Analysis and Approaches Higher Level for IBDP Mathematics Practice Paper Set 1 – Paper 3 (60 Minutes)

Question – Answer Book

Instructions

1. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

2. A graphic display calculator is needed.

3. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.

4. Supplementary answer sheets and graph papers will be supplied on request.

5. Unless otherwise specified, **ALL** working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

7. The diagrams in this paper are **NOT** necessarily drawn to scale.

8. Information to be read before you start the exam:

	Marker's Use Only	Examiner's Use Only	
Question Number	**Marks**	**Marks**	**Maximum Mark**
1			**27**
2			**28**
Overall			
Paper 3 Total			**55**

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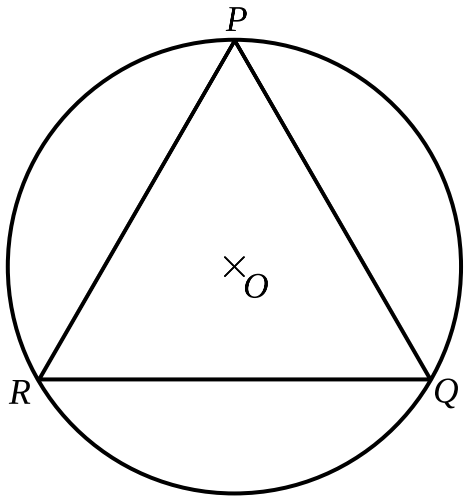
1. You are asked to investigate the difference between the area of a unit circle and the area of an inscribed polygon.

An inscribed equilateral triangle PQR is constructed in a unit circle, as shown in the following diagram:



P, Q and R are fixed point on the circumference such that QR is horizontal.

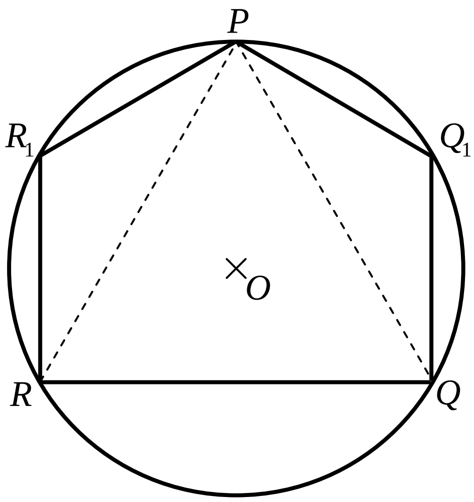
Let A_1 be the difference between the area of the unit circle and the area of the equilateral triangle PQR.

(a) (i) Write down $\mathrm{P\hat{O}Q}$.

(ii) Show that A_1 can be expressed as $\left(\frac{1}{2}+1\right)\left(\frac{2\pi}{3}-\sin\frac{2\pi}{3}\right)$.

[4]

The points Q_1 and R_1 are constructed on the arc PQ and PR respectively such that $\mathrm{PQ_1}=\mathrm{Q_1Q}=\mathrm{RR_1}=\mathrm{R_1P}$, as shown in the following diagram:



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Let A_2 be the difference between the area of the unit circle and the area of the inscribed pentagon $\mathrm{PQ_1QRR_1}$.

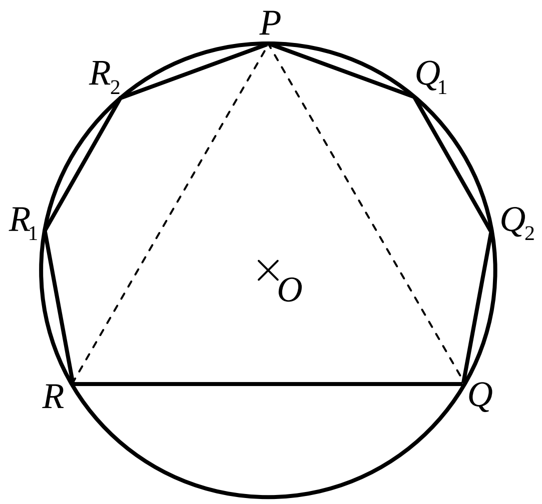
(b) (i) Write down $\mathrm{Q_1\hat{O}Q}$.

(ii) Write down the exact area of the triangle $\mathrm{Q_1OQ}$ in terms of π.

(iii) Hence, show that $A_2 = \frac{1}{2}\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right) + 2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$.

[5]

The points $\mathrm{Q_1}$, $\mathrm{Q_2}$ and R_1, R_2 are constructed on the arc PQ and PR respectively such that $\mathrm{PQ_1 = Q_1Q_2 = Q_2Q = RR_1 = R_1R_2 = R_2P}$, as shown in the following diagram:



Let A_3 be the difference between the area of the unit circle and the area of the inscribed heptagon $\mathrm{PQ_1Q_2QRR_1R_2}$.

(c) (i) Find $\mathrm{Q_2\hat{O}Q}$.

(ii) Express A_3 in the form similar to the expression of A_2 in (b)(iii).

[6]

The points $\mathrm{Q_1}$,..., Q_{n-1} and R_1,..., R_{n-1} are constructed on the arc PQ and PR respectively such that $\mathrm{PQ_1 = Q_1Q_2 = ... = Q_{n-1}Q = RR_1 = R_1R_2 = ... = R_{n-1}P}$.

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Let A_n be the difference between the area of the unit circle and the area of the inscribed polygon $PQ_1Q_2...Q_{n-1}QRR_1R_2...R_{n-1}$. It is given that $A_n = \frac{1}{2}\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right) + n \cdot f(n)$, where $f(n)$ is a function of n.

(d) (i) Find $f(n)$.

(ii) Interpret the meaning of $f(n)$ geometrically.

[6]

(e) Find $\lim_{n\to\infty} f(n)$.

[2]

It is given that $v < A_n \le u$.

(f) (i) Write down u.

(ii) Find the maximum possible value of v.

[4]

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2. You are asked to investigate the factorization of a complex polynomial equation of even degree.

(a) (i) Solve the quadratic equation $w^2 - w + 1 = 0$, giving the roots in the form $\cos\theta + \mathrm{i}\sin\theta$, $-\pi < \theta \le \pi$.

(ii) Hence, show that the roots of the equation $u^4 - u^2 + 1 = 0$ is $\cos\left(-\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(-\frac{5\pi}{6}\right)$, $\cos\left(-\frac{\pi}{6}\right) + \mathrm{i}\sin\left(-\frac{\pi}{6}\right)$, $\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$ and $\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}$.

(iii) Solve the quadratic equation $z^{2n} - z^n + 1 = 0$, giving the roots in the form $\cos\theta + \mathrm{i}\sin\theta$.

[12]

(b) (i) Express $(z - (\cos\theta + \mathrm{i}\sin\theta))(z - (\cos(-\theta) + \mathrm{i}\sin(-\theta)))$ as a quadratic expression of z, giving the answer in terms of z and θ.

(ii) Hence, show that

$$u^4 - u^2 + 1 = \left(u^2 - 2u\cos\frac{\pi}{6} + 1\right)\left(u^2 - 2u\cos\frac{5\pi}{6} + 1\right).$$

(iii) Express $z^6 - z^3 + 1$ as the product of three quadratic expressions of z.

(iv) Assume that n is even. Suggest an expression for $z^{2n} - z^n + 1 = 0$ as the product of n quadratic expressions of z.

[9]

(c) By using (b)(ii), verify that $\cos\frac{\pi}{6}\cos\frac{5\pi}{6} = -\frac{3}{4}$.

[3]

(d) Find $\cos\frac{\pi}{9}\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}$.

[4]

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