Chapter 8 Solution

Exercise 20

1. (a) $r = \frac{1}{4}$ A1 N1

[1]

(b) $u_8 = u_1 \times r^{8-1}$

$u_8 = 1024 \times \left(\frac{1}{4}\right)^{8-1}$ (A1) for substitution

$u_8 = \frac{1}{16}$ A1 N2

[2]

(c) $S_{12} = \frac{u_1(1-r^{12})}{1-r}$ (M1) for valid approach

$S_{12} = \frac{1024\left(1-\left(\frac{1}{4}\right)^{12}\right)}{1-\frac{1}{4}}$ (A1) for substitution

$S_{12} = 1365.333252$

$S_{12} = 1365$ A1 N3

[3]

2. (a) $r = \frac{4}{3}$ A1 N1

[1]

(b) $\sum_{n=1}^{7} u_n = \frac{u_1(1-r^7)}{1-r}$ (M1) for valid approach

$\sum_{n=1}^{7} u_n = \frac{576\left(1-\left(\frac{4}{3}\right)^7\right)}{1-\frac{4}{3}}$ (A1) for substitution

$\sum_{n=1}^{7} u_n = 11217.38272$

$\sum_{n=1}^{7} u_n = 11217$ A1 N3

[3]

(c) $243u_n = 1048576$

$243\left(576 \times \left(\frac{4}{3}\right)^{n-1}\right) = 1048576$ (M1) for substitution

$243\left(576 \times \left(\frac{4}{3}\right)^{n-1}\right) - 1048576 = 0$ (M1) for setting equation

By considering the graph of

$y = 243\left(576 \times \left(\frac{4}{3}\right)^{n-1}\right) - 1048576$,

$n = 8$. A1 N3

[3]

3. (a) $r = \dfrac{1.28}{1.024}$ (M1) for finding r

$r = 1.25$ A1 N2

[2]

(b) $u_n > 5$

$1.024 \times 1.25^{n-1} > 5$ (M1) for correct formula

$1.024 \times 1.25^{n-1} - 5 > 0$

By considering the graph of $y = 1.024 \times 1.25^{n-1} - 5$,

$n > 8.1062837$. (A1) for correct value

$\therefore$ The least value of n is 9. A1 N3

[3]

(c) $\sum_{n=1}^{10} u_n = S_{10}$

$\sum_{n=1}^{10} u_n = \dfrac{1.024(1-1.25^{10})}{1-1.25}$ (A1) for substitution

$\sum_{n=1}^{10} u_n = 34.05097266$

$\sum_{n=1}^{10} u_n = 34.1$ A1 N2

[2]

4. (a) $r = \dfrac{2.4}{1.5}$ (M1) for valid approach

$r = 1.6$ A1 N2

[2]

(b) $u_8 = u_1 \times r^{8-1}$

$u_8 = 1.5 \times 1.6^{8-1}$ (A1) for correct substitution

$u_8 = 40.2653184$

$u_8 = 40.27$ A1 N2

[2]

(c) $S_n < 100$

$\dfrac{1.5(1-1.6^n)}{1-1.6} < 100$ (M1) for correct formula

$-2.5(1-1.6^n) < 100$

$-2.5(1-1.6^n) - 100 < 0$

By considering the graph of

$y = -2.5(1-1.6^n) - 100$, $n < 7.9011562$. (A1) for correct value

Thus, the greatest value of n is 7. A1 N3

[3]

Exercise 21

1. (a) 1.331 m A1 N1

[1]

(b) The length of the longest side

$= u_{10}$ (M1) for valid approach

$= 1 \times 1.1^{10-1}$

$= 2.357947691$

$= 2.36$ m A1 N2

[2]

(c) The perimeter

$= S_{10}$ (M1) for valid approach

$= \frac{1(1-1.1^{10})}{1-1.1}$ (A1) for substitution

$= 15.9374246$

$= 15.9$ m A1 N3

[3]

2. (a) The required price

$= u_6$ (M1) for valid approach

$= 100 \times 0.9^{6-1}$

$= 59.049$

$= \$59$ A1 N2

[2]

(b) The required price

$= u_n$ (M1) for valid approach

$= 100 \times 0.9^{n-1}$ A1 N2

[2]

(c) The total income

$= 30S_{12}$ (M1) for valid approach

$= 30\left(\frac{100(1-0.9^{12})}{1-0.9}\right)$ (A1) for substitution

$= 21527.11391$

$= \$21527$ A1 N3

[3]

3. (a) The total volume

$= S_{20}$ (M1) for valid approach

$= \frac{24000(1-0.95^{20})}{1-0.95}$ (A1) for substitution

$= 307926.7572$

$= 308000 \text{ cm}^3$ A1 N3

[3]

(b) $u_n < 10000$

$24000 \times 0.95^{n-1} < 10000$ (M1) for correct formula

$24000 \times 0.95^{n-1} - 10000 < 0$

By considering the graph of

$y = 24000 \times 0.95^{n-1} - 10000$, $n > 18.067898$. (A1) for correct value

$\therefore$ The number of wooden dolls is 2. A1 N3

[3]

4. (a) The distance travelled

$= u_5$ (M1) for valid approach

$= 120 \times 0.9^{5-1}$

$= 78.732$

$= 78.7$ km A1 N2

[2]

(b) The difference

$= u_5 - u_6$

$= u_5 - 0.9u_5$ (M1) for valid approach

$= 78.732 - 0.9(78.732)$

$= 7.8732$

$= 7.87$ km A1 N2

[2]

(c) $S_n = 1000$

$\frac{120(1-0.9^n)}{1-0.9} = 1000$ (M1) for correct formula

$1200(1-0.9^n) = 1000$

$1200(1-0.9^n) - 1000 = 0$

By considering the graph of

$y = 1200(1-0.9^n) - 1000$, $n = 17.005986$. (A1) for correct value

$\therefore$ The date is 17th February. A1 N3

[3]

Exercise 22

1. (a) The amount of insurance premium

$=1200+(4-1)(-15)$ (A1) for substitution

$=1155$ EUR A1 N2

[2]

(b) The exact value of the car

$=24000\times(1-15\%)^{6-1}$ (A1) for substitution

$=10648.9275$ EUR A1 N2

[2]

(c) $24000\times(1-15\%)^{n-1}<8000$ (M1) for setting inequality

$24000\times0.85^{n-1}-8000<0$ (A1) for correct inequality

By considering the graph of

$y=24000\times0.85^{n-1}-8000$, $n>7.7599036$.

