Chapter 4 Solution

Exercise 6

1. (a) $f(x)=0$ (M1) for setting equation

$x^2-6x+8=0$

$(x-2)(x-4)=0$ A1

$x=2$ or $x=4$

Hence, the x-intercepts are 2 and 4 respectively. A2 N4

[4]

(b) (i) $x=3$ A1 N1

(ii) The y-coordinate of the vertex

$=3^2-6(3)+8$ (M1) for substitution

$=-1$ A1 N2

[3]

2. (a) $f(x)=0$ (M1) for setting equation

$x^2-11x+10=0$

$(x-10)(x-1)=0$ A1

$x=10$ or $x=1$

Hence, the x-intercepts are 1 and 10 respectively. A2 N4

[4]

(b) (i) $x=5.5$ A1 N1

(ii) The y-coordinate of the vertex

$=5.5^2-11(5.5)+10$ (M1) for substitution

$=-20.25$ A1 N2

[3]

3.	(a)	$f(x)=0$		(M1) for setting equation	
		$-2x^2-14x=0$			
		$-2x(x+7)=0$		A1	
		$x=0$ or $x=-7$			
		Hence, the *x*-intercepts are 0 and -7 respectively.		A2	N4
					[4]
	(b)	(i)	$x=-3.5$	A1	N1
		(ii)	The y-coordinate of the vertex		
			$=-2(-3.5)^2-14(-3.5)$	(M1) for substitution	
			$=24.5$	A1	N2
					[3]
4.	(a)	$f(x)=0$		(M1) for setting equation	
		$13.5-1.5x^2=0$			
		$1.5(9-x^2)=0$		A1	
		$1.5(3+x)(3-x)=0$			
		$x=-3$ or $x=3$			
		Hence, the *x*-intercepts are -3 and 3 respectively.		A2	N4
					[4]
	(b)	(i)	$x=0$	A1	N1
		(ii)	The y-coordinate of the vertex		
			$=13.5-1.5(0)^2$	(M1) for substitution	
			$=13.5$	A1	N2
					[3]

Exercise 7

1.	(a)	$f(x)=(x-7)(x+5)$	A2	N2	
					[2]
	(b)	$x=-5$ and $x=7$	A2	N2	
					[2]
	(c)	$\{y: y\geq -36\}$	A1	N1	
					[1]
2.	(a)	$f(x)=-2(x+1)(x+6)$	A2	N2	
					[2]
	(b)	$x=-1$ and $x=-6$	A2	N2	
					[2]
	(c)	$\{y: y\leq 12.5\}$	A1	N1	
					[1]
3.	(a)	$p=5$ and $q=11$	A2	N2	
					[2]
	(b)	$-7.5=a(10-5)(10-11)$	M1A1		
		$-7.5=-5a$			
		$a=1.5$	A1	N3	
					[3]
	(c)	$\{y: y\geq -13.5\}$	A1	N1	
					[1]
4.	(a)	$p=0$ and $q=18$	A2	N2	
					[2]
	(b)	$30=a(0-15)(15-18)$	M1A1		
		$30=45a$			
		$a=\frac{2}{3}$	A1	N3	
					[3]
	(c)	$\{y: y\leq 54\}$	A1	N1	
					[1]

Exercise 8

1.	(a)	$x = 2$	A2	N2	
					[2]
	(b)	$(2, 17)$	A1	N1	
					[1]
	(c)	$\{y : y \le 17\}$	A2	N2	
					[2]
2.	(a)	4	A2	N2	
					[2]
	(b)	$(7, -2)$	A1	N1	
					[1]
	(c)	$\{y : y \ge -2\}$	A2	N2	
					[2]
3.	(a)	$(-5, 12.5)$	A1	N1	
					[1]
	(b)	Note that the another x-intercept is $-r$.	(A1) for correct value		
		$\frac{-r+0}{2} = -5$	(M1) for valid approach		
		$-r = -10$			
		$r = 10$	A1	N3	
		$12.5 = a(-5)(-5+10)$	(M1) for substitution		
		$12.5 = -25a$			
		$a = -0.5$	A1	N2	
					[5]
4.	(a)	-5	A1	N1	
					[1]
	(b)	Note that the x-intercept are $-r$ and -1.			
		$\frac{-r+(-1)}{2} = -2.5$	(M1) for valid approach		
		$-r - 1 = -5$			
		$-r = -4$			
		$r = 4$	A1	N2	
		$-4 = a(0+4)(0+1)$	(M1) for substitution		
		$-4 = 4a$			
		$a = -1$	A1	N2	
					[4]
	(c)	t	A1	N1	
					[1]

Exercise 9

1. (a) The y-intercept

$= -(0-100)^2 + 80$ (M1) for substitution

$= -9920$ A1 N2

[2]

(b) $-(x-100)^2 + 80 \geq 16$

$-(x-100)^2 + 64 \geq 0$ (M1) for setting inequality

By considering the graph of $y = -(x-100)^2 + 64$,

$92 \leq x \leq 108$.

