Chapter 16 Solution

Exercise 60

1. $f(x) = \int (6x^2 - 2x - 8)\mathrm{d}x$ (M1) for indefinite integral

$f(x) = 6\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) - 8x + C$ (A2) for correct integration

$f(x) = 2x^3 - x^2 - 8x + C$

$7 = 2(0)^3 - 0^2 - 8(0) + C$ (M1) for substitution

$C = 7$

$\therefore f(x) = 2x^3 - x^2 - 8x + 7$ A1 N5

[5]

2. (a) $f(x) = \int \left(x^2 - 36 - \frac{3}{x^2}\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x) = \frac{1}{3}x^3 - 36x - 3(-x^{-1}) + C$ (A2) for correct integration

$f(x) = \frac{1}{3}x^3 - 36x + \frac{3}{x} + C$

$0 = \frac{1}{3}(3)^3 - 36(3) + \frac{3}{3} + C$ (M1) for substitution

$C = 98$

$\therefore f(x) = \frac{1}{3}x^3 - 36x + \frac{3}{x} + 98$ A1 N5

[5]

(b) $\frac{377}{8} = \frac{1}{3}a^3 - 36a + \frac{3}{a} + 98$ (M1) for setting equation

$\frac{1}{3}a^3 - 36a + \frac{3}{a} + \frac{407}{8} = 0$

By considering the graph of

$y = \frac{1}{3}a^3 - 36a + \frac{3}{a} + \frac{407}{8}$,

$a = -11.03439$ (*Rejected*), $a = 1.5$ or

$a = 9.591087$ (*Rejected*). A1 N2

[2]

3. (a) $f(x)=\int(x^2-64x+12)\mathrm{d}x$ (M1) for indefinite integral

$f(x)=\frac{1}{3}x^3-64\left(\frac{x^2}{2}\right)+12x+C$ (A2) for correct integration

$f(x)=\frac{1}{3}x^3-32x^2+12x+C$

$0=\frac{1}{3}(6)^3-32(6)^2+12(6)+C$ (M1) for substitution

$C=1008$

$\therefore f(x)=\frac{1}{3}x^3-32x^2+12x+1008$ A1 N5

[5]

(b) 1008 A1 N1

[1]

(c) The area of the triangle OAB

$=\frac{(6)(1008)}{2}$ (M1) for valid approach

$=3024$ A1 N2

[2]

4. (a) $f(x)=\int\left(-\frac{200}{x^3}-\frac{20}{x^2}\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x)=-200\left(\frac{x^{-2}}{-2}\right)-20\left(\frac{x^{-1}}{-1}\right)+C$ (A2) for correct integration

$f(x)=\frac{100}{x^2}+\frac{20}{x}+C$

$0=\frac{100}{(-10)^2}+\frac{20}{-10}+C$ (M1) for substitution

$C=1$

$\therefore f(x)=\frac{100}{x^2}+\frac{20}{x}+1$ A1 N5

[5]

(b) $\frac{(0-(-10))(b-0)}{2}=100$ (M1)(A1) for correct formula

$5b=100$

$b=20$ A1 N3

[3]

Exercise 61

1. (a) (i) 10 A1 N1

(ii) (7.67, 50.8) A2 N2

[3]

(b) (i) $\int_3^{10}(-x^3+16x^2-69x+90)\mathrm{d}x$ A1 N1

(ii) $\frac{2401}{12}$ A1 N1

[2]

2. (a) (i) -5 A1 N1

(ii) (5, 0) A2 N2

[3]

(b) (i) $\int_{-5}^{0}(x^3-5x^2-25x+125)\mathrm{d}x$ A1 N1

(ii) $\frac{6875}{12}$ A1 N1

[2]

3. (a) (i) $a=-7,\ b=3$ A2 N2

(ii) $(-2, 50)$ A2 N2

[4]

(b) $\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x=\frac{28}{3}$ (M1) for setting equation

$\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x-\frac{28}{3}=0$

By considering the graph of

$y=\int_{-7}^{c}(-2x^2-8x+42)\mathrm{d}x-\frac{28}{3},\ c=-6.$ A1 N2

[2]

4. (a) (i) $a = 20,\ b = 40$ A2 N2

(ii) $\int_{20}^{40}(-2x^2 + 120x - 1600)\mathrm{d}x$ A1 N1

(iii) $\frac{8000}{3}$ A1 N1

[4]

(b) $(117 - 17)(c - 30) = \frac{8000}{3}$ (M1)(A1) for correct equation

$c - 30 = \frac{80}{3}$

$c - 30 = \frac{80}{3}$

$c = \frac{170}{3}$ A1 N3

[3]

Exercise 62

1. (a) (i) $\int_{-2}^{10} 1.5(x+2)(x-10)^2 \mathrm{d}x$ A1 N1

(ii) 2592 A1 N1

[2]

(b) The volume of the pyramid

$= \frac{1}{3}(2592)(15)$ (A1) for substitution

$= 12960$ A1 N2

[2]

2. (a) The area of R

$= \int_0^6 \pi(9-(x-3)^2)\mathrm{d}x$ (A1) for correct integral

$= 36\pi$ A1 N2

[2]

(b) $4\pi r^2 = 36\pi$ (M1) for setting equation

$r^2 = 9$

$r = -3$ (*Rejected*) or $r = 3$

Thus, the radius of the sphere is 3. A1 N2

[2]

3. (a) (i) $\int_4^8 (x-4)^2(8-x)\mathrm{d}x$ A1 N1

(ii) $\frac{64}{3}$ A1 N1

[2]

