AI HL Practice Set 4 Paper 1 Solution

1. (a) $V = \frac{1}{3}\pi r^2 h$

$\therefore 128\pi = \frac{1}{3}\pi r^2 (6)$ (A1) for correct equation

$r^2 = 64$

$r = 8$

Thus, the required radius is $8\,\text{cm}$. A1

[2]

(b) l

$= \sqrt{r^2 + h^2}$ (M1) for valid approach

$= \sqrt{8^2 + 6^2}$

$= 10$

Thus, the required slant height is $10\,\text{cm}$. A1

[2]

(c) The total surface area

$= \pi r^2 + \pi rl$

$= \pi(8)^2 + \pi(8)(10)$ (A1) for substitution

$= 144\pi\ \text{cm}^2$ A1

[2]

2. (a) (i) 20 hours A1

(ii) 15 hours A1

[2]

(b) 5 workers worked for more than 30 hours. (R1) for correct argument
Therefore, 12.5% of the workers worked for more than 30 hours.

$\therefore k = 30$ A1

[2]

3. (a) (i) c_n A1

(ii) b_n A1

[2]

(b) (i) 1.25 A1

(ii) $\frac{3125}{128}$ A1

(iii) S_8

$= \frac{10(1.25^8 - 1)}{1.25 - 1}$ (A1) for substitution

$= 198.4185791$

$= 198$ A1

[4]

4. (a) (i) The radius

$= \sqrt{(10-6)^2 + (12-14)^2}$ (A1) for substitution

$= 4.472135955 \text{ km}$

$= 4.47 \text{ km}$ A1

(ii) 4 km A1

(iii) The apartment at P A1

[4]

(b) $x + y - 20 = 0$ A2

[2]

5. (a) $\mathrm{E}(X) = 8.64$

$\therefore 0.72n = 8.64$ (A1) for correct equation

$n = 12$ A1

[2]

(b) $\mathrm{Var}(X)$

$= (12)(0.72)(1-0.72)$ (A1) for substitution

$= 2.4192$ A1

[2]

(c) $\mathrm{P}(X \geq 11)$

$= 1 - \mathrm{P}(X \leq 10)$ (A1) for substitution

$= 0.1099809898$

$= 0.110$ A1

[2]

6. (a) By TVM Solver:

N = 120
I% = 4.5
PV = 0
PMT = −200
FV = ?
P / Y = 12
C / Y = 1
PMT : END

(A2) for correct values

FV = 30095.13482

Thus, the value of the investment after ten years is \$30100. A1

[3]

(b) By TVM Solver:

N = 144
I% = 4.5
PV = 0
PMT = ?
FV = 5 × 30095.13482
P / Y = 12
C / Y = 1
PMT : END

(A2) for correct values

PMT = −794.6316652

Thus, the new amount of deposit is \$795. A1

[3]

7. (a) x

$= -\dfrac{b}{2a}$

$= -\dfrac{100}{2(-1)}$ (A1) for substitution

$= 50$ A1

[2]

(b) The required maximum height

$= -50^2 + 100(50) - 1600$ A1

$= -2500 + 5000 - 1600$

$= 900 \text{ m}$ AG

[1]

(c) $V = 0$

$-x^2 + 100x - 1600 = 0$

$x = 20$ or $x = 80$ (A1) for correct values

The required horizontal distance

$= 80 - 20$ (M1) for valid approach

$= 60 \text{ m}$ A1

[3]

8. (a) $\dfrac{\sin \hat{ACB}}{AB} = \dfrac{\sin \hat{ABC}}{AC}$ (M1) for sine rule

$\dfrac{\sin \hat{ACB}}{13.9} = \dfrac{\sin 60.8^\circ}{17.7}$ (A1) for substitution

$\hat{ACB} = 43.27612856^\circ$

$\hat{ACB} = 43.3^\circ$ A1

[3]

(b) The area of the triangle ABC

$= \dfrac{1}{2}(AB)(AC)\sin \hat{BAC}$ (M1) for area formula

$= \dfrac{1}{2}(13.9)(17.7)\sin(180^\circ - 60.8^\circ - 43.27612856^\circ)$ (A1) for substitution

$= 119.3212815 \text{ cm}^2$

$= 119 \text{ cm}^2$ A1

[3]

9. (a) $\frac{\mathrm{d}x}{\mathrm{d}t} = \pi x^2 \cos \pi t$

$\frac{1}{x^2}\mathrm{d}x = \pi \cos \pi t \mathrm{d}t$ (M1) for valid approach

$\therefore \int \frac{1}{x^2}\mathrm{d}x = \int \pi \cos \pi t \mathrm{d}t$ A1

[2]

(b) Let $u = \pi t$.

