Chapter 2 Solution

Exercise 6

1.	$a(-2)^3-(-2)^2+b(-2)+3=-13$	(M1) for remainder theorem	
	$-8a-4-2b+3=-13$		
	$-8a+12=2b$		
	$b=6-4a$	A1	
	$a(3)^3-3^2+3b+3=27$		
	$27a-9+3b+3=27$		
	$\therefore 27a-9+3(6-4a)+3=27$	(M1) for substitution	
	$27a-9+18-12a+3=27$		
	$15a=15$		
	$a=1$	A1	
	$b=6-4(1)$		
	$b=2$	A1	
			[5]
2.	$4(2)^3-2k(2)^2-5=3(2)^3-k^2(2)^2+15$	M1A1	
	$32-8k-5=24-4k^2+15$		
	$27-8k=24-4k^2+15$	(A1) for simplification	
	$4k^2-8k-12=0$	A1	
	$k^2-2k-3=0$		
	$(k+1)(k-3)=0$		
	$k+1=0$ or $k-3=0$		
	$k=-1$ or $k=3$	A2	
			[6]
3.	(a) $4(2)^3+p(2)^2+q(2)=4(-2)^3+p(-2)^2+q(-2)$	M1A1	
	$32+4p+2q=-32+4p-2q$		
	$4q=-64$		
	$q=-16$	A1	
			[3]
	(b) $p\in\mathbb{R}$	A1	
			[1]

4.	$f(p) = q$	
	$p^4 + 4p^2 - 3p + 2 = q$	(M1) for remainder theorem
	$f(-p) = q + 12$	
	$(-p)^4 + 4(-p)^2 - 3(-p) + 2 = q + 12$	(M1) for remainder theorem
	$p^4 + 4p^2 + 3p + 2 = q + 12$	
	$\therefore p^4 + 4p^2 + 3p + 2 = p^4 + 4p^2 - 3p + 2 + 12$	(M1) for substitution
	$3p + 2 = -3p + 14$	
	$6p = 12$	
	$p = 2$	A1
	$q = 2^4 + 4(2)^2 - 3(2) + 2$	
	$q = 28$	A1
		[5]

Exercise 7

1. $f(2)=0$

$a(2)^3+b(2)^2-13(2)+6=0$ (M1) for factor theorem

$8a+4b-26+6=0$

$4b=20-8a$

$b=5-2a$ A1

$f(-3)=0$

$a(-3)^3+b(-3)^2-13(-3)+6=0$

$-27a+9b+39+6=0$

$\therefore -27a+9(5-2a)+39+6=0$ (M1) for substitution

$-27a+45-18a+39+6=0$

$-45a=-90$

$a=2$ A1

$b=5-2(2)$

$b=1$ A1

[5]

2. $f(4)=0$

$4^3+p(4)^2+q(4)+48=0$ (M1) for factor theorem

$64+16p+4q+48=0$

$4q=-16p-112$

$q=-4p-28$ A1

$f(-3)=105$

$(-3)^3+p(-3)^2+q(-3)+48=105$

$-27+9p-3q+48=105$

$\therefore -27+9p-3(-4p-28)+48=105$ (M1) for substitution

$-27+9p+12p+84+48=105$

$21p=0$

$p=0$ A1

$q=-4(0)-28$

$q=-28$ A1

[5]

3. (a) $p(5)=0$

$5^3+5m-30=0$ (M1) for factor theorem

$5m=-95$

$m=-19$ A1

[2]

(b) $p(x)=x^3-19x-30$

$$\begin{array}{r}
x^2+5x+6 \\
x-5\overline{)\,x^3+0x^2-19x-30} \\
\underline{x^3-5x^2} \\
5x^2-19x \\
\underline{5x^2-25x} \\
6x-30 \\
\underline{6x-30}
\end{array}$$

M1

By using the long division,

$p(x)=(x-5)(x^2+5x+6)$ A1

$p(x)=(x-5)(x+2)(x+3)$ A1

[3]

