Exercise 3.1

(a) The coordinates of the mid-point

$= \left(\frac{0+7}{2}, \frac{0+24}{2}, \frac{0+0}{2} \right)$ $\quad \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$ (M1)

$= (3.5, 12, 0)$ (A1)

(b) (i) $(7, 24, h)$ (A1)

(ii) $\sqrt{(7-0)^2+(24-0)^2+(h-0)^2} = \sqrt{1025}$ Correct equation (A1)

$49+576+h^2 = 1025$

$h^2 = 400$

$h = 20$ (A1)

Exercise 3.2

(a) (i)

$\sin^2 A+\cos^2 A=1$	$\sin^2 A+\cos^2 A\equiv 1$ (M1)
$\therefore \sin^2 A+\left(-\frac{1}{2}\right)^2=1$	$\cos A=-\frac{1}{2}$ (A1)
$\sin^2 A=\frac{3}{4}$	
$\sin A=\sqrt{\frac{3}{4}}$ (*Rejected*) or $\sin A=-\sqrt{\frac{3}{4}}$	
$\sin A=-\frac{\sqrt{3}}{2}$	(A1)

(ii)

$\tan A$	
$=\frac{\sin A}{\cos A}$	$\tan A\equiv\frac{\sin A}{\cos A}$ (M1)
$=-\frac{\sqrt{3}}{2}\div-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$ & $-\frac{1}{2}$ (A1)
$=\sqrt{3}$	(A1)

(iii)

$\sin 2A$	
$=2\sin A\cos A$	$\sin 2A\equiv 2\sin A\cos A$ (M1)
$=2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$	$-\frac{\sqrt{3}}{2}$ & $-\frac{1}{2}$ (A1)
$=\frac{\sqrt{3}}{2}$	(A1)

(b) (i)

$\cos 2B$	
$=1-2\sin^2 B$	$\cos 2B\equiv 1-2\sin^2 B$ (M1)
$=1-2\left(-\frac{1}{3}\right)^2$	$\sin B=-\frac{1}{3}$ (A1)
$=\frac{7}{9}$	(A1)

(ii) $\sin^2 2B + \cos^2 2B = 1$ — $\sin^2 2B + \cos^2 2B \equiv 1$ (M1)

$\therefore \sin^2 2B + \left(\frac{7}{9}\right)^2 = 1$ — $\cos 2B = \frac{7}{9}$ (A1)

$\sin^2 2B = \frac{32}{81}$

$\sin 2B = \sqrt{\frac{32}{81}}$ (*Rejected*) or

$\sin 2B = -\sqrt{\frac{32}{81}}$

$\sin 2B = -\frac{\sqrt{32}}{9}$ (A1)

Exercise 3.3

(a) $0 < x < \frac{2\pi}{3} \Rightarrow 0 < 3x < 2\pi$ — $0 < 3x < 2\pi$ (M1)

$\cos 3x = -\frac{1}{2}$

$\because \cos\left(\pi - \frac{\pi}{3}\right)$ and $\cos\left(\pi + \frac{\pi}{3}\right) = -\frac{1}{2}$ — 2 possible cases (M1)

$\therefore 3x = \pi - \frac{\pi}{3}$ or $\pi + \frac{\pi}{3}$ — 2 possible angles (A1)

$3x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$x = \frac{2\pi}{9}$ or $\frac{4\pi}{9}$ — (A1)(A1)

(b) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$ — $(2\sin x - 1)(\sin x - 1)$ (M1)

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 1$ — $\frac{1}{2}$ & 1 (A1)

$\because \sin\left(\pi - \frac{\pi}{6}\right) = \frac{1}{2}$ and $\sin x \neq 1$ for $\frac{\pi}{2} < x < \pi$ — 1 possible case (M1)

$\therefore x = \frac{5\pi}{6}$ — (A1)

Exercise 3.4

(a) (i) 8 m (A1)

(ii) 62 m (A1)

(b) (i) a

$= \frac{62-8}{2}$ $\quad \frac{y_{\max}-y_{\min}}{2}$ (M1)