$\frac{\mathrm{d}u}{\mathrm{d}t} = \pi \Rightarrow \mathrm{d}u = \pi \mathrm{d}t$ A1

$\therefore \int \frac{1}{x^2}\mathrm{d}x = \int \cos u \mathrm{d}u$ (A1) for correct working

$-\frac{1}{x} = \sin u + C$

$\frac{1}{x} = -\sin \pi t + C$ A1

[3]

(c) $\frac{1}{1} = -\sin 2.5\pi + C$ (M1) for substitution

$1 = -1 + C$

$C = 2$ (A1) for correct value

$\therefore \frac{1}{x} = -\sin \pi t + 2$

$x = \frac{1}{-\sin \pi t + 2}$ A1

[3]

10. (a) An unbiased estimate

$= \bar{X}$ (A1) for correct approach

$= \frac{18.95 + 25.15}{2}$

$= 22.05$ A1

[2]

(b) $25.15 - 18.95 = 2(1.959963986)\left(\frac{\sigma}{\sqrt{10}}\right)$ M1A1

$\sigma = 5.001653508$

$\sigma = 5.00$ A1

[3]

11. (a) $y = \sqrt{3-x}$

$\Rightarrow x = \sqrt{3-y}$ (M1) for swapping variables

$10 = \sqrt{3-y}$

$100 = 3-y$

$y = -97$ (M1) for valid approach

$\therefore f^{-1}(10) = -97$ A1

[3]

(b) (i) 5 A1

(ii) $(f^{-1} \circ g^{-1})(\pi)$

$= f^{-1}(5)$

$5 = \sqrt{3-x}$ (M1) for valid approach

$25 = 3-x$

$x = -22$

$\therefore f^{-1}(5) = -22$ A1

[3]

12. (a) (i) $a = 32$ A1

$b = 20.6$ A1

(ii) The estimated number of oil refills

$= 32(2.5) + 20.6$ (A1) for substitution

$= 100.6$ A1

[4]

(b) (i) $r = 0.9765724246$

$r = 0.977$ A1

(ii) $R^2 = 0.9536937004$

$R^2 = 0.954$ A1

(iii) 95.4% of the variability of the data is explained by the regression model. A1

[3]

13. (a) CE — A1

[1]

(b) For any two edges correct — A1

For all edges correct — A1

1. Choose BE of weight 22
2. Choose DE of weight 24
3. Choose AD of weight 10
4. Choose AC of weight 20

Thus, the minimum spanning tree is a tree containing BE, DE, AD and AC. — A1

[3]

(c) 76 — A1

[1]

14. (a) (i) $H_0: p = 0.25$ — A1

(ii) $H_1: p > 0.25$ — A1

[2]

(b) $\mathrm{P}(X \geq 39) = 0.4193193762$ — (M1) for valid approach

Thus, the p-value is 0.419. — A1

[2]

(c) The null hypothesis is not rejected. — A1

As p-value > 0.05. — R1

[2]

15. (a) $y = e^{5x}$

$\Rightarrow x = e^{5y}$ (M1) for swapping variables

$5y = \ln x$

$y = \frac{1}{5}\ln x$ (A1) for changing subject

$\therefore f^{-1}(x) = \frac{1}{5}\ln x$ A1

[3]

(b) $\{y : y \in \mathbb{R}\}$ A1

[1]

(c) $(g \circ f)(x)$

$= g(f(x))$

$= (3 + \ln f(x))^2$

$= (3 + \ln e^{5x})^2$ (M1) for substitution

$= (3 + 5x)^2$ (A1) for correct approach

$= 25x^2 + 30x + 9$ A1

[3]

16. (a) Rotation anticlockwise of $\frac{5\pi}{6}$ radians about the origin. A1

[1]

(b) $\begin{pmatrix} 8 \\ 0 \end{pmatrix} = \mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{T}^{-1}\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6.92820323 \\ -4 \end{pmatrix}$

Thus, the coordinates of P are $(-6.93, -4)$. A1

[3]

(c) 12 A2

[2]

17. (a) $f'(x)$

$=\left(\frac{1}{x^2+4}\right)(2x)$ (M1) for chain rule

$=\frac{2x}{x^2+4}$ A1

[2]

(b) $\frac{6}{13}$ A1

[1]

(c) $13x+my=39+m\ln 13$

$my=-13x+39+m\ln 13$

$y=-\frac{13}{m}x+\frac{39+m\ln 13}{m}$ (M1) for valid approach

$\therefore -\frac{13}{m}\times\frac{6}{13}=-1$ (A1) for correct equation

$m=6$

$13x+6(0)=39+6\ln 13$ (M1) for substitution

$x=3+\frac{6}{13}\ln 13$

Thus, the x-intercept of the normal is

$x=3+\frac{6}{13}\ln 13$. A1

[4]

18. (a) By considering the graph of $y = \det(\mathbf{T} - \lambda\mathbf{I})$,

$\lambda = 0.42$ or $\lambda = 1$. (M1) for valid approach

$\therefore \lambda_1 = 0.42$, $\lambda_2 = 1$ A2

[3]

(b) $\mathbf{v}_{10}$

$= \begin{pmatrix} 0.73 & 0.31 \\ 0.27 & 0.69 \end{pmatrix}^{10} \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$ (M1) for valid approach

$= \begin{pmatrix} 0.5344597887 \\ 0.4655402113 \end{pmatrix}$

$= \begin{pmatrix} 0.534 \\ 0.466 \end{pmatrix}$ A1

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to $\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{31}{58} \\ \frac{27}{58} \end{pmatrix}$ A1

[2]