$\therefore p = 108$ A1 N2

[2]

(c) 100 A1 N1

[1]

2. (a) 5400 A1 N1

[1]

(b) $5064 \leq x^2 - 40x + 5400 \leq 5400$

$5064 \leq x^2 - 40x + 5400$ and $x^2 - 40x + 5400 \leq 5400$ (M1) for valid approach

$x^2 - 40x + 336 \geq 0$ and $x^2 - 40x \leq 0$

By considering the graphs of $y = x^2 - 40x + 336$

and $y = x^2 - 40x$, $28 \leq x \leq 40$.

$\therefore p = 28$, $q = 40$ A2 N3

[3]

(c) (20, 5000) A2 N2

[2]

3. (a) The required cost

$= 0.5(200-60)^2 + 40$ (M1) for substitution

$= 9840$ A1 N2

[2]

(b) $0.5(x-60)^2 + 40 \leq 240$

$0.5(x-60)^2 - 200 \leq 0$ (M1) for setting inequality

By considering the graph of $y = 0.5(x-60)^2 - 200$,

$40 \leq x \leq 80$. A1 N2

[2]

(c) 40 A1 N1

[1]

(d) 60 A1 N1

[1]

4. (a) The average profit

$= -0.25(22-20)^2 + 21$ (M1) for substitution

$= 20$ A1 N2

[2]

(b) $-0.25(x-20)^2 + 21 \geq 17$

$-0.25(x-20)^2 + 4 \geq 0$ (M1) for setting inequality

By considering the graph of $y = -0.25(x-20)^2 + 4$,

$16 \leq x \leq 24$. A1 N2

[2]

(c) $x \leq 18$

By considering the graph of $y = -0.25(x-20)^2 + 21$,

$y \leq 20$. (M1) for valid approach

Therefore, the maximum average profit is 20. A1 N2

[2]

Exercise 10

1.	(a)	$A=(30-2x)(20-2x)$	(M1) for valid approach	
		$A=600-60x-40x+4x^2$		
		$A=600-100x+4x^2$	A1	N2
				[2]
	(b)	$A=299$		
		$600-100x+4x^2=299$	(M1) for setting equation	
		$4x^2-100x+301=0$	(A1) for correct equation	
		$(2x-43)(2x-7)=0$		
		$2x-43=0$ or $2x-7=0$		
		$x=21.5$ (*Rejected*) or $x=3.5$	A1	N3
				[3]
	(c)	1046.5 cm^3	A1	N1
				[1]
2.	(a)	$x^2=(x-1)^2+(x-18)^2$	(M1) for valid approach	
		$x^2=x^2-2x+1+x^2-36x+324$		
		$x^2-38x+325=0$	A1	N2
				[2]
	(b)	$x^2-38x+325=0$		
		$(x-13)(x-25)=0$	(M1) for factorization	
		$x-13=0$ or $x-25=0$		
		$x=13$ (*Rejected*) or $x=25$	A1	N2
				[2]
	(c)	The painting cost per cm^2		
		$=\dfrac{1680}{\frac{1}{2}(25-18)(25-1)}$	M1	
		$=\$20/\text{cm}^2$	A1	N2
				[2]

3. (a) $Q \geq 7.5$

$75 - 15r \geq 7.5$ (M1) for setting inequality

$67.5 \geq 15r$

$r \leq 4.5$

Therefore, the maximum cost is \$4.5. A1 N2

[2]

(b) $P = (75 - 15r)(r + 2)$ (M1) for valid approach

$P = 75r + 150 - 15r^2 - 30r$

$P = -15r^2 + 45r + 150$ A1 N2

[2]

(c) $P = -15r^2 + 45r + 150$

By considering the graph of

$y = -15r^2 + 45r + 150$, the maximum value of P

is 183.75. (M1) for valid approach

Therefore, the maximum weekly profit is \$183.75. A1 N2

[2]

4. (a) By considering the graph of $y = -0.5x^2 + 2x + 10$,

the maximum value of y is 12. (M1) for valid approach

Therefore, the maximum height is 12 m. A1 N2

[2]

(b) 4 m A1 N1

[1]

(c) $-0.5x^2 + 2x + 10 = 0$ (M1) for setting equation

By considering the graph of $y = -0.5x^2 + 2x + 10$,

the x-intercept is 6.8989795. (M1) for valid approach

Therefore, the horizontal distance is 6.90 m. A1 N3

[3]