$= 27$ (A1)

(ii) d

$= \frac{62+8}{2}$ $\quad \frac{y_{\max}+y_{\min}}{2}$ (M1)

$= 35$ (A1)

(iii) b

$= \frac{2\pi}{\text{Period}}$ $\quad \frac{2\pi}{\text{Period}}$ (M1)

$= \frac{2\pi}{\frac{60}{6}}$

$= \frac{\pi}{5}$ (A1)

(c) $y = 27\sin\left(\frac{\pi}{5}(t-c)\right)+35$

$\therefore 8 = 27\sin\left(\frac{\pi}{5}(0-c)\right)+35$ $\quad t=0$ & $y=8$ (M1)

$-27 = 27\sin\left(-\frac{\pi}{5}c\right)$

$\sin\left(-\frac{\pi}{5}c\right) = -1$

$-\frac{\pi}{5}c = -\frac{\pi}{2}$ $\quad \sin\left(-\frac{\pi}{2}\right) = -1$ (A1)

$C = 2.5$ (AG)

(d) $27\sin\left(\frac{\pi}{5}(t-2.5)\right)+35\geq 50$ — Correct inequality (A1)

$27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15\geq 0$

By considering the graph of

$y=27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15$, the graph is above

The horizontal axis when

$3.4374719<x<6.5625281$,

$13.437472<x<16.562528$

or $23.437472<x<26.562528$. — GDC approach (M1)

The amount of time

$=3(6.5625281-3.4374719)$ — $3(t_2-t_1)$ (A1)

$=9.3751686$ min

$=9.38$ min — (A1)

Exercise 3.5

(a) (i) $\overset{\frown}{ACB}$

$=(8)\left(\frac{3\pi}{4}\right)$ — $s=r\theta$ (A1)

$=6\pi$ m — (A1)

(ii) The exact perimeter

$=6\pi+8+8$ — Sum of three sides (M1)

$=(6\pi+16)$ m — (A1)

(b) (i) The area of OACB

$=\frac{1}{2}(8)^2\left(\frac{3\pi}{4}\right)$ — $A=\frac{1}{2}r^2\theta$ (A1)

$=24\pi$ m^2 — (A1)

(ii) The area of OAB

$=\frac{1}{2}(8)(8)\sin\frac{3\pi}{4}$ — $A=\frac{1}{2}ab\sin C$ (A1)

$=32\left(\frac{\sqrt{2}}{2}\right)$ — $\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ (A1)

$=16\sqrt{2}$ m^2 — (A1)

(iii) The area of ACB

$=24\pi-16\sqrt{2}$ — Difference of areas (M1)

$=8(3\pi-2\sqrt{2})$ m^2 — (A1)

Exercise 3.6

$QR^2=PQ^2+PR^2-2(PQ)(PR)\cos Q\hat{P}R$ — Cosine rule (M1)

$12^2=19^2+PR^2-2(19)(PR)\cos 0.483$ — 12, 19 & 0.483 (A1)

$PR^2-(38\cos 0.483)PR+217=0$

$PR=8.694390054$ or $PR=24.95862259$ — Two values (A1)

The required difference

$=24.95862259-8.694390054$

$=16.26423254$

$=16.3$ — (A1)

Exercise 3.7

(a) $\frac{\text{AC}}{\sin \text{A}\hat{\text{B}}\text{C}} = \frac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}}$ — Sine rule (M1)

$\frac{\text{AC}}{\sin\left((360^\circ - 310^\circ) \times \frac{\pi}{180^\circ}\right)} = \frac{60}{\sin 1.45}$ — $50^\circ \times \frac{\pi}{180^\circ}$, 60 & 1.45 (A1)

$\frac{\text{AC}}{\sin \frac{5\pi}{18}} = \frac{60}{\sin 1.45}$

$\text{AC} = 46.30005551$ km

$\text{AC} = 46.3$ km — (A1)

(b) The area of ABC

$= \frac{1}{2}(\text{AB})(\text{AC}) \sin \text{B}\hat{\text{A}}\text{C}$ — $A = \frac{1}{2}ab\sin \text{C}$ (M1)