(b) $\frac{64}{3}h = 320$ (M1) for setting equation

$h = 15$ A1 N2

[2]

4. (a) The exact area of R

$= \int_{12}^{16} (x-8)^2 (x-16)^2 \, \mathrm{d}x$ (A1) for correct integral

$= \frac{8192}{15}$ A1 N2

[2]

(b) $\frac{1}{3}\left(\frac{8192}{15} \times 5\right) h = 1024$ (M1) for setting equation

$h = 1.125$ A1 N2

[2]

Exercise 63

1. (a) The width of each trapezium

$= \frac{7-1}{4}$ (A1) for correct substitution

$= 1.5$ A1 N2

[2]

(b) The estimate of $\int_1^7 x^{0.3}\mathrm{d}x$

$= \frac{1}{2}(1.5)\left[1^{0.3} + 7^{0.3} + 2(2.5^{0.3} + 4^{0.3} + 5.5^{0.3})\right]$ (A2) for substitution

$= 8.844238086$

$= 8.84$ A1 N3

[3]

2. (a) $\frac{0.8-0.5}{n} = 0.1$ (A1) for substitution

$\frac{0.3}{n} = 0.1$

$n = 3$ A1 N2

[2]

(b) The estimate of $\int_{0.5}^{0.8} 4^x\mathrm{d}x$

$= \frac{1}{2}(0.1)\left[4^{0.5} + 4^{0.8} + 2(4^{0.6} + 4^{0.7})\right]$ (A2) for substitution

$= 0.7452129098$

$= 0.745$ A1 N3

[3]

3. (a) $\frac{6-a}{6} = 0.4$ (A1) for correct substitution

$6 - a = 2.4$

$a = 3.6$ A1 N2

[2]

(b) The estimate of $\int_a^6 e^x\mathrm{d}x$

$= \frac{1}{2}(0.4)\left[e^{3.6} + e^6 + 2(e^4 + e^{4.4} + e^{4.8} + e^{5.2} + e^{5.6})\right]$ (A2) for substitution

$= 371.7086398$

$= 372$ A1 N3

[3]

4. (a) $\int_2^b \frac{1}{2x}\,dx$ A1 N1

[1]

(b) $\frac{b-2}{8} = 0.25$ (A1) for correct substitution

$b-2=2$

$b=4$ A1 N2

[2]

(c) The estimate of $\int_2^b \frac{1}{2x}\,dx$

$$= \frac{1}{2}(0.25)\left[\begin{array}{l}\frac{1}{2(2)}+\frac{1}{2(4)} \\ +2\left(\begin{array}{l}\frac{1}{2(2.25)}+\frac{1}{2(2.5)}+\frac{1}{2(2.75)} \\ +\frac{1}{2(3)}+\frac{1}{2(3.25)}+\frac{1}{2(3.5)} \\ +\frac{1}{2(3.75)}\end{array}\right)\end{array}\right]$$ (A2) for substitution

$=0.3470609252$

$=0.347$ A1 N3

[3]

Exercise 64

1. (a) The estimate of $\int_{-11}^{-8} \frac{3}{\sqrt{x+12}}\,dx$

$$= \frac{1}{2}(0.75)\left[\begin{array}{l}\frac{3}{\sqrt{-11+12}}+\frac{3}{\sqrt{-8+12}} \\ +2\left(\frac{3}{\sqrt{-10.25+12}}+\frac{3}{\sqrt{-9.5+12}}\right. \\ \left.+\frac{3}{\sqrt{-8.75+12}}\right)\end{array}\right]$$ (A2) for substitution

$= 6.059440517$

$= 6.06$ A1 N3

[3]

(b) The percentage error

$= \left|\frac{6.059440517-6}{6}\right| \times 100\%$ (A1) for correct substitution

$= 0.9906752854\%$

$= 0.991\%$ A1 N2

[2]

2. (a) 0.6 A1 N1

[1]

(b) The estimate of $\int_0^3 4e^x dx$

$= \frac{1}{2}(0.6)\left[4e^0+4e^3+2(4e^{0.6}+4e^{1.2}+4e^{1.8}+4e^{2.4})\right]$ (A2) for substitution

$= 78.61878727$

$= 78.6$ A1 N3

[3]

(c) The percentage error

$= \left|\frac{78.61878727-4(e^3-1)}{4(e^3-1)}\right| \times 100\%$ (A1) for correct substitution

$= 2.982152908\%$

$= 2.98\%$ A1 N2

[2]

3. (a) (i) $\frac{5}{3}$ A1 N1

(ii) $f(5.75)=\frac{14}{3},\ f(5.875)=\frac{19}{3}$ A2 N2

[3]

(b) The estimate of $\int_5^6 f(x)\mathrm{d}x$

$=\frac{1}{2}(0.125)\left[8+8+2\left(7+6+5+4+3+\frac{14}{3}+\frac{19}{3}\right)\right]$ (A2) for substitution

$=5.5$ A1 N3

[3]

(c) Overestimate A1 N1

[1]

4. (a) $a=0.35,\ b=0.5$ A2 N2

[2]

(b) The estimate of $\int_{0.3}^{0.6} f(x)\mathrm{d}x$

$=\frac{1}{2}(0.05)\begin{bmatrix}0.25^{0.3}+0.25^{0.6}\\+2(0.25^{0.35}+0.25^{0.4}\\+0.25^{0.45}+0.5+0.25^{0.55})\end{bmatrix}$ (A2) for substitution

$=0.1619919615$

$=0.162$ A1 N3

[3]

(c) Overestimate A1 N1

[1]