4. (a) $q(-3)=0$

$2(-3)^3+(k+9)(-3)^2+k(-3)+(k+1)=0$ (M1) for factor theorem

$-54+9k+81-3k+k+1=0$

$7k=-28$

$k=-4$ A1

[2]

(b) $q(x)=2x^3+5x^2-4x-3$

$$\begin{array}{r}
2x^2-x-1 \\
x+3\overline{)\,2x^3+5x^2-4x-3} \\
\underline{2x^3+6x^2} \\
-x^2-4x \\
\underline{-x^2-3x} \\
-x-3 \\
\underline{-x-3}
\end{array}$$

M1

By using the long division,

$p(x)=(x+3)(2x^2-x-1)$ A1

$p(x)=(x+3)(2x+1)(x-1)$ A1

[3]

Exercise 8

1. $2x^3+30x^2+kx+20=x-5$

$2x^3+30x^2+(k-1)x+25=0$ (M1) for valid approach

$r_1r_2+r_2r_3+r_1r_3=2r_1r_2r_3$

$\therefore \frac{k-1}{2}=-\frac{25}{2}$ M1A2

$k-1=-25$

$k=-24$ A1

[5]

2. $x^4+kx^3+3x^2+2x+1=-x^4+4x^3+13$

$2x^4+(k-4)x^3+3x^2+2x-12=0$ (M1) for valid approach

$r_1r_2r_3r_4+r_1+r_2+r_3+r_4=0$

$\therefore \frac{-12}{2}+\left(-\frac{k-4}{2}\right)=0$ M1A2

$-12-k+4=0$

$k=-8$ A1

[5]

3. (a) $\frac{1}{r_1r_2}+\frac{1}{r_2r_3}+\frac{1}{r_1r_3}=\frac{r_3}{r_1r_2r_3}+\frac{r_1}{r_1r_2r_3}+\frac{r_2}{r_1r_2r_3}$ (M1) for valid approach

$\frac{1}{r_1r_2}+\frac{1}{r_2r_3}+\frac{1}{r_1r_3}=\frac{r_1+r_2+r_3}{r_1r_2r_3}$

$\frac{1}{r_1r_2}+\frac{1}{r_2r_3}+\frac{1}{r_1r_3}=\frac{-\frac{-18}{6}}{-\frac{72}{6}}$ A2

$\frac{1}{r_1r_2}+\frac{1}{r_2r_3}+\frac{1}{r_1r_3}=-\frac{1}{4}$ A1

[4]

(b) The sum of the roots

$=-\frac{-18}{6}+3(3)$ M1

$=12$ A1

[2]

4. (a) $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{r_2r_3}{r_1r_2r_3}+\frac{r_1r_3}{r_1r_2r_3}+\frac{r_1r_2}{r_1r_2r_3}$ (M1) for valid approach

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{r_2r_3+r_1r_3+r_1r_2}{r_1r_2r_3}$$

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{\frac{-10}{-1}}{-\frac{-15}{-1}}$$ A2

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=-\frac{2}{3}$$ A1

[4]

(b) The product of the roots

$$=-\frac{-15}{-1}$$ M1

$$=-15$$ A1

[2]

Exercise 9

1. (a) The required real root

$= 1.25 - 6$ (M1) for valid approach

$= -4.75$ A1

[2]

(b) The required real root

$= -1(1.25)$ (M1) for valid approach

$= -1.25$ A1

[2]

2. (a) The required real root

$= -1(-2.25)$ (M1) for valid approach

$= 2.25$ A1

[2]

(b) The required real root

$= \frac{-2.25}{3}$ (M1) for valid approach

$= -0.75$ A1

[2]

3. $g(x) = f(x-4)$

$g(x) = -(x-4)^4 + 4(x-4)^3 + 5(x-4) + 3$ (A1) for substitution

$g(x) = -(x^4 - 16x^3 + 96x^2 - 256x + 256)$
$+4(x^3 - 12x^2 + 48x - 64) + 5x - 20 + 3$ M1A1

