Chapter 5 Solution

Exercise 17

1. When $n=1$,

$$\text{L.H.S.}=\sum_{r=1}^{1}(2r+1)^2$$

$$\text{L.H.S.}=9$$

$$\text{R.H.S.}=\frac{(1)(4(1)^2+12(1)+11)}{3}$$

$$\text{R.H.S.}=9$$

Thus, the statement is true when $n=1$. R1

Assume that the statement is true when $n=k$. M1

$$\sum_{r=1}^{k}(2r+1)^2=\frac{k(4k^2+12k+11)}{3}$$

When $n=k+1$,

$$\sum_{r=1}^{k+1}(2r+1)^2=\sum_{r=1}^{k}(2r+1)^2+(2(k+1)+1)^2 \quad \text{M1}$$

$$\sum_{r=1}^{k+1}(2r+1)^2=\frac{k(4k^2+12k+11)}{3}+(4k^2+12k+9) \quad \text{A1}$$

$$\sum_{r=1}^{k+1}(2r+1)^2=\frac{4k^3+12k^2+11k+12k^2+36k+27}{3} \quad \text{A1}$$

$$\sum_{r=1}^{k+1}(2r+1)^2=\frac{4k^3+24k^2+47k+27}{3}$$

$$\sum_{r=1}^{k+1}(2r+1)^2=\frac{(k+1)(4k^2+20k+27)}{3} \quad \text{A1}$$

$$\sum_{r=1}^{k+1}(2r+1)^2=\frac{(k+1)(4(k+1)^2+12(k+1)+11)}{3} \quad \text{A1}$$

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[8]

2. When $n=1$,

$\text{L.H.S.}=1\times 1!$

$\text{L.H.S.}=1$

$\text{R.H.S.}=(1+1)!-1$

$\text{R.H.S.}=1$

Thus, the statement is true when $n=1$. R1

Assume that the statement is true when $n=k$. M1

$1\times 1!+2\times 2!+\ldots+k\times k!=(k+1)!-1$

When $n=k+1$,

$1\times 1!+2\times 2!+\ldots+k\times k!+(k+1)\times(k+1)!$

$=(k+1)!-1+(k+1)\times(k+1)!$ M1A1

$=(k+1)!(1+(k+1))-1$ A1

$=(k+1)!(k+2)-1$

$=(k+2)!-1$ A1

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[7]

3. When $n=1$,

L.H.S. $=\binom{1}{1}$

L.H.S. $=1$

R.H.S. $=\frac{1(1+1)}{2}$

R.H.S. $=1$

Thus, the statement is true when $n=1$. R1

Assume that the statement is true when $n=k$. M1

$$\binom{1}{1}+\binom{2}{1}+\ldots+\binom{k}{1}=\frac{k(k+1)}{2}$$

When $n=k+1$,

$$\binom{1}{1}+\binom{2}{1}+\ldots+\binom{k}{1}+\binom{k+1}{1}$$

$$=\frac{k(k+1)}{2}+\binom{k+1}{1}$$ M1A1

$$=\frac{k(k+1)}{2}+(k+1)$$ A1

$$=(k+1)\left[\frac{k}{2}+1\right]$$

$$=\frac{(k+1)(k+2)}{2}$$

$$=\frac{(k+1)((k+1)+1)}{2}$$ A1

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[7]

4. When $n=1$,

L.H.S. $=1^3$

L.H.S. $=1$

R.H.S. $=1^4$

R.H.S. $=1$

Thus, the statement is true when $n=1$. R1

Assume that the statement is true when $n=k$. M1

$1^3+2^3+\ldots+k^3 \le k^4$

When $n=k+1$,

$1^3+2^3+\ldots+k^3+(k+1)^3$

$\le k^4+(k+1)^3$ M1A1

$=k^4+k^3+3k^2+3k+1$ A1

$\le k^4+4k^3+6k^2+4k+1$ A1

$=(k+1)^4$

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n \in \mathbb{Z}^+$. R1

[7]

Exercise 18

1. When $n = 1$,

$9^{1+1} + 11 = 92$

$9^{1+1} + 11 = 4(23)$ A1

Thus, the statement is true when $n = 1$.

Assume that the statement is true when $n = k$. M1

$9^{k+1} + 11 = 4M$, where $M \in \mathbb{Z}$.

When $n = k + 1$,

$9^{(k+1)+1} + 11 = 9(9^{k+1}) + 11$ M1

$9^{(k+1)+1} + 11 = 9(4M - 11) + 11$ A1

$9^{(k+1)+1} + 11 = 36M - 88$ M1

$9^{(k+1)+1} + 11 = 4(9M - 22)$, where $9M - 22 \in \mathbb{Z}$. A1

Thus, the statement is true when $n = k + 1$.

Therefore, the statement is true for all $n \in \mathbb{Z}^+$. R1

[7]

2. When $n = 1$,

$9^1 - 4^1 = 5$

$9^1 - 4^1 = 5(1)$ A1

Thus, the statement is true when $n = 1$.

Assume that the statement is true when $n = k$. M1

$9^k - 4^k = 5M$, where $M \in \mathbb{Z}$.

When $n = k + 1$,

$9^{k+1} - 4^{k+1} = 9(9^k) - 4^{k+1}$ M1

$9^{k+1} - 4^{k+1} = 9(5M + 4^k) - 4^{k+1}$ A1

$9^{k+1} - 4^{k+1} = 45M + 9(4^k) - 4(4^k)$ M1

$9^{k+1} - 4^{k+1} = 5(9M + 4^k)$, where $9M + 4^k \in \mathbb{Z}$. A1

Thus, the statement is true when $n = k + 1$.

Therefore, the statement is true for all $n \in \mathbb{Z}^+$. R1

[7]

3. When $n=1$,

$1^3-4(1)=-3$

$1^3-4(1)=3(-1)$ A1

Thus, the statement is true when $n=1$.

Assume that the statement is true when $n=k$. M1

$k^3-4k=3M$, where $M\in\mathbb{Z}$.

When $n=k+1$,

$(k+1)^3-4(k+1)=k^3+3k^2+3k+1-4k-4$ M1

$(k+1)^3-4(k+1)=(3M+4k)+3k^2+3k+1-4k-4$ A1

$(k+1)^3-4(k+1)=3M+3k^2+3k-3$ M1

$(k+1)^3-4(k+1)=3(M+k^2+k-1)$, where

$M+k^2+k-1\in\mathbb{Z}$. A1

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[7]

4. When $n=1$,

$16^1+12(1)+8=36$

$16^1+12(1)+8=9(4)$ A1

Thus, the statement is true when $n=1$.

Assume that the statement is true when $n=k$. M1

$16^k+12k+8=9M$, where $M\in\mathbb{Z}$.

When $n=k+1$,

$16^{k+1}+12(k+1)+8=16(16^k)+12k+20$ M1

$16^{k+1}+12(k+1)+8=16(9M-12k-8)+12k+20$ A1

$16^{k+1}+12(k+1)+8=144M-180k-108$ M1

$16^{k+1}+12(k+1)+8=9(16M-20k-12)$, where

$16M-20k-12\in\mathbb{Z}$. A1

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[7]