Exercise 2.1

(a) 0 (A1)

(b) (i) $x^2 - 16x = 0$

$x(x-16) = 0$ — $x(x-16)$ (M1)

$x = 0$ or $x - 16 = 0$

$x = 0$ or $x = 16$ (A1)(A1)

(ii) $y = x(x-16)$ (A1)

(c) (i) h

$= \frac{0+16}{2}$ — $h = \frac{p+q}{2}$ (M1)

$= 8$ (A1)

(ii) k

$= 8^2 - 16(8)$ — $k = ah^2 + bh + c$ (M1)

$= -64$ (A1)

(iii) $x = 8$ (A1)

Exercise 2.2

(a) $\begin{cases} y = x^2 + 2kx + 5 \\ y = 3kx - 4 \end{cases}$

$\therefore x^2 + 2kx + 5 = 3kx - 4$

$x^2 - kx + 9 = 0$ — $x^2 - kx + 9 = 0$ (A1)

The two graphs have two intersection points.

$\therefore \Delta > 0$ — $\Delta > 0$ (R1)

$(-k)^2 - 4(1)(9) > 0$ — $\Delta = b^2 - 4ac$ (M1)

$k^2 - 36 > 0$ — $k^2 - 36 > 0$ (A1)

$k^2 > 36$

$\therefore k < -6$ or $k > 6$ — (A1)

(b) $x^2 - 7x + 9 = 0$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$ — $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (M1)

$x = \dfrac{7 \pm \sqrt{13}}{2}$

$\therefore m + r = 20$ — (A1)

Exercise 2.3

(a) (i) $f(x)=0$ — $f(x)=0$ (M1)

$\frac{x-3}{2x+4}=0$

$x-3=0$

$x=3$ — (A1)

(ii) The y-intercept

$=\frac{0-3}{2(0)+4}$ — Substitute $x=0$ (M1)

$=-\frac{3}{4}$ — (A1)

(b) (i) $x=-2$ — (A1)

(ii) $y=\frac{1}{2}$ — (A1)

(iii) $y\neq\frac{1}{2},\ y\in\mathbb{R}$ — (A1)

(c) (i) $y\neq-2,\ y\in\mathbb{R}$ — (A1)

(ii) $y=\frac{x-3}{2x+4}$

$\to x=\frac{y-3}{2y+4}$ — Interchange x and y (M1)

$x(2y+4)=y-3$

$2xy+4x=y-3$

$2xy-y=-4x-3$ — Combine like terms (M1)

$y(2x-1)=-4x-3$

$y=\frac{-4x-3}{2x-1}$

$\therefore f^{-1}(x)=\frac{-4x-3}{2x-1}$ — (A1)

(d) $(f^{-1} \circ g)(x)$

$= f^{-1}(g(x))$

$= \dfrac{-4(0.5-5x)-3}{2(0.5-5x)-1}$ $\qquad \dfrac{-4(0.5-5x)-3}{2(0.5-5x)-1}$ (M1)

$= \dfrac{-5+20x}{-10x}$

$= \dfrac{1}{2x} - 2$

$\therefore a = \dfrac{1}{2}$ (A1)

Exercise 2.4

(a) Reflection about the x-axis (A1)
followed by an upward translation by 5 units (A1)

(b) $h(x)$

$= g(3(x+4))$ — $h(x) = g(3(x+4))$ (M1)

$= g(3x+12)$

$= (3x+12)^2 + 5$ — $g(3x+12) = (3x+12)^2 + 5$ (A1)

$= 9x^2 + 72x + 144 + 5$

$= 9x^2 + 72x + 149$ (A1)

(c) The image after transformed to g

$= (-3, -9+5)$ — $-y$ (A1) & $y+5$ (A1)

$= (-3, -4)$

The coordinates of P

$= (-3 \div 3 - 4, -4)$ — $x \div 3$ (A1) & $x-4$ (A1)

$= (-5, -4)$ (A1)

Exercise 2.5

(a) (i) $\log_3 x^6 = 6\log_3 x$ (A1)

(ii) $\log_3(16x)$

$= \log_3 16 + \log_3 x$ — $\log_3(pq) = \log_3 p + \log_3 q$ (M1)

$= \log_3 2^4 + \log_3 x$ — $16 = 2^4$ (M1)

$= 4\log_3 2 + \log_3 x$ (A1)

(iii) $\log_2 3$

$= \dfrac{\log_3 3}{\log_3 2}$ — $\log_2 a = \dfrac{\log_3 a}{\log_3 2}$ (M1)

$= \dfrac{1}{\log_3 2}$ (A1)

(b) $\dfrac{2}{3}\log_3 x^6 + \dfrac{1}{\log_2 3} + \log_3(16x) = 0$

$\therefore \dfrac{2}{3}(6\log_3 x) + \log_3 2 + 4\log_3 2 + \log_3 x = 0$ — Substitution (M1)

$4\log_3 x + \log_3 2 + 4\log_3 2 + \log_3 x = 0$

$5\log_3 x + 5\log_3 2 = 0$ — Combine like terms (M1)

$\log_3 x + \log_3 2 = 0$

$\log_3 x = -\log_3 2$

$\log_3 x = \log_3 2^{-1}$ — $n\log_3 p = \log_3 p^n$ (M1)

$x = \dfrac{1}{2}$ (A1)

Exercise 2.6

(a) $P_1 = P_0 e^{k(1)}$

$\therefore P_0(1-10\%) = P_0 e^{k(1)}$ — $P_1 = P_0(1-10\%)$ & $t=1$ (A1)

$0.9 = e^k$

$k = \ln 0.9$ — $b = e^x \Leftrightarrow x = \ln b$ (M1)

$k = -0.1053605157$

$\therefore k = -0.1054$ — (A1)

(b) The growth rate is negative. — (R1)

(c) $0.5P_0 = P_0 e^{-0.1053605157t}$ — Correct equation (A1)

$0.5 = e^{-0.1053605157t}$

$e^{-0.1053605157t} - 0.5 = 0$

By considering the graph of

$y = e^{-0.1053605157t} - 0.5$, the horizontal intercept is

6.5788135. — GDC approach (M1)

$\therefore$ The least number of complete years is 66. — (A1)

Exercise 2.7

(a) The gradient

$=\frac{1-(-9)}{-8-0}$ — $m=\frac{y_2-y_1}{x_2-x_1}$ (M1)

$=-\frac{5}{4}$ — (A1)

(b) (i) $(-4,-4)$ — (A1)

(ii) The exact distance

$=\sqrt{(-4-(-8))^2+(-4-1)^2}$ — $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (M1)

$=\sqrt{41}$ — (A1)

(c) The required slope

$=-1\div-\frac{5}{4}$ — $m_1\times m_2=-1$ (M1)

$=\frac{4}{5}$

The equation:

$y-1=\frac{4}{5}(x-(-8))$ — $y-y_1=m(x-x_1)$ (M1)

$5(y-1)=4(x+8)$

$5y-5=4x+32$

$4x-5y+37=0$ — (A1)