Exercise 2.1

(a) 0 (A1)

(b) (i) $x^2 - 16x = 0$

$x(x-16) = 0$ — $x(x-16)$ (M1)

$x = 0$ or $x - 16 = 0$

$x = 0$ or $x = 16$ — (A1)(A1)

(ii) $y = x(x-16)$ — (A1)

(c) (i) h

$= \dfrac{0+16}{2}$ — $h = \dfrac{p+q}{2}$ (M1)

$= 8$ — (A1)

(ii) k

$= 8^2 - 16(8)$ — $k = ah^2 + bh + c$ (M1)

$= -64$ — (A1)

(iii) $x = 8$ — (A1)

Exercise 2.2

(a) $\begin{cases} y = x^2 + 2kx + 5 \\ y = 3kx - 4 \end{cases}$

$\therefore x^2 + 2kx + 5 = 3kx - 4$

$x^2 - kx + 9 = 0$ $\quad$ $x^2 - kx + 9 = 0$ (A1)

The two graphs have two intersection points.

$\therefore \Delta > 0$ $\quad$ $\Delta > 0$ (R1)

$(-k)^2 - 4(1)(9) > 0$ $\quad$ $\Delta = b^2 - 4ac$ (M1)

$k^2 - 36 > 0$ $\quad$ $k^2 - 36 > 0$ (A1)

$k^2 > 36$

$\therefore$ $k < -6$ or $k > 6$ $\quad$ (A1)

(b) $x^2 - 7x + 9 = 0$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$ $\quad$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (M1)

$x = \dfrac{7 \pm \sqrt{13}}{2}$

$\therefore$ $m + r = 20$ $\quad$ (A1)

Exercise 2.3

(a) (i)

$f(x)=0$ — $f(x)=0$ (M1)

$\frac{x-3}{2x+4}=0$

$x-3=0$

$x=3$ — (A1)

(ii) The y-intercept

$=\frac{0-3}{2(0)+4}$ — Substitute $x=0$ (M1)

$=-\frac{3}{4}$ — (A1)

(b) (i) $x=-2$ — (A1)

(ii) $y=\frac{1}{2}$ — (A1)

(iii) $y\neq\frac{1}{2},\ y\in\mathbb{R}$ — (A1)

(c) (i) $y\neq-2,\ y\in\mathbb{R}$ — (A1)

(ii)

$y=\frac{x-3}{2x+4}$

$\rightarrow x=\frac{y-3}{2y+4}$ — Interchange x and y (M1)

$x(2y+4)=y-3$

$2xy+4x=y-3$

$2xy-y=-4x-3$ — Combine like terms (M1)

$y(2x-1)=-4x-3$

$y=\frac{-4x-3}{2x-1}$

$\therefore f^{-1}(x)=\frac{-4x-3}{2x-1}$ — (A1)

(d) $(f^{-1}\circ g)(x)$

$=f^{-1}(g(x))$

$=\dfrac{-4(0.5-5x)-3}{2(0.5-5x)-1}$ $\dfrac{-4(0.5-5x)-3}{2(0.5-5x)-1}$ (M1)

$=\dfrac{-5+20x}{-10x}$

$=\dfrac{1}{2x}-2$

$\therefore a=\dfrac{1}{2}$ (A1)

Exercise 2.4

(a) Reflection about the x-axis (A1)
followed by an upward translation by 5 units (A1)

(b) $h(x)$
$= g(3(x+4))$ — $h(x) = g(3(x+4))$ (M1)
$= g(3x+12)$
$= (3x+12)^2 + 5$ — $g(3x+12) = (3x+12)^2 + 5$ (A1)
$= 9x^2 + 72x + 144 + 5$
$= 9x^2 + 72x + 149$ (A1)

(c) The image after transformed to g
$= (-3, -9+5)$ — $-y$ (A1) & $y+5$ (A1)
$= (-3, -4)$
The coordinates of P
$= (-3 \div 3 - 4, -4)$ — $x \div 3$ (A1) & $x-4$ (A1)
$= (-5, -4)$ (A1)

Exercise 2.5

(a) $f(-x)$

$=-(-x)^2\sqrt{9-(-x)^2}$ — $f(-x)$ (A1)

$=-x^2\sqrt{9-x^2}$

$=f(x)$ — (A1)