$g(x) = -x^4 + 16x^3 - 96x^2 + 256x - 256$
$+4x^3 - 48x^2 + 192x - 256 + 5x - 20 + 3$ M1

$g(x) = -x^4 + 20x^3 - 144x^2 + 453x - 529$ A1

[5]

4. $g(x) = 2f(x+1)$

$g(x) = 2(2(x+1)^4 + 5(x+1)^2 - 2)$ (A1) for substitution

$g(x) = 2(2(x^4 + 4x^3 + 6x^2 + 4x + 1) + 5(x^2 + 2x + 1) - 2)$ M1A1

$g(x) = 2(2x^4 + 8x^3 + 12x^2 + 8x + 2 + 5x^2 + 10x + 5 - 2)$ M1

$g(x) = 2(2x^4 + 8x^3 + 17x^2 + 18x + 5)$

$g(x) = 4x^4 + 16x^3 + 34x^2 + 36x + 10$ A1

[5]

Exercise 10

1. $-2x^3-3x^2+12x+r-7=0$

$r=2x^3+3x^2-12x+7$ (M1) for valid approach

By considering the graph of $y=2x^3+3x^2-12x+7$, the local maximum is $(-2, 27)$ and the local minimum is $(1, 0)$. (M1) for valid approach

Thus, $0<r<27$. A2

[4]

2. $x^4-8x^3+16x^2=k-4x$

$x^4-8x^3+16x^2+4x=k$ (M1) for valid approach

By considering the graph of $y=x^4-8x^3+16x^2+4x$, the global minimum is $(-0.114908, -0.236058)$. (M1) for valid approach

Thus, $k<-0.236$. A2

[4]

3. $x^2+180=\dfrac{r+24x^2}{x}$

$x^3+180x=r+24x^2$

$x^3-24x^2+180x=r$ (M1) for valid approach

By considering the graph of $y=x^3-24x^2+180x$, the local maximum is $(6, 432)$ and the local minimum is $(10, 400)$. (M1) for valid approach

Thus, $r\le 400$ or $r\ge 432$. A2

[4]

4. $8x^2(2x+1)=k+192x+x^4$

$16x^3+8x^2=k+192x+x^4$

$-x^4+16x^3+8x^2-192x=k$ (M1) for valid approach

By considering the graph of $y=-x^4+16x^3+8x^2-192x$, the local maxima are $(-2, 272)$ and $(12, 5760)$, and the local minimum is $(2, -240)$. (M1) for valid approach

Thus, $-240<k<272$. A2

[4]

Exercise 11

1. (a) Let $\frac{x+1}{(x-4)(3x-1)} \equiv \frac{A}{x-4} + \frac{B}{3x-1}$, where A and B are constants.

$$\frac{x+1}{(x-4)(3x-1)} \equiv \frac{A(3x-1)}{(x-4)(3x-1)} + \frac{B(x-4)}{(x-4)(3x-1)}$$ M1

$$\frac{x+1}{(x-4)(3x-1)} \equiv \frac{3Ax-A+Bx-4B}{(x-4)(3x-1)}$$

$x+1 \equiv (3A+B)x+(-A-4B)$ A1

$1 = 3A+B$

$B = 1-3A$

$1 = -A-4B$

$\therefore 1 = -A-4(1-3A)$ A1

$1 = -A-4+12A$

$5 = 11A$

$A = \frac{5}{11}$

$\therefore B = 1-3\left(\frac{5}{11}\right)$

$B = -\frac{4}{11}$

$$\therefore \frac{x+1}{(x-4)(3x-1)} \equiv \frac{5}{11(x-4)} - \frac{4}{11(3x-1)}$$ A1

[4]

(b) -1 A1

[1]

(c) $x = 4$, $x = \frac{1}{3}$ A2

[2]

2. (a) $a = -2$, $b = -\frac{1}{2}$ A2

[2]

(b) Let $\frac{3-x}{2x^2+5x+2} \equiv \frac{A}{2x+1} + \frac{B}{x+2}$, where A and B are constants.

