \dfrac{11}{\sin 21^\circ}$ (M1)(A1) for substitution

$OC = 26.84619758$

$OC = 26.8\ \text{cm}$ A1 N4

[4]

(c) The area of the sector OBA

$= \pi(11)^2 \times \dfrac{40^\circ}{360^\circ}$ (A1) for substitution

$= 42.2369679$ (A1) for correct value

The area of the triangle OAC

$= \dfrac{1}{2}(11)(26.84619758)\sin 40^\circ$ (A1) for substitution

$= 94.91021744$ (A1) for correct value

The required area

$= 94.91021744 - 42.2369679$ (M1) for valid approach

$= 52.67324954$

$= 52.7\ \text{cm}^2$ A1 N6

[6]

4. (a) $\frac{\sin A\hat{C}B}{28} = \frac{\sin 63^\circ}{47}$ (M1)(A1) for substitution

$\sin A\hat{C}B = 0.530812397$

$A\hat{C}B = 32.06036169^\circ$

$A\hat{C}B = 32.1^\circ$ A1 N3

[3]

(b) $O\hat{A}C = 180^\circ - 63^\circ - 32.06036169^\circ$ (M1) for valid approach

$O\hat{A}C = 84.93963831^\circ$

$\frac{OC}{\sin 84.93963831^\circ} = \frac{47}{\sin 63^\circ}$ (M1)(A1) for substitution

$OC = 52.54373345$

$OC = 52.5$ cm A1 N4

[4]

(c) The length of the arc ADB

$= 2\pi(28) \times \frac{63^\circ}{360^\circ}$ (A1) for substitution

$= 30.78760801$ (A1) for correct value

The required perimeter

$= 30.78760801 + (52.54373345 - 28) + 47$ (M1) for valid approach

$= 102.3313415$

$= 102$ cm A1 N4

[4]