

# AI HL Practice Set 1 Paper 3 Solution

1. (a) (i)  $\tan \frac{\pi}{6} = \frac{DE}{30}$  (M1) for tangent ratio  
 $DE = 17.32050808 \text{ m}$   
 $DE = 17.3 \text{ m}$  A1
- (ii) The area of the triangle ODE  
 $= \frac{(30)(17.32050808)}{2}$  A1  
 $= 259.8076212 \text{ m}^2$   
 $= 260 \text{ m}^2$  AG
- (iii) 1.46 A1
- (b) (i)  $\frac{(30)(DE)}{2} = \frac{(30)(30)}{3}$  (M1) for setting equation  
 $DE = 20 \text{ m}$  A1
- (ii)  $\tan \hat{D}OE = \frac{20}{30}$  (M1) for tangent ratio  
 $\hat{D}OE = 0.5880026035 \text{ rad}$   
 $\hat{D}OE = 0.588 \text{ rad}$  A1
- (iii) 0.395 rad A1
- (c) (i) BD and CF are perpendicular. A1
- (ii) The required coordinates  
 $= \left( \frac{20+30}{2}, \frac{30+20}{2} \right)$  (A1) for substitution  
 $= (25, 25)$  A1
- (iii) (20, 20) A2

[4]

[5]

[5]

(d) 
$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$
 A3 [3]

(e) 
$$\mathbf{M} + \mathbf{M}^2 + \mathbf{M}^3 = \begin{pmatrix} 10 & 8 & 12 & 6 & 12 & 8 \\ 8 & 4 & 8 & 4 & 6 & 4 \\ 12 & 8 & 10 & 8 & 12 & 6 \\ 6 & 4 & 8 & 4 & 8 & 4 \\ 12 & 6 & 12 & 8 & 10 & 8 \\ 8 & 4 & 6 & 4 & 8 & 4 \end{pmatrix}$$
 (M1) for valid approach [3]

Thus, the total number of walks of length at most 3 from C to E is 4. A1

(f) (i) 46.1 A1 [2]

(ii) 54.1 A1

(g) For any three edges correct A1 [2]  
For all edges correct A1

1. Choose OA of distance 30
  2. Choose AB of distance 20
  3. Choose BC of distance 10
  4. Choose CD of distance 10
  5. Choose DE of distance 20
  6. Choose EO of distance 30
- Thus, the required upper bound is 120 m. A1

[3]

- (h) For any two edges correct A1  
For all edges correct A1
1. Choose BD of distance 14.1
  2. Choose AB of distance 20
  3. Choose DE of distance 20
  4. Choose OA of distance 30
- Therefore, the distance of a minimum spanning tree after deleting the vertex C is 84.1. A1
- The required lower bound  
 $= 84.1 + 10 + 10$   
 $= 104.1 \text{ m}$  A1

[4]

2. (a) (i) The required probability  

$$= \left( \frac{45 + 35 + 20}{300} \right) \left( \frac{45 + 35 + 20 - 1}{300 - 1} \right)$$
 (M1) for valid approach  

$$= \frac{33}{299}$$
 A1
- (ii) The required probability  

$$\left( \frac{45}{300} \right) \left( \frac{45 - 1}{300 - 1} \right) + \left( \frac{35}{300} \right) \left( \frac{35 - 1}{300 - 1} \right)$$
  

$$+ \left( \frac{20}{300} \right) \left( \frac{20 - 1}{300 - 1} \right)$$
  

$$= \frac{\frac{33}{299}}{\frac{33}{299}}$$
 M1A1  

$$= \frac{71}{198}$$
 A1
- (b) (i)  $H_0: p = 0.18$  A1
- (ii)  $H_1: p > 0.18$  A1
- (iii)  $P(X \geq 7)$   
 $= 1 - P(X \leq 6)$  (M1) for valid approach  
 $= 0.148763448$   
 Thus, the  $p$ -value is 0.149. A1
- (iv) The null hypothesis is not rejected. A1  
 As  $p$ -value  $> 0.05$ . R1
- (c) (i) 48.6 A1
- (ii) 19.6 A1
- (iii) 385 A1

[5]

[6]

[3]

(d)	(i)	$H_0$ : The data follows a normal distribution with parameters $N(48.6, 19.6126367^2)$ .	A1	
	(ii)	16.4	A1	
	(iii)	2	A1	
	(iv)	$p$ -value = 0.0004378451724 $p$ -value = 0.000438	(A1) for correct value A1	
	(v)	The null hypothesis is rejected. As $p$ -value < 0.05.	A1 R1	[7]
(e)	(i)	$H_0: \lambda = 11$	A1	
	(ii)	$H_1: \lambda < 11$	A1	[2]
(f)		The required probability = $P(X \leq 5   \lambda = 11)$ = 0.0375198141 = 0.0375	(M1) for valid approach A1	[2]
(g)		The required probability = $P(X \geq 6   \lambda = 7)$ = $1 - P(X \leq 5   \lambda = 7)$ = 0.6992917238 = 0.699	(M1) for valid approach A1	[2]