$\frac{3-x}{2x^2+5x+2} \equiv \frac{A(x+2)}{(2x+1)(x+2)} + \frac{B(2x+1)}{(2x+1)(x+2)}$ M1

$\frac{3-x}{2x^2+5x+2} \equiv \frac{Ax+2A+2Bx+B}{(2x+1)(x+2)}$

$-x+3 \equiv (A+2B)x+(2A+B)$ A1

$-1 = A+2B$

$A = -1-2B$

$3 = 2A+B$

$\therefore 3 = 2(-1-2B)+B$ A1

$3 = -2-4B+B$

$5 = -3B$

$B = -\frac{5}{3}$

$\therefore A = -1-2\left(-\frac{5}{3}\right)$

$A = \frac{7}{3}$

$\therefore \frac{3-x}{2x^2+5x+2} \equiv \frac{7}{3(2x+1)} - \frac{5}{3(x+2)}$ A1

[4]

3. (a) Let $\dfrac{x^2-x-1}{(x+3)(x+7)} \equiv A+\dfrac{B}{x+3}+\dfrac{C}{x+7}$, where A, B and C are constants.

$$\frac{x^2-x-1}{(x+3)(x+7)} \equiv \frac{A(x+3)(x+7)}{(x+3)(x+7)} + \frac{B(x+7)}{(x+3)(x+7)} + \frac{C(x+3)}{(x+3)(x+7)} \quad \text{M1}$$

$$\frac{x^2-x-1}{(x+3)(x+7)} \equiv \frac{Ax^2+10Ax+21A+Bx+7B+Cx+3C}{(x+3)(x+7)}$$

$x^2-x-1 \equiv Ax^2+(10A+B+C)x+(21A+7B+3C)$ A1

$A=1$ A1

$-1=10(1)+B+C$

$C=-11-B$

$-1=21A+7B+3C$

$\therefore -1=21(1)+7B+3(-11-B)$ A1

$-1=21+7B-33-3B$

$11=4B$

$B=\dfrac{11}{4}$ A1

$\therefore C=-11-\dfrac{11}{4}$

$C=-\dfrac{55}{4}$ A1

[6]

(b) $y=1$ A1

[1]

4. (a) Let $\frac{x^2+3}{(2-x)(5-3x)} \equiv A+\frac{B}{2-x}+\frac{C}{5-3x}$, where A, B and C are constants.

$\frac{x^2+3}{(2-x)(5-3x)} \equiv \frac{A(2-x)(5-3x)}{(2-x)(5-3x)}$ M1

$+\frac{B(5-3x)}{(2-x)(5-3x)}+\frac{C(2-x)}{(2-x)(5-3x)}$

$\frac{x^2+3}{(2-x)(5-3x)}$

$\equiv \frac{10A-11Ax+3Ax^2+5B-3Bx+2C-Cx}{(2-x)(5-3x)}$

$x^2+3 \equiv 3Ax^2+(-11A-3B-C)x+(10A+5B+2C)$ A1

$3A=1$

$A=\frac{1}{3}$ A1

$0=-11\left(\frac{1}{3}\right)-3B-C$

$C=-\frac{11}{3}-3B$

$3=10A+5B+2C$

$\therefore 3=10\left(\frac{1}{3}\right)+5B+2\left(-\frac{11}{3}-3B\right)$ A1

$3=\frac{10}{3}+5B-\frac{22}{3}-6B$

$3=-4-B$

$B=-7$ A1

$\therefore C=-\frac{11}{3}-3(-7)$

$C=\frac{52}{3}$ A1

[6]

(b) $g(x)=\frac{(2-x)(5-3x)}{x^2+3}$

The discriminant of x^2+3

$=0^2-4(1)(3)$ A1

$=-12$

<0

Therefore, the denominator is always nonzero.

Thus, $g(x)$ has no vertical asymptote. AG

[1]