Thus, the year is 2018. A1 N3

[3]

(d) $1200+(n-1)(-15)>24000\times(1-15\%)^{n-1}$ (M1) for setting inequality

$1200-15n+15>24000\times0.85^{n-1}$

$1215-15n-24000\times0.85^{n-1}>0$ (A1) for correct inequality

By considering the graph of

$y=1215-15n-24000\times0.85^{n-1}$, $n>21.22625$.

Thus, the year is 2032. A1 N3

[3]

(e) The total amount of insurance premium

$=\frac{21}{2}[2(1200)+(21-1)(-15)]$ M1A1

$=22050$ EUR A1 N3

[3]

2. (a) (i) v_n A1 N1

(ii) t_n A1 N1

(iii) u_n A1 N1

(iv) w_n A1 N1

[4]

(b) (i) $v_{100} = v_1 + (100-1)d$ (M1) for valid approach

$v_{100} = 50 + (100-1)(1000)$

$v_{100} = 99050$ A1 N2

(ii) The sum of the first 25 terms

$= \frac{25}{2}\left[2v_1 + (25-1)d\right]$ (M1) for valid approach

$= \frac{25}{2}\left[2(50) + (25-1)(1000)\right]$ (A1) for substitution

$= 301250$ A1 N3

[5]

(c) (i) t_7

$= t_1 \times r^{7-1}$ (M1) for valid approach

$= 50 \times 2^{7-1}$

$= 3200$ A1 N2

(ii) The sum of the first 14 terms

$= \frac{t_1(1-r^{14})}{1-r}$ (M1) for valid approach

$= \frac{50(1-2^{14})}{1-2}$ (A1) for substitution

$= 819150$ A1 N3

[5]

(d) $v_m \geq t_m$

$50 + (m-1)(1000) \geq 50 \times 2^{m-1}$ (M1) for setting inequality

$1000m - 950 \geq 50 \times 2^{m-1}$

$1000m - 950 - 50 \times 2^{m-1} \geq 0$ (A1) for correct inequality

By considering the graph of

$y = 1000m - 950 - 50 \times 2^{m-1}$, $m \leq 8.1749071$.

Thus, $m = 8$. A1 N3

[3]

3. (a) Giselle's running distance

$= 2400 + (n-1)(200)$ (M1) for valid approach

$= (200n + 2200)$ m A1 N2

[2]

(b) $200x + 2200 > 10000$ (M1) for setting inequality

$200x > 7800$

$x > 39$ (A1) for correct value

Thus, $x = 40$. A1 N3

[3]

(c) The total running distance

$= \frac{16}{2}\left[2(2400) + (16-1)(200)\right]$ (M1) for valid approach

$= 62400$ m

$= 62.4$ km (A1) for correct unit

$= 6.24 \times 10^1$ km A1 N3

[3]

(d) Helena's running distance

$= 2000 \times (1 + 5\%)^{10-1}$ (M1) for valid approach

$= 3102.656432$

$= 3100$ m A1 N2

[2]

(e) $\frac{w}{2}\left[2(2400) + (w-1)(200)\right] < \frac{2000(1-1.05^w)}{1-1.05}$ M1A1

$\frac{w}{2}(200w + 4600) < -40000(1-1.05^w)$

$100w^2 + 2300w + 40000(1-1.05^w) < 0$

By considering the graph of

$y = 100w^2 + 2300w + 40000(1-1.05^w)$,

$w > 42.551642$. (A1) for correct value

Thus, $w = 43$. A1 N4

[4]

4. (a) The coffee shop's profit

$= 2000 \times (1 + 15\%)^{9-1}$ — (M1) for valid approach

$= 6118.045725$

$= 6120$ EUR — A1 N2

[2]

(b) The coffee shop's total profit

$= \frac{2000(1 - 1.15^{10})}{1 - 1.15}$ — (M1)(A1) for substitution

$= 40607.43648$

$= 40600$ EUR — A1 N3

[3]

(c) The fast food shop's profit

$= 4000 + (16 - 1)(1100)$ — (M1) for valid approach

$= 20500$ EUR — A1 N2

[2]

(d) $4000 + (m - 1)(1100) > 30000$ — (M1) for setting inequality

$1100m + 2900 > 30000$

$1100m > 27100$

$m > 24.63636364$ — (A1) for correct value

Thus, $m = 25$. — A1 N3

[3]

(e) $\frac{n}{2}\left[2(4000) + (n - 1)(1100)\right] < \frac{2000(1 - 1.15^n)}{1 - 1.15}$ — M1A1

$\frac{n}{2}(1100n + 6900) < -\frac{40000(1 - 1.15^n)}{3}$

$550n^2 + 3450n + \frac{40000(1 - 1.15^n)}{3} < 0$

By considering the graph of

$y = 550n^2 + 3450n + \frac{40000(1 - 1.15^n)}{3}$,

$n > 25.142701$. — (A1) for correct value

Thus, $n = 26$. — A1 N4

[4]