$= \frac{1}{2}(60)(46.30005551) \sin\left(\pi - 1.45 - \frac{5\pi}{18}\right)$ — 60, 46.30005551 & $\pi - 1.45 - \frac{5\pi}{18}$ (A1)

$= 1014.546384$ km^2

$= 1010$ km^2 — (A1)

(c) The area of ACD

$= (1014.546384)(1.44)$

$= 1460.946793$ — 1460.946793 (A1)

$\frac{1}{2}(\text{AC})(\text{CD}) \sin \text{A}\hat{\text{C}}\text{D} = 1460.946793$ — $A = \frac{1}{2}ab\sin \text{C}$ (M1)

$\frac{1}{2}(46.30005551)(75) \sin \theta = 1460.946793$ — 46.30005551 & 75 (A1)

$\sin \theta = 0.8414370289$

$\theta = 0.999937157$ radian

$\theta = 1.00$ radian — (A1)

(d) $\frac{BC}{\sin B\hat{A}C} = \frac{AB}{\sin A\hat{C}B}$ Sine rule (M1)

$\frac{BC}{\sin\left(\pi - 1.45 - \frac{5\pi}{18}\right)} = \frac{60}{\sin 1.45}$ $\pi - 1.45 - \frac{5\pi}{18}$, 60 & 1.45 (A1)

$BC = 44.14654149$

$BD^2 = CD^2 + BC^2 - 2(CD)(BC)\cos B\hat{C}D$ Cosine rule (M1)

$BD^2 = 75^2 + 44.14654149^2$

$-2(75)(44.14654149)\cos(1.45 + 0.999937157)$

75, 44.14654149 & $1.45 + 0.999937157$ (A1)

$BD = 112.5793436$

$\frac{BD}{\text{Speed of Car Q}} = \frac{BC + CD}{60}$ Same time (M1)

$\frac{112.5793436}{\text{Speed of Car Q}} = \frac{44.14654149 + 75}{60}$

Speed of $Q = 56.69288031$

Thus, the speed of Car Q is 56.7 km/h. (A1)

Exercise 3.8

(a) The volume

$=\frac{2}{3}\pi r^3$ — $V=\frac{2}{3}\pi r^3$ (M1)

$=\frac{2}{3}\pi(20)^3$ — $r=20$ (A1)

$=16755.16082$ cm³

$=16800$ cm³ — (A1)

(b) The total surface area

$=3\pi r^2$ — $A=3\pi r^2$ (M1)

$=3\pi(20)^2$ — $r=20$ (A1)

$=3769.911184$ cm²

$=3770$ cm² — (A1)

(c) $V=\frac{1}{3}Ah$ — $V=\frac{1}{3}Ah$ (M1)

$(16755.16082)(4)=\frac{1}{3}(\text{AD})^2(47)$ — $V=67020.64328$ & $h=47$ (A1)

$\text{AD}^2=4277.913401$

$\text{AD}=65.40575969$ cm

$\text{AD}=65.4$ cm — (A1)

(d) $\tan \text{O}\hat{\text{M}}\text{V}=\frac{\text{OV}}{\text{OM}}$ — Tangent ratio (M1)

$\tan \text{O}\hat{\text{M}}\text{V}=\frac{47}{65.40575969\div 2}$

$\text{O}\hat{\text{M}}\text{V}=0.9628907079$ radians

Thus, the required angle is 0.963 radians. — (A1)

(e) OD

$= \sqrt{OM^2 + DM^2}$

$= \sqrt{\left(\frac{65.40575969}{2}\right)^2 + \left(\frac{65.40575969}{2}\right)^2}$

$= 46.24885621$ 46.24885621 (A1)

$\tan O\hat{D}V = \frac{OV}{OD}$ Tangent ratio (M1)

$\tan O\hat{D}V = \frac{47}{46.24885621}$

$O\hat{D}V = 0.7934532493$ radians

Thus, the required angle is 0.793 radians. (A1)