Thus, f is an even function. — (AG)

(b) By considering the graph of $y=-x^2\sqrt{9-x^2}$, the coordinates of the maximum point and the two minimum points are $(0, 0)$, $(-2.4494904, -10.3923)$ and $(2.4494904, -10.3923)$ respectively. — GDC approach (M1)

Thus, the range of f is $-10.4 \le y \le 0$, $y \in \mathbb{R}$. — (A1)

(c) $c=-2.45$ — (A1)

(d) $|x|-2 \ge f(x)$

$\therefore |x|-2 \ge -x^2\sqrt{9-x^2}$

$|x|-2+x^2\sqrt{9-x^2} \ge 0$ — Correct inequality (A1)

By considering the graph of $y=|x|-2+x^2\sqrt{9-x^2}$, the graph is above the horizontal axis when $-3 < x < -0.6735888$ or $0.6735888 < x < 3$. — GDC approach (M1)

$\therefore -3 < x \le -0.674$ or $0.674 \le x < 3$ — (A1)

Exercise 2.6

(a) (i) $\log_3 x^6 = 6\log_3 x$ (A1)

(ii) $\log_3(16x)$

$= \log_3 16 + \log_3 x$ $\qquad \log_3(pq) = \log_3 p + \log_3 q$ (M1)

$= \log_3 2^4 + \log_3 x$ $\qquad 16 = 2^4$ (M1)

$= 4\log_3 2 + \log_3 x$ (A1)

(iii) $\log_2 3$

$= \dfrac{\log_3 3}{\log_3 2}$ $\qquad \log_2 a = \dfrac{\log_3 a}{\log_3 2}$ (M1)

$= \dfrac{1}{\log_3 2}$ (A1)

(b) $\dfrac{2}{3}\log_3 x^6 + \dfrac{1}{\log_2 3} + \log_3(16x) = 0$

$\therefore \dfrac{2}{3}(6\log_3 x) + \log_3 2 + 4\log_3 2 + \log_3 x = 0$ $\qquad$ Substitution (M1)

$4\log_3 x + \log_3 2 + 4\log_3 2 + \log_3 x = 0$

$5\log_3 x + 5\log_3 2 = 0$ $\qquad$ Combine like terms (M1)

$\log_3 x + \log_3 2 = 0$

$\log_3 x = -\log_3 2$

$\log_3 x = \log_3 2^{-1}$ $\qquad n\log_3 p = \log_3 p^n$ (M1)

$x = \dfrac{1}{2}$ (A1)

Exercise 2.7

(a) $P_1 = P_0 e^{k(1)}$

$\therefore P_0(1-10\%) = P_0 e^{k(1)}$ — $P_1 = P_0(1-10\%)$ & $t = 1$ (A1)

$0.9 = e^k$

$k = \ln 0.9$ — $b = e^x \Leftrightarrow x = \ln b$ (M1)

$k = -0.1053605157$

$\therefore k = -0.1054$ — (A1)

(b) The growth rate is negative. — (R1)

(c) $0.5P_0 = P_0 e^{-0.1053605157t}$ — Correct equation (A1)

$0.5 = e^{-0.1053605157t}$

$e^{-0.1053605157t} - 0.5 = 0$

By considering the graph of $y = e^{-0.1053605157t} - 0.5$, the horizontal intercept is 6.5788135. — GDC approach (M1)

$\therefore$ The least number of complete years is 66. — (A1)

Exercise 2.8

(a) The gradient

$=\dfrac{1-(-9)}{-8-0}$ $\quad m=\dfrac{y_2-y_1}{x_2-x_1}$ (M1)

$=-\dfrac{5}{4}$ (A1)

(b) (i) $(-4,-4)$ (A1)

(ii) The exact distance

$=\sqrt{(-4-(-8))^2+(-4-1)^2}$ $\quad d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (M1)

$=\sqrt{41}$ (A1)

(c) The required slope

$=-1\div-\dfrac{5}{4}$ $\quad m_1\times m_2=-1$ (M1)

$=\dfrac{4}{5}$

The equation:

$y-1=\dfrac{4}{5}(x-(-8))$ $\quad y-y_1=m(x-x_1)$ (M1)

$5(y-1)=4(x+8)$

$5y-5=4x+32$

$4x-5y+37=0$ (A1)

Exercise 2.9

(a) $f(-1)$

$= 2(-1)^3 + k(-1)^2 + 6(-1) + 8$

$= k$ — k (A1)

$g(-1)$

$= (-1)^4 - 6(-1)^2 - 5k(-1) - 3k$

$= 2k - 5$ — $2k - 5$ (A1)

$f(-1) = g(-1)$ — $f(-1) = g(-1)$ (M1)

$\therefore k = 2k - 5$

$-k = -5$

$k = 5$ — (A1)

(b) Let α_1 and α_2 be the two complex roots.

Sum of roots $= -\frac{5}{2}$ — $r_1 + r_2 + r_3 = -\frac{a_2}{a_3}$ (A1)

$-2 + \alpha_1 + \alpha_2 = -\frac{5}{2}$

$\alpha_1 + \alpha_2 = -\frac{1}{2}$ — (A1)

Product of roots $= (-1)^3 \frac{8}{2}$ — $r_1 r_2 r_3 = (-1)^3 \frac{a_0}{a_3}$ (A1)

$(-2)(\alpha_1)(\alpha_2) = -4$

$\alpha_1 \alpha_2 = 2$ — (A1)

Exercise 2.10

(a) (i) $x = 2$, $x = 4$ (A1)(A1)

(ii) x-intercept $= -5$ (A1)

y-intercept $= \frac{5}{8}$ (A1)

(b) By considering the graph of $y = \frac{x+5}{x^2 - 6x + 8}$,

the coordinates of the minimum point are $(-12.93725, -0.031373)$. GDC approach (M1)

Thus, the coordinates are $(-12.9, -0.0314)$. (A1)

(c) By considering the graph of $y = \frac{x+5}{x^2 - 6x + 8}$,

the coordinates of the maximum point are $(2.9372539, -7.968627)$. GDC approach (M1)

Thus, the range of f is

$y \le -7.97$ or $y \ge -0.0314$, $y \in \mathbb{R}$. $y \le -7.97$ (A1) & $y \ge -0.0314$ (A1)

(d) Let $\frac{x+5}{x^2 - 6x + 8} = \frac{A}{x-2} + \frac{B}{x-4}$.

$\frac{x+5}{x^2 - 6x + 8} = \frac{A(x-4)}{(x-2)(x-4)} + \frac{B(x-2)}{(x-4)(x-2)}$

$x + 5 = Ax - 4A + Bx - 2B$ Expansion (M1)

$\therefore \begin{cases} x = Ax + Bx \\ 5 = -4A - 2B \end{cases}$ Compare coefficients (M1)

$1 = A + B$

$A = 1 - B$

$5 = -4A - 2B$

$\therefore 5 = -4(1 - B) - 2B$ Substitute $A = 1 - B$ (M1)

$5 = -4 + 4B - 2B$

$9 = 2B$

$B = 4.5$ (A1)

$\therefore A = 1 - 4.5$

$A = -3.5$ (A1)

(e) (i) $x=-5$ (A1)

(ii) x-intercepts $=2$ or 4 (A1)

y-intercept $=\frac{8}{5}$ (A1)

(f) $g(x)=\frac{x^2-6x+8}{x+5}$

Let $\frac{x^2-6x+8}{x+5}=x+C+\frac{D}{x+5}$. $x+C$ (A1)

$\frac{x^2-6x+8}{x+5}=\frac{(x+C)(x+5)}{x+5}+\frac{D}{x+5}$

$x^2-6x+8=x^2+5x+Cx+5C+D$ Expansion (M1)

$\therefore \begin{cases} -6x=5x+Cx \\ 8=5C+D \end{cases}$ Compare coefficients (M1)

$-6=5+C$

$C=-11$

Thus, the equation is $y=x-11$. (A1)

(g) By considering the graph of $y=\frac{x^2-6x+8}{x+5}$,

the coordinates of the maximum and the minimum point are $(-12.93725,-31.87451)$ and $(2.9372558,-0.125492)$ respectively. GDC approach (M1)

Thus, the range of g is

$y\le -31.9$ or $y\ge -0.125$, $y\in\mathbb{R}$. $y\le -31.9$ (A1) & $y\ge -0.